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Grid Optimization for Matrix-based Source Localization under Inhomogeneous Sensor Topology

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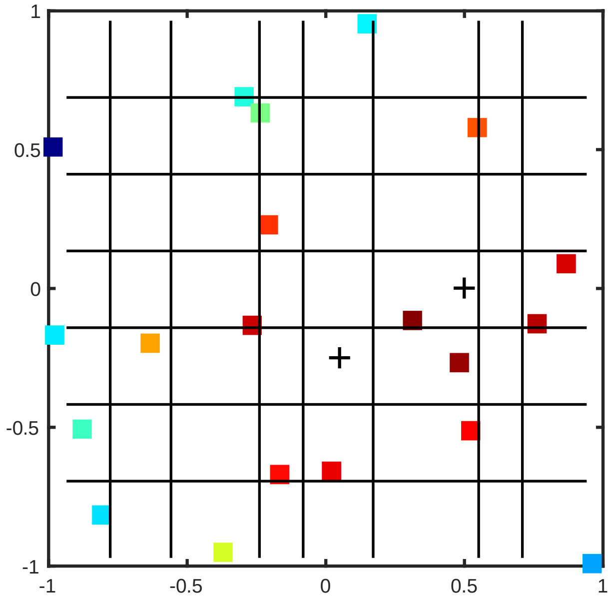

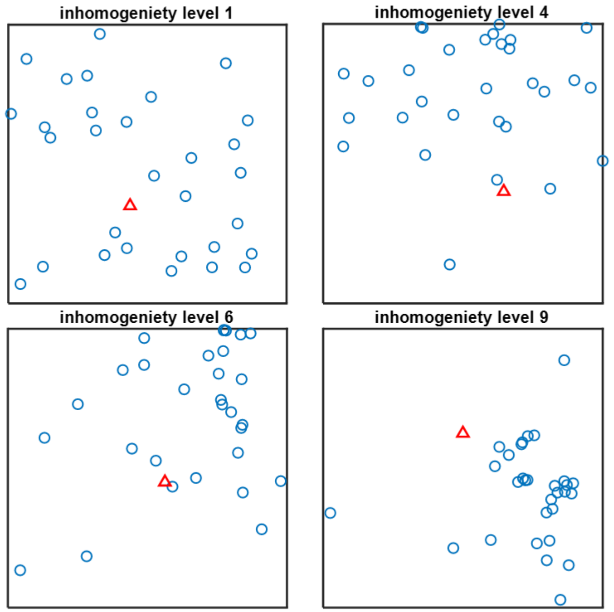

Acknowledgement

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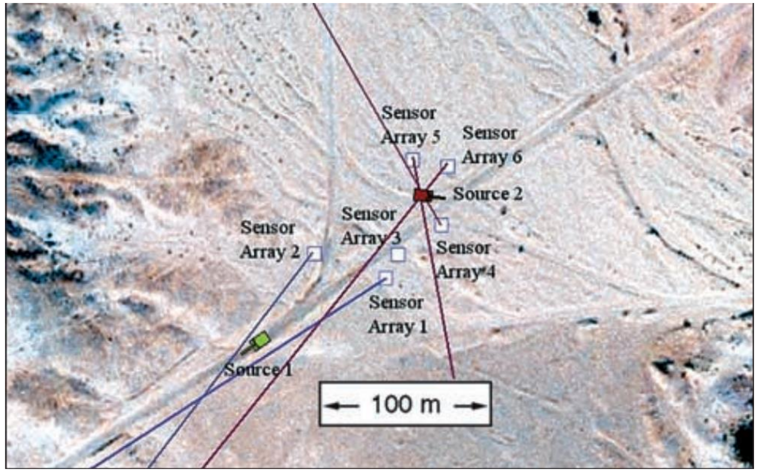
Source localization is important in many areas.



