Efficient Real-time Video Stabilization with A Novel Least Squares Formulation

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Content

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- Other real-time methods rely heavily on Kalman filter where the covariance matrix and the state transfer matrix must be assumed.
- We present a simple and efficient real-time online video stabilization algorithm(LSstab) with minimum latency.
- LSstab is based on a novel Least squared formulation of the smoothing cost function. A recursive solver is derived for optimizing the cost function.

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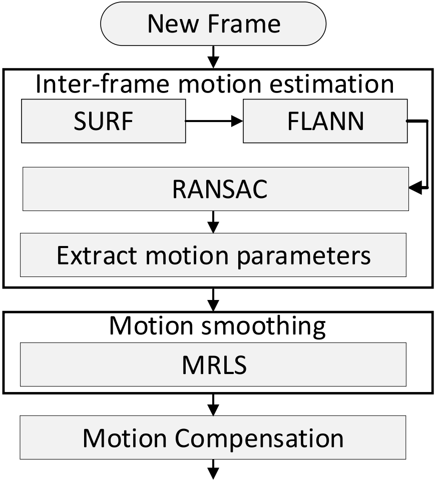
Algorithm Flowchart



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Least Squared based Motion Smoothing

- Assume the inter-frame motion model is the simplified affine transformation $A = \begin{bmatrix} s \cdot \cos(\theta) & -s \cdot \sin(\theta) & t_x \\ s \cdot \sin(\theta) & s \cdot \cos(\theta) & t_y \\ 0 & 0 & 1 \end{bmatrix}$

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- s, θ, t_x, t_y are the scale, rotation, and translations in x, y axis.
- Once A is estimated by RANSAC, the four parameters are extracted and will be smoothed.

Least Squared based Motion Smoothing

- Start by considering motion smoothing as

$$\underset{\overline{m}_1,\ldots,\overline{m}_n}{\arg\min} \sum_{i=1}^{n}(m_i - \overline{m}_i)^2 + \lambda \sum_{i=2}^{n}(\overline{m}_i - \overline{m}_{i-1})^2 \tag{1}$$

where m_i and $\overline{m}_i$ are the original and smoothed motion for the i^{th} frame.

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- Find all stable motions $\overline{m}_1, \ldots, \overline{m}_n$ at once. Real-time solver needed.

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- Find all stable motions $\overline{m}_1, \ldots, \overline{m}_n$ at once. Real-time solver needed.
- Even it can be solved in real-time, fail to capture the relationship between successive solutions $(\overline{m}_1^n, \ldots, \overline{m}_n^n)$ and $(\overline{m}_1^{n+1}, \ldots, \overline{m}_{n+1}^{n+1})$

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A direct approach

- A direct approach

$$\overline{m}_n = \arg\min_{\overline{m}_n} \sum_{i=1}^{n-1} (m_i - \widehat{m}_i)^2 + (m_n - \overline{m}_n)^2 + \\ \lambda \sum_{i=2}^{n} (\widehat{m}_i - \widehat{m}_{i-1})^2 + \lambda(\overline{m}_n - \widehat{m}_{n-1})^2 \tag{2}$$

where $\widehat{m}_1, \ldots, \widehat{m}_{n-1}$ are the previous smoothed motions. $\overline{m}_n$ is the current smoothed estimate.

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where $\widehat{m}_1, \ldots, \widehat{m}_{n-1}$ are the previous smoothed motions. $\overline{m}_n$ is the current smoothed estimate.

- We can further simplify (2) to:

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- Since (3) is quadratic, the minimum occurs when the derivative is zero. The solution is

$$\overline{m}_n = \frac{m_n + \lambda\widehat{m}_{n-1}}{1+\lambda} \tag{4}$$

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- However, (4) only involves the last stabilized motion $\hat{m}_{n-1}$ and the current motion m_n.
- Thus has a limited ability to stabilizing the current frame.

