An Efficient Active Set Algorithm for Covariance Based Joint Data and Activity Detection for Massive Random Access with Massive MIMO

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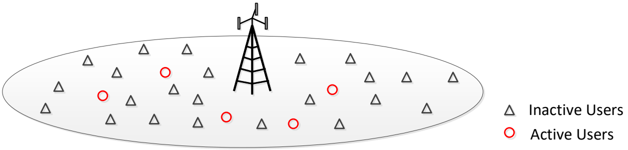
Massive Random Access

Let us focus on a fundamental problem in mMTC:



- A large number of devices with sporadic activity
- Low latency random access scheme for massive users is required
- Non-orthogonal signature sequences are preferred to orthogonal sequences
- User activity detection (user identification) performed at base station (BS)

System Model

- Single cell with one BS equipped with M antennas
- N single-antenna devices, K of which are active at a time
- Each active device wishes to transmit J bits of data to the BS
- Each device n has a unique signature sequence set $\mathcal{S}_n = \{\mathrm{s}_{n,1}, \mathrm{s}_{n,2}, \ldots, \mathrm{s}_{n,Q}\}$, where $\mathrm{s}_{n,q} \in \mathbb{C}^{L\times 1}, 1 \leq q \leq Q \triangleq 2^J$, and L is the signature sequence length
- Channel $\sqrt{g_n}\mathrm{h}_n \in \mathbb{C}^{M\times 1}$ of user n includes both
  - ◇ Large-scale fading component $g_n \geq 0$
  - ◇ Rayleigh fading component $\mathrm{h}_n \in \mathbb{C}^{M\times 1}$ following the i.i.d. complex Gaussian distribution

System Model

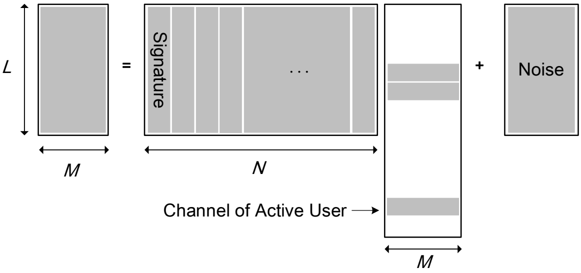
- Whether or not $s_{n,q}$ is transmitted is indicated as $\chi_{n,q} \in \{0, 1\}$, which satisfies $\sum_{q=1}^{Q} \chi_{n,q} \in \{0, 1\}$
  - ◇ $\sum_{q=1}^{Q} \chi_{n,q} = 1$ indicates that device n is active
  - ◇ $\sum_{q=1}^{Q} \chi_{n,q} = 0$ indicates that device n is inactive
- Define
  - $S_n = [s_{n,1}, \ldots, s_{n,Q}] \in \mathbb{C}^{L \times Q}$, and $S = [S_1, \ldots, S_N] \in \mathbb{C}^{L \times NQ}$;
  - $D_n = \sqrt{g_n}\,\mathrm{diag}\{\chi_{n,1}, \ldots, \chi_{n,Q}\} \in \mathbb{C}^{Q \times Q}$, and $\mathbf{\Gamma}^{1/2} = \mathrm{diag}\{D_1, \ldots, D_N\} \in \mathbb{C}^{NQ \times NQ}$;
  - $H_n = [h_n, \ldots, h_n]^T \in \mathbb{C}^{Q \times M}$ for all n, and $H = [H_1^T, \ldots, H_N^T]^T \in \mathbb{C}^{NQ \times M}$.

System Model

- The received signal $\mathsf{Y} \in \mathbb{C}^{L\times M}$ at the BS can be expressed as

$$\mathsf{Y} = \sum_{n=1}^{N}\sum_{q=1}^{Q}\chi_{n,q}\mathsf{s}_{n,q}\sqrt{g_n}\mathsf{h}_n^T + \mathsf{W}$$
$$= \mathsf{S}\boldsymbol{\Gamma}^{1/2}\mathsf{H} + \mathsf{W}. \quad (1)$$

where $\mathsf{W} \in \mathbb{C}^{L\times M}$ is the effective i.i.d. Gaussian noise whose variance σ_w^2 is the background noise power normalized by the device transmit power.



System Model

- The joint activity and data detection problem can be formulated as the maximum likelihood estimation (MLE) problem [Haghighatshoar-Jung-Caire '18]

$$\begin{aligned} \min_{\boldsymbol{\gamma}} \quad & \log\left|\mathsf{S}\boldsymbol{\Gamma}\mathsf{S}^H + \sigma_w^2\mathsf{I}\right| + \operatorname{Tr}\left(\left(\mathsf{S}\boldsymbol{\Gamma}\mathsf{S}^H + \sigma_w^2\mathsf{I}\right)^{-1}\hat{\boldsymbol{\Sigma}}\right) && \text{(2a)} \\ \text{s.t.} \quad & \boldsymbol{\gamma} \geq 0. && \text{(2b)} \end{aligned}$$

- The sample covariance matrix $\hat{\boldsymbol{\Sigma}} = \mathsf{Y}\mathsf{Y}^H/M$ is computed by averaging over different antennas.