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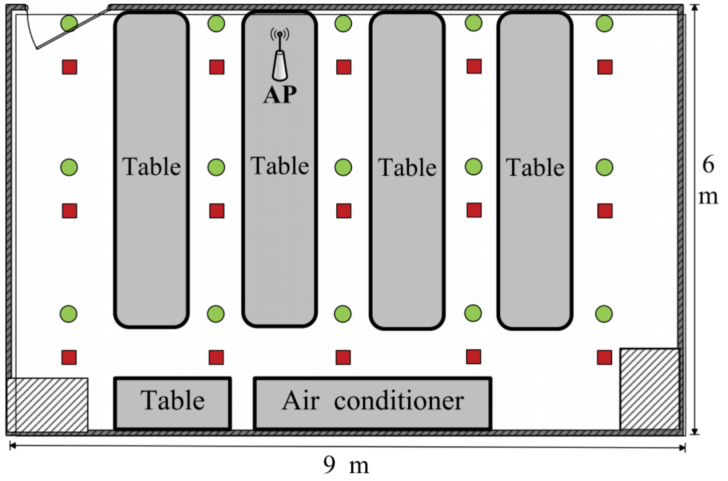

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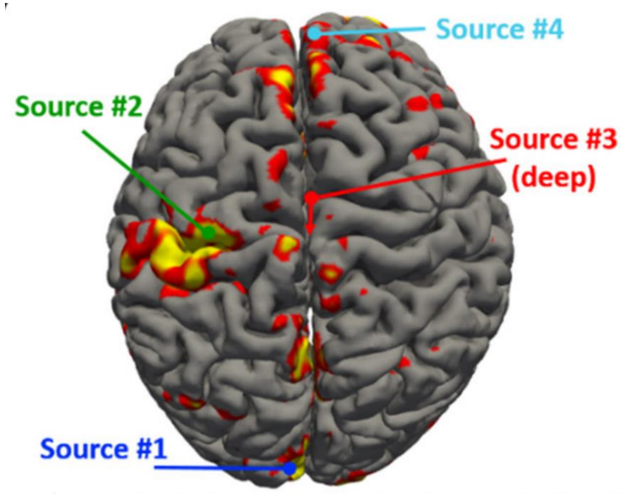

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Outdoor source localization:

Sensor networks monitor an area, including detecting, identifying, localizing, and tracking objects

Indoor source localization

Wi-Fi provides ubiquitous access in indoor environments

Neuroimaging source localization

Dynamic changes of Electroencephalography (EEG) and functional magnetic resonance imaging (fMRI)

1 Chen J C , Yao K , Hudson R E . Source localization and beamforming. Signal Processing Magazine IEEE, 2002.
2 X. Wang, X. Wang and S. Mao, Deep Convolutional Neural Networks for Indoor Localization with CSI Images. IEEE Transactions on Network Science and Engineering, 2020.
3 Thinh, Nguyen, et al. Characterization of dynamic changes of current source localization based on spatiotemporal fMRI constrained EEG source imaging. Journal of neural engineering, 2018.

The problem: how to improve the localization accuracy according to different sensor topologies

