Deep Weighted MMSE Downlink Beamforming

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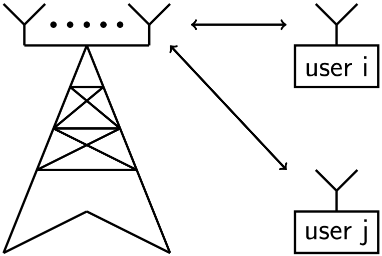

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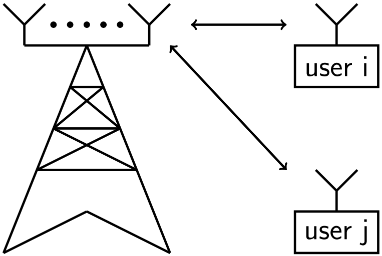

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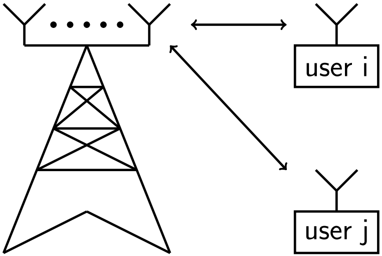

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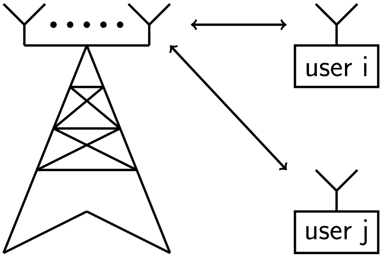

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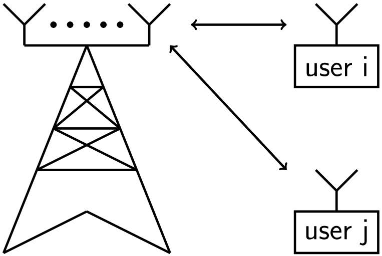

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1. Maximize **weighted sum rate** subject to **transmit power constraint**
2. Satisfy the **power consumption** and **latency** requirements at the base station

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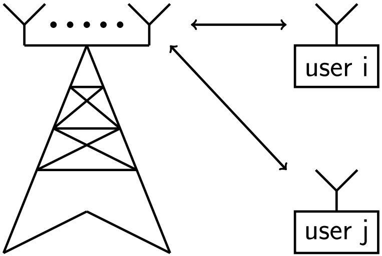

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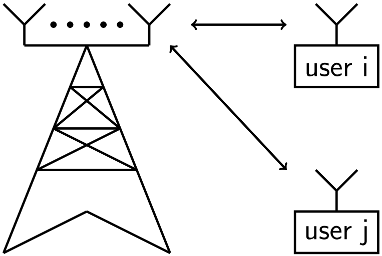

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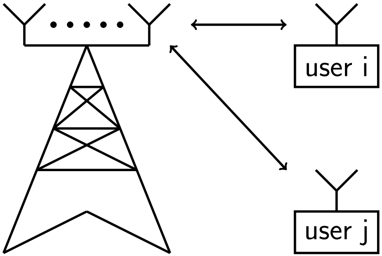

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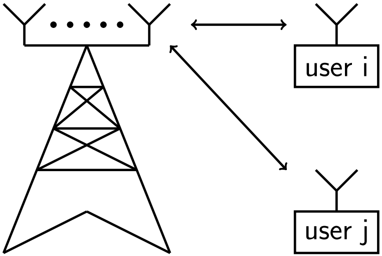

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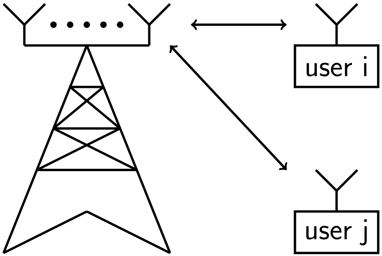

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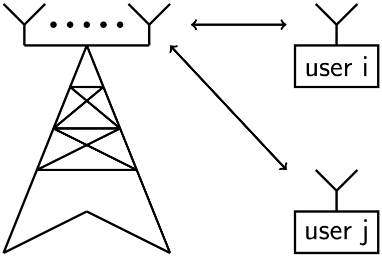