- Let $\boldsymbol{\gamma} \in \mathbb{C}^{NQ\times 1}$ denote the diagonal entries of $\boldsymbol{\Gamma}$, i.e., $\boldsymbol{\gamma} = [\boldsymbol{\gamma}_1^T, \ldots, \boldsymbol{\gamma}_N^T]^T$, where $\boldsymbol{\gamma}_n = [\gamma_{n,1}, \ldots, \gamma_{n,Q}]^T \in \mathbb{C}^{Q\times 1}$ with $\gamma_{n,q} = g_n\chi_{n,q} \geq 0$

Problem Formulation and Analysis

- Let $f(\gamma)$ denote the objective function of problem (2). The gradient of $f(\gamma)$ with respect to $\gamma_{n,q}$ is

$$[\nabla f(\gamma)]_{n,q} = \mathbf{s}_{n,q}^H \boldsymbol{\Sigma}^{-1} \mathbf{s}_{n,q} - \mathbf{s}_{n,q}^H \boldsymbol{\Sigma}^{-1} \hat{\boldsymbol{\Sigma}} \boldsymbol{\Sigma}^{-1} \mathbf{s}_{n,q}.$$

- The first-order (necessary) optimality condition of problem (2) is

$$[\nabla f(\gamma)]_{n,q} \begin{cases} = 0, & \text{if } \gamma_{n,q} > 0; \\ \geq 0, & \text{if } \gamma_{n,q} = 0, \end{cases} \forall\, q, n, \tag{3}$$

- Let $[\cdot]_+$ denote the projection operator onto the nonnegative orthant. Then (3) is equivalent to

$$[\gamma - \nabla f(\gamma)]_+ - \gamma = \mathbf{0}.$$

Motivation

- Existing algorithms for solving problem (2):
  - Coordinate descent (CD) [Haghighatshoar-Jung-Caire '18]: Iteratively update every single coordinate (it admits a closed-form solution for each update)
  - Expectation-maximization (EM) [Wipf-Rao '07]
  - Sparse iterative covariance-based estimation (SPICE) [Yang-Li-Stoica-Xie '18]
- None of the above algorithms take advantage of the sparsity of true solution
- This paper proposes the active set algorithm
  - It fully exploits the sparsity of its true solution
  - At each iteration, it first judiciously selects an active set
  - It solves the small dimensional subproblem defined over the active set
  - It is more effective than the CD algorithm (the state-of-the-art algorithm)

Active Set Algorithm

- To fully exploit the sparsity of the true solution of (2), the active set should
  - contain the indices of active sequences
  - have the smallest possible cardinality
- At the k-th iteration, the proposed selection strategy of the active set $\mathcal{A}^k$ is

$$\mathcal{A}^k = \Big\{(i,q) \mid \gamma_{i,q}^k > \omega_k \text{ or } [\nabla f(\gamma^k)]_{i,q} < -\nu_k\Big\}, \tag{4}$$

where $\omega_k, \nu_k > 0$ and $\omega_k \downarrow 0$ and $\nu_k \downarrow 0$ (monotonically decrease and converge to zero).

Active Set Algorithm

- Once the active set $\mathcal{A}^k$ is selected, solve the following subproblem

$$\begin{aligned} \min \quad & \hat{f}(\gamma_{\mathcal{A}^k}) && (5a) \\ \text{s. t.} \quad & \gamma_{\mathcal{A}^k} \geq 0, && (5b) \end{aligned}$$

  where $\gamma_{\mathcal{A}^k}$ is the subvector of γ indexed by $\mathcal{A}^k$ and $\hat{f}(\gamma_{\mathcal{A}^k})$ is $f(\gamma)$ defined over $\gamma_{\mathcal{A}^k}$ with all the other variables fixed being zero.

- If the set $\mathcal{A}^k$ in (5) is properly chosen, the dimension of problem (5) is potentially much smaller than that of problem (2).

- Apply the spectral PG algorithm [Birgin-Martínez-Raydan '00] to solve the subproblem in (5) until $\gamma_{\mathcal{A}^k}^{k+1}$ satisfying

$$\left\| \left[[\gamma_{\mathcal{A}^k}^{k+1} - \nabla \hat{f}(\gamma_{\mathcal{A}^k}^{k+1})]_+ - \gamma_{\mathcal{A}^k}^{k+1} \right] \right\| < \varepsilon_k, \qquad (6)$$

  where $\varepsilon_k > 0$ is the solution tolerance at the k-th iteration.