Information available:
Sensor location and energy measurement pairs

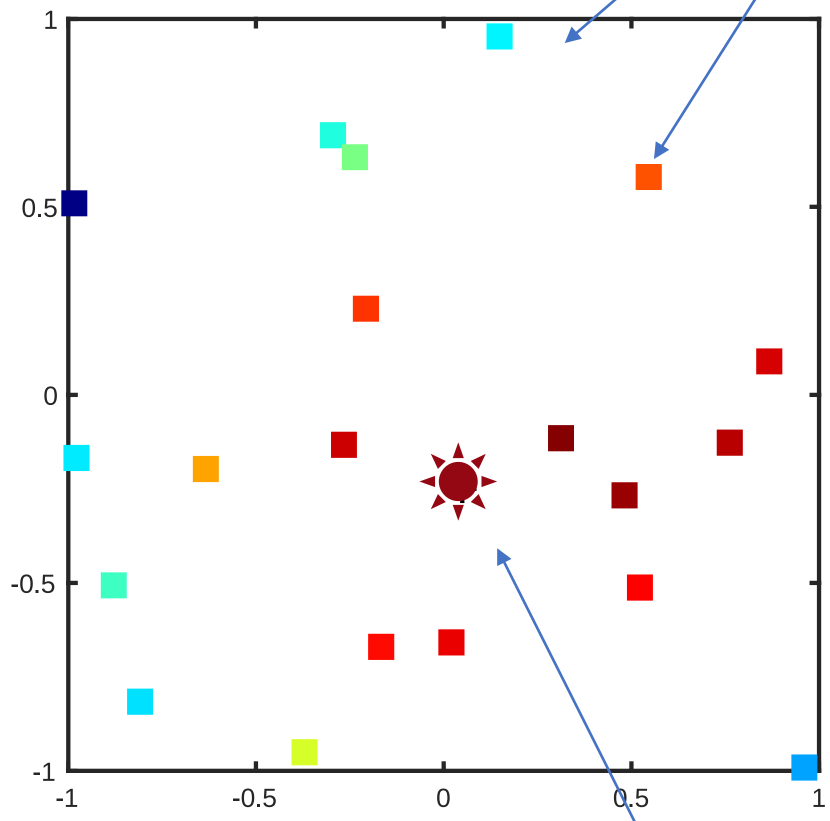
$$\{(\boldsymbol{z}_m, \gamma_m)\}$$

$m = 1,2, \ldots, M.$

$$\gamma_m = f(d(\boldsymbol{s}, \boldsymbol{z}_m)) + \xi_m$$

$f(d)$ is an *unknown* non-increasing function in distance d

Sensor measurements

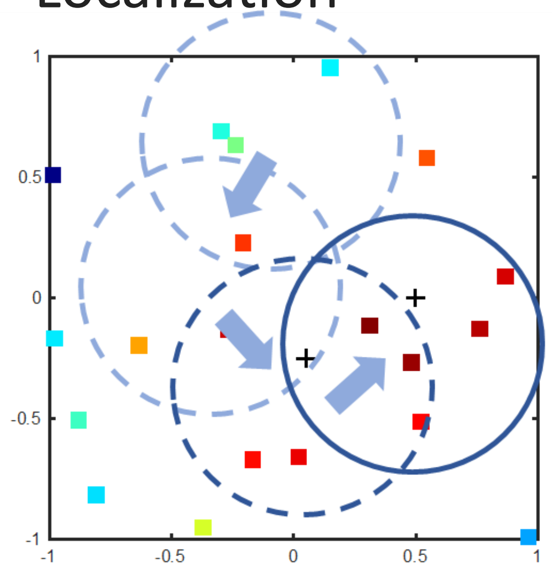


Source to localize

Challenge:
- No ranging information
- No information on the power decay law

Existing model free methods for localization

Weighted Centroid Localization

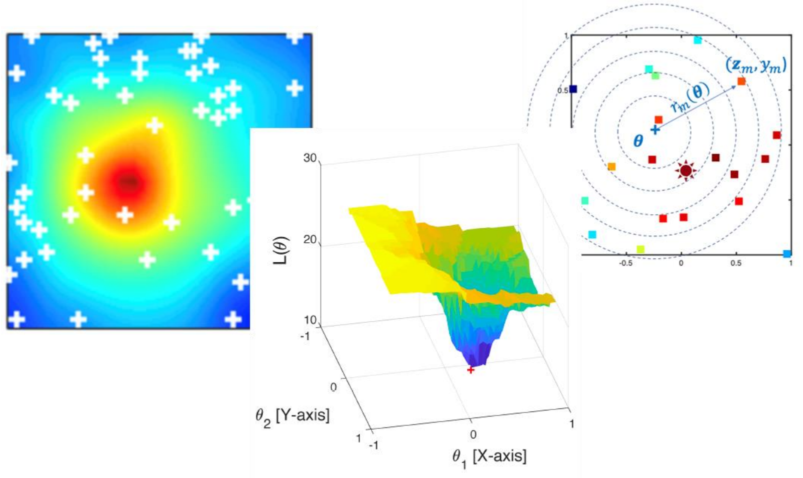


[4]