Modified Recursive Least Squares Stabilization

- Modified Recursive Least Squares formulation (MRLS)

$$\underset{\overline{m}_1,\ldots,\overline{m}_n}{\arg\min} \sum_{i=1}^{n} (m_i - \overline{m}_i)^2 + \lambda_1 \sum_{i=2}^{n} (\overline{m}_i - \overline{m}_{i-1})^2 + \lambda_2 \sum_{i=1}^{n-1} (\overline{m}_i - \widehat{m}_i)^2 \quad (5)$$

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- Similar to (1), (5) also tries to estimate an augmented path $(\overline{m}_1, \ldots, \overline{m}_n)$.

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- Similar to (1), (5) also tries to estimate an augmented path $(\overline{m}_1,\ldots,\overline{m}_n)$.
- $(\overline{m}_1,\ldots,\overline{m}_n)$ is guaranteed to be smooth by the minimization of $\lambda_1\sum_{i=2}^{n}(\overline{m}_i-\overline{m}_{i-1})^2$.

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$$\arg\min_{\overline{m}_1,\dots,\overline{m}_n} \sum_{i=1}^{n}(m_i - \overline{m}_i)^2 + \lambda_1 \sum_{i=2}^{n}(\overline{m}_i - \overline{m}_{i-1})^2 + \lambda_2 \sum_{i=1}^{n-1}(\overline{m}_i - \widehat{m}_i)^2 \quad (5)$$

- Similar to (1), (5) also tries to estimate an augmented path $(\overline{m}_1, \dots, \overline{m}_n)$.
- $(\overline{m}_1, \dots, \overline{m}_n)$ is guaranteed to be smooth by the minimization of $\lambda_1 \sum_{i=2}^{n}(\overline{m}_i - \overline{m}_{i-1})^2$.
- In (5), $\lambda_2 \sum_{i=1}^{n-1}(\overline{m}_i - \widehat{m}_i)^2$ secures that $(\overline{m}_1, \dots, \overline{m}_n)$ is similar to $(\widehat{m}_1, \dots, \widehat{m}_{n-1})$, making sure that $(\widehat{m}_1, \dots, \widehat{m}_{n-1}, \widehat{m}_n = \overline{m}_n)$ will also form a smooth curve.

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- First attempt

$$\underset{\overline{m}_1,\ldots,\overline{m}_n}{\arg\min} \sum_{i=1}^{n} (m_i - \overline{m}_i)^2 + \lambda \sum_{i=2}^{n} (\overline{m}_i - \overline{m}_{i-1})^2 \tag{1}$$

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$$\underset{\overline{m}_1,\ldots,\overline{m}_n}{\arg\min} \sum_{i=1}^{n} (m_i - \overline{m}_i)^2 + \lambda \sum_{i=2}^{n} (\overline{m}_i - \overline{m}_{i-1})^2 \quad (1)$$

- Second attempt

$$\overline{m}_n = \underset{\overline{m}_n}{\arg\min} (m_n - \overline{m}_n)^2 + \lambda (\overline{m}_n - \widehat{m}_{n-1})^2 \quad (3)$$

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- MRLS

$$\underset{\overline{m}_1,\ldots\overline{m}_n}{\arg\min} \sum_{i=1}^{n} (m_i - \overline{m}_i)^2 + \lambda_1 \sum_{i=2}^{n} (\overline{m}_i - \overline{m}_{i-1})^2 + \lambda_2 \sum_{i=1}^{n-1} (\overline{m}_i - \widehat{m}_i)^2 \tag{5}$$