Active Set Algorithm

- The pseudocodes of the proposed algorithm are given in Algorithm 1.

Algorithm 1 Proposed active set PG algorithm for solving problem (2)

1: **Initialize:** $\gamma^0 = 0$, $k = 0$, $\{\omega_k, \nu_k, \varepsilon_k\}_{k \geq 0}$, and $\varepsilon > 0$;
2: **repeat**
3: Select the active set $\mathcal{A}^k$ according to (4);
4: Apply the spectral PG algorithm to solve the subproblem (5) until (6) is satisfied;
5: Set $k \leftarrow k + 1$;
6: **until** $\|[\gamma^k - \nabla f(\gamma^k)]_+ - \gamma^k\| < \varepsilon$
7: **Output:** γ^k

Active Set Algorithm

Theorem

For any given tolerance $\varepsilon > 0$, suppose that the parameters ω_k and ν_k in (4) satisfy $\omega_k \downarrow 0$ and $\nu_k \downarrow 0$ and the parameter ε_k in (6) satisfy $\lim_{k \to \infty} \varepsilon_k < \varepsilon$, then the active set PG Algorithm 1 will terminate within a finite number of iterations.

- The convergence property is mainly because of
  - the activity set selection strategy in (4) (choices of parameters ω_k and ν_k)
  - the convergence property of the spectral PG algorithm
- A not careful selection of the active set might lead to oscillation or divergence

Simulation Results

- The power spectrum density of the background noise is -169dBm/Hz over 10 MHz and the transmit power of each device is 25dBm
- A single cell of radius 1000m, all devices are located in the cell edge, g_n's are the same for all devices
- All signature sequences from i.i.d. complex Gaussian distribution with zero mean and unit variance
- Parameters setting: $M = 256$, $L = 150$, and $J = 1$ (and thus $Q = 2$), $K/N = 0.1$ (10% of the total devices are active)

Simulation Results

- Compare the proposed Algorithm 1 with
  - random CD algorithm: apply the random CD algorithm to solve problem (2)
  - Ideal CD algorithm: apply the CD algorithm to solve problem (2) defined over the indices of active sequences
  - Ideal PG algorithm: apply the PG algorithm to solve problem (2) defined over the indices of active sequences
- Parameters setting:

$$\omega_k = 10^{-6-k}, \varepsilon_k = \max\left\{10^{-k}, 0.8 * 10^{-3}\right\},$$
$$\nu_k = \min\left\{10^{4-k}, 0.5\left|\min_{n,q}\left\{\left[\nabla f(\gamma^k)\right]_{n,q}\right\}\right|\right\}.$$

- Average over 500 Monte-Carlo runs.

Simulation Results

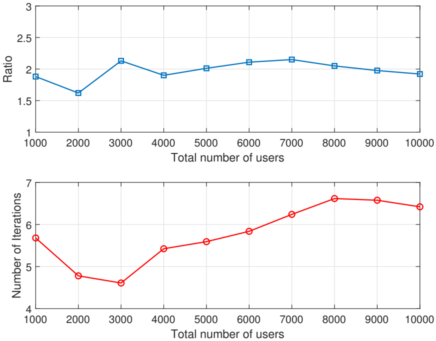


Figure: Performance of the proposed active set PG algorithm.

- The average ratio of $|\mathcal{A}^k|/K$ is in the interval $[1.5, 2.5]$.
- Algorithm 1 will generally terminate within 4–7 iterations.
- The proposed active set selection strategy (4) is very efficient.

Simulation Results

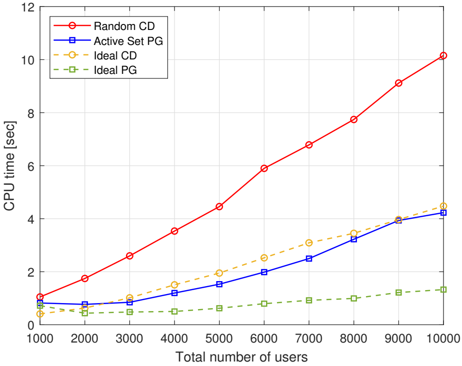


Figure: Average CPU time comparison.

- Algorithm 1 significantly outperforms the random CD algorithm.
- Algorithm 1 even achieves slightly better efficiency than the ideal CD algorithm.

Summary

The main contribution of this paper is

- Propose a computationally efficient active set algorithm for activity detection
- Provide the convergence guarantee for the proposed algorithm
- Show the efficiency of the proposed algorithm by simulations

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