$$\hat{s}_{\mathrm{WCL}} = \sum_{m=1}^{M} w_m z_m / \sum_{m=1}^{M} w_m$$

$$w_m = \gamma_m$$

Symmetry and Unimodality: Curve fitting method



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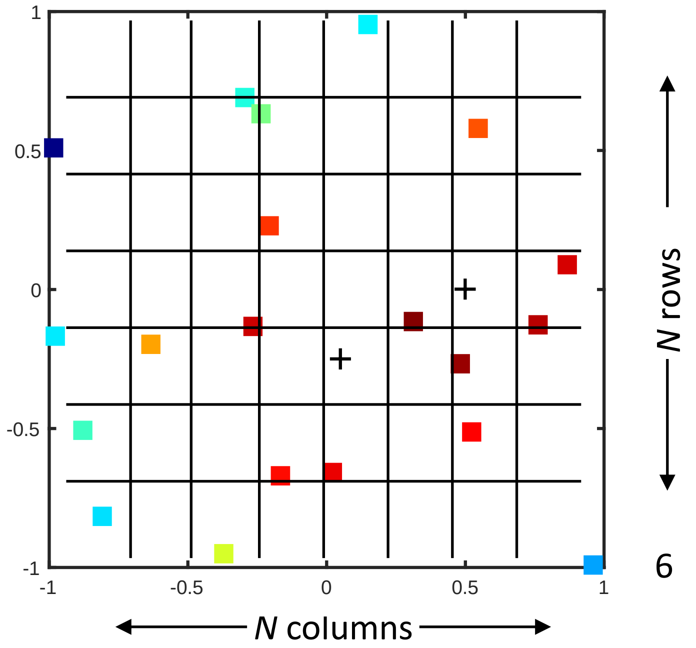
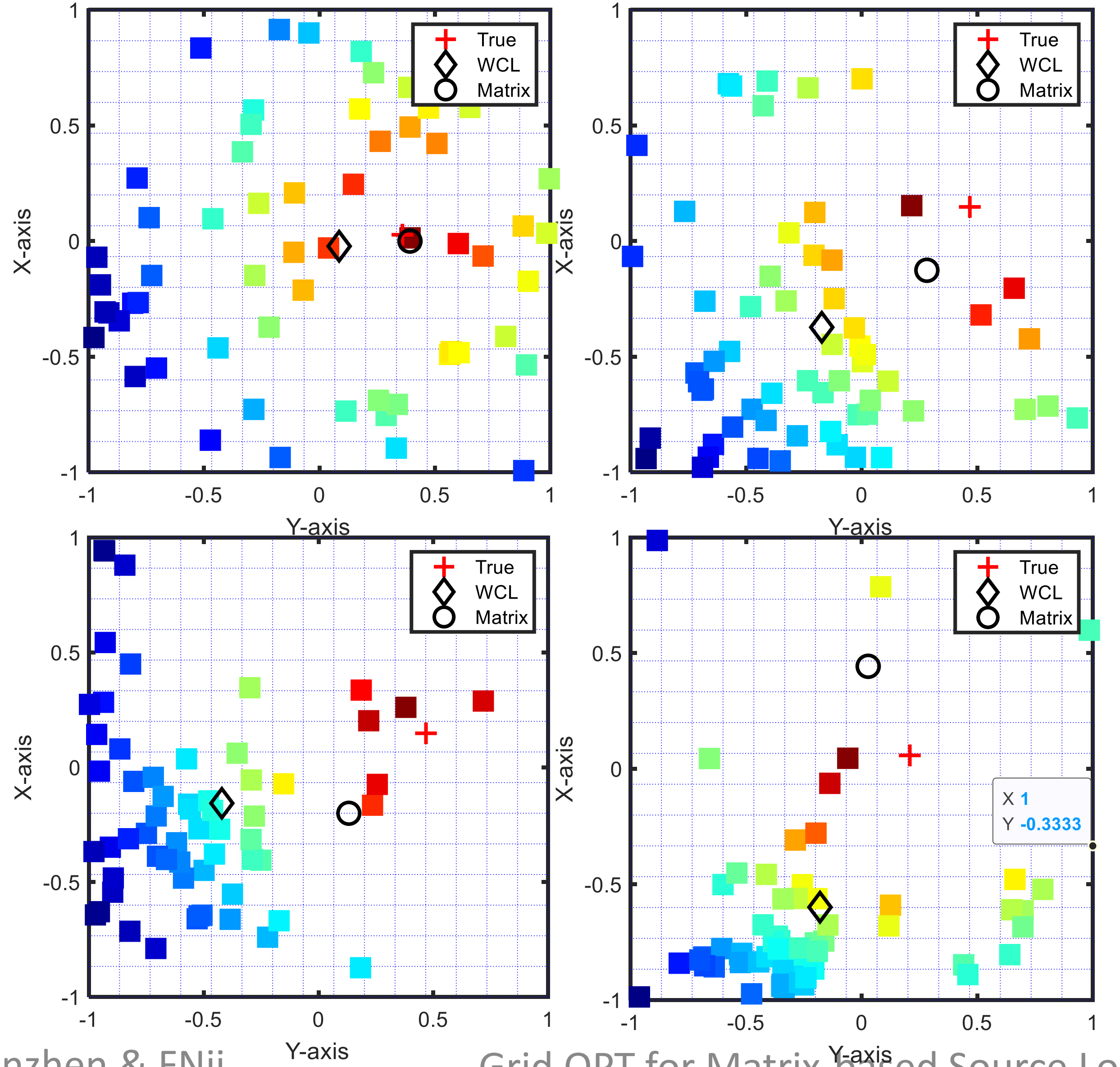
Matrix Factorization