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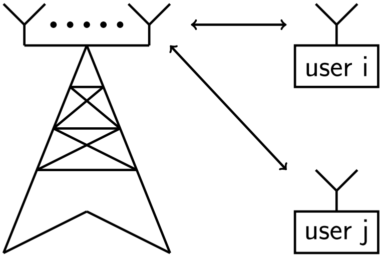

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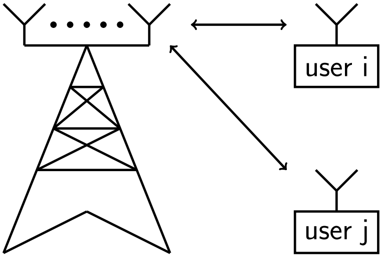

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Beamformer matrix

Beamforming problem

- We address the weighted sum rate (WSR) maximization problem

$$\max_{\boldsymbol{V}} \quad \sum_{i=1}^{N} \alpha_i \log_2 (1 + \mathrm{SINR}_i) \tag{1a}$$

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- Problem (1) is known to be **NP-hard**[1]

[1] Luo et al., "Dynamic spectrum management: Complexity and duality," *IEEE Journal of Selected Topics in Signal Processing*, 2008.

WMSE algorithm

- The weighted minimum mean square error (WMMSE) algorithm[2] finds a **local optimum** by working on an **equivalent reformulation** of the WSR problem, i.e.,

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- It exhibits a **relatively high computational complexity**

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  - Train the network with gradient-based methods and back-propagation
- It incorporates **domain knowledge** in the structure of the network

Deep unfolding

Advantages with respect to standard neural network solutions

- No architecture selection
- Explainability
- Fewer parameters to train

WMMSE algorithm - Deep unfolding

- The WMMSE algorithm involves operations that are **hard to map to neural network layers as acknowledged by Sun et al.**[3]

[3] Sun et al., "Learning to Optimize: Training Deep Neural Networks for Interference Management," *IEEE Transactions on Signal Processing*, 2018

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WMMSE steps	Unfolded steps
$\boldsymbol{u}_j = \text{argmin}_{\xi} f(\xi, \boldsymbol{w}_{j-1}, \boldsymbol{V}_{j-1})$	$\boldsymbol{u}_j = \Omega(\boldsymbol{w}_{j-1}, \boldsymbol{V}_{j-1})$
$\boldsymbol{w}_j = \text{argmin}_{\xi} f(\boldsymbol{u}_j, \xi, \boldsymbol{V}_{j-1})$	$\boldsymbol{w}_j = \Psi(\boldsymbol{u}_j, \boldsymbol{V}_{j-1})$
$\boldsymbol{V}_j = \text{argmin}_{\xi} f(\boldsymbol{u}_j, \boldsymbol{w}_j, \xi)$ s.t. $\text{Tr}(\xi\xi^H) \leq P$	**?**

j is the iteration index

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- The update equation of $\boldsymbol{V}$ is obtained by solving

$$\min_{\boldsymbol{\xi}} \quad f(\boldsymbol{u}, \boldsymbol{w}, \boldsymbol{\xi}) \tag{3a}$$

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- We truncate the sequence of PGD steps to K

Unfoldable WMMSE algorithm

- At each iteration:
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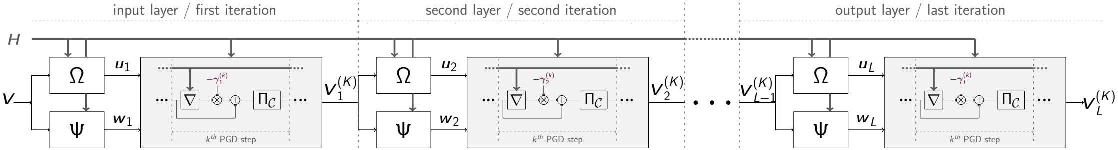
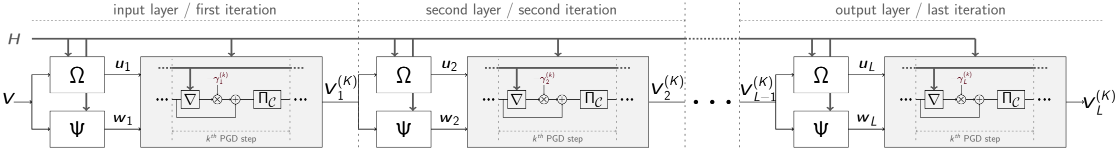
Convergence