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An Recursive Solver

- (5) is convex. Setting all partial derivatives to zero results in

$$\underbrace{\begin{bmatrix} (1+\lambda_1+\lambda_2) & -\lambda_1 & 0 & \cdots & \cdots & 0 \\ -\lambda_1 & (1+2\lambda_1+\lambda_2) & -\lambda_1 & \ddots & \ddots & \vdots \\ 0 & -\lambda_1 & (1+2\lambda_1+\lambda_2) & -\lambda_1 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & -\lambda_1 & (1+2\lambda_1+\lambda_2) & -\lambda_1 \\ 0 & \cdots & \cdots & 0 & -\lambda_1 & (1+\lambda_1) \end{bmatrix}}_{\mathbf{S}_n} \underbrace{\begin{bmatrix} \overline{m}_1 \\ \overline{m}_2 \\ \overline{m}_3 \\ \vdots \\ \overline{m}_{n-1} \\ \overline{m}_n \end{bmatrix}}_{\overline{\mathbf{m}_n}} = \underbrace{\begin{bmatrix} m_1+\lambda_2\widehat{m}_1 \\ m_2+\lambda_2\widehat{m}_2 \\ m_3+\lambda_2\widehat{m}_3 \\ \vdots \\ m_{n-1}+\lambda_2\widehat{m}_{n-1} \\ m_n \end{bmatrix}}_{\mathbf{Y}_n}$$

- The key idea is the last row of S_n^{-1} can be derived from S_{n-1}^{-1} in O(n).

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Theorem 1

Let $\mathcal{S}_{n-1} = [\boldsymbol{S}_{n-1}|I]$ be an augmented matrix, where I is the identity matrix. $\mathcal{S}^e_{n-1} = [S^e_{n-1}|A_{n-1}]$ is the echelon form of $\mathcal{S}_{n-1}$ with $b = -\lambda_1$ as leading term for each row. Then $\{S^e_{n-1}\}_{n-2} \in \mathbb{R}^{2(n-1)}$ and $\{S^e_n\}_n \in \mathbb{R}^{2n}$ have the form:

$$\{\mathcal{S}^e_{n-1}\}_{n-2} = \begin{bmatrix} 0 & \cdots & b & x_1^{(n-1)} & y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_{n-3}^{(n-1)} & 0 \end{bmatrix} \tag{6}$$

$$\{\boldsymbol{S}_n^{-1}\}_n = \begin{bmatrix} \frac{y_1^{(n-1)}b}{q_n} & \cdots & \frac{y_{n-3}^{(n-1)}b}{q_n} & \frac{-b}{q_n} & \frac{c-x_1^{(n-1)}}{q_n} \end{bmatrix} \tag{7}$$

where $\{\cdot\}_i$ denotes the i^{th} row of a matrix and $a = (1 + \lambda_1 + \lambda_2), b = -\lambda_1, c = (1 + 2\lambda_1 + \lambda_2), d = (\lambda_1 + 1)$.

Timing performance

Table: Overall timing performance of the proposed algorithm on public data set in [6, 8, 18]

Resolution	SURF and FLANN	RANSAC	Motion Smoothing	Total(ms)
320×240	6.83	2.29	0.14	9.26
640×360	8.48	3.30	0.56	12.34
1280×720	17.30	5.28	1.04	23.62

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Stabilization performance

- ITF(Inter-frame Transformation Fidelity) is a popular evaluation metric of stabilization quality.
- ITF defined as

$$ITF = \frac{1}{N-1} \sum_{k=1}^{N-1} PSNR(F_{k+1}, F_k) \tag{8}$$

where N is the total number of frames.

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Stabilization performance

Table 2. Comparison of ITF for different video stabilization methods

	Offline			Real-time		
Videos	[6]	[8]	[18]	[19]	[12]	Ours
18AF	21.33	27.09	27.55	21.54	23.15	**24.98**
Fountain	27.16	32.27	31.87	27.62	28.14	**28.68**
Talking	23.30	25.94	25.97	23.40	23.52	**23.64**
Dancing	22.62	22.40	22.49	21.32	22.96	**21.87**
Traffic	23.74	26.87	26.77	21.59	24.09	**24.18**
Street	22.48	24.32	27.98	21.85	23.78	**24.33**
Park	21.66	27.47	27.49	22.04	23.19	**23.56**
Parking lot	28.81	25.95	27.02	18.52	21.76	**22.38**
Dynamic	21.12	22.04	22.03	20.24	21.42	**21.44**
Degenerate	22.37	23.18	22.63	20.78	22.02	**22.72**

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More results: https://ieee-dataport.org/open-access/lsstab-results
(DOI: 10.21227/e1dj-g876)

Thank you!