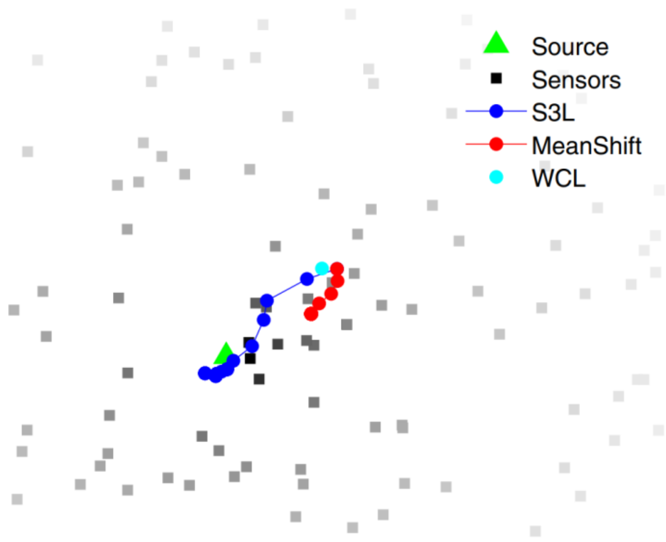
[6]

4 Wang J, Urriza P, et al. Weighted Centroid Localization Algorithm: Theoretical Analysis and Distributed Implementation. IEEE Trans. on Wireless Communications, 2011, 10(10):3403-3413.
5 Chen J, Exploiting Two-Dimensional Symmetry and Unimodality for Model-Free Source Localization in Harsh Environment. ICASSP 2020
6 Chen J, Mitra U. Unimodality-Constrained Matrix Factorization for Non-Parametric Source Localization. 2017.

Inhomogeneity influence the localization accuracy