We can prove that the unfoldable WMMSE algorithm retains the **same convergence guarantees** of the original WMMSE

Deep unfolded WMMSE

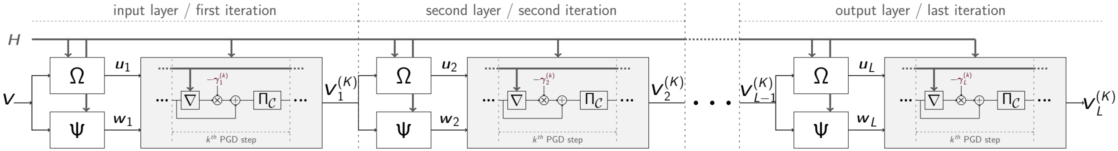


Deep unfolded WMMSE



- We select the step sizes of the PGD (Γ) to be the trainable parameters

Deep unfolded WMMSE

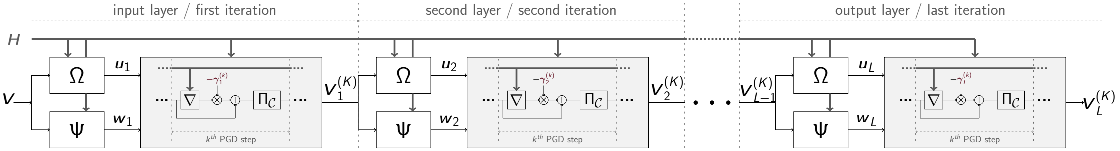


- We select the step sizes of the PGD (Γ) to be the trainable parameters
- We minimize the following loss function

$$\mathcal{L}(\Gamma) = -\frac{1}{N_\mathrm{s}} \sum_{n=1}^{N_\mathrm{s}} \sum_{l=1}^{L} f_\mathrm{WSR}(\boldsymbol{H}_n, \boldsymbol{V}_l; \Gamma) \tag{4}$$

where N_s is the size of the training set

Deep unfolded WMMSE



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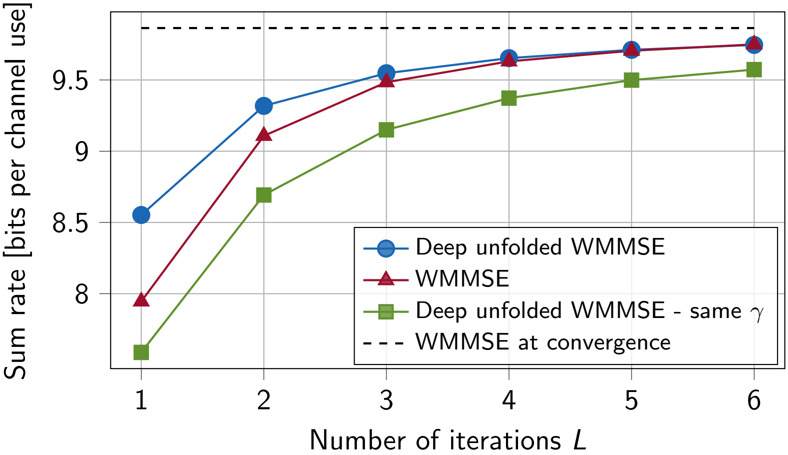
Weighted Sum Rate

where N_s is the size of the training set

Numerical results

- $M = 4$
- $N = 4$
- $\frac{P}{\sigma^2} = 10$ dB
- 4 PGD steps



Conclusion

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Conclusion

- We addressed the **trade-off between complexity and performance** for the WSR maximization beamforming problem
- To this end, we provided a **variant of the WMMSE** algorithm that
  - allows for the novel application of deep unfolding
  - retains the same convergence guarantees of the original WMMSE algorithm
- **Numerical results** confirmed that the deep unfolded WMMSE successfully addresses the trade-off

Thank you for your attention!

https://github.com/lpkg/ WMMSE-deep-unfolding/tree/ICASSP2021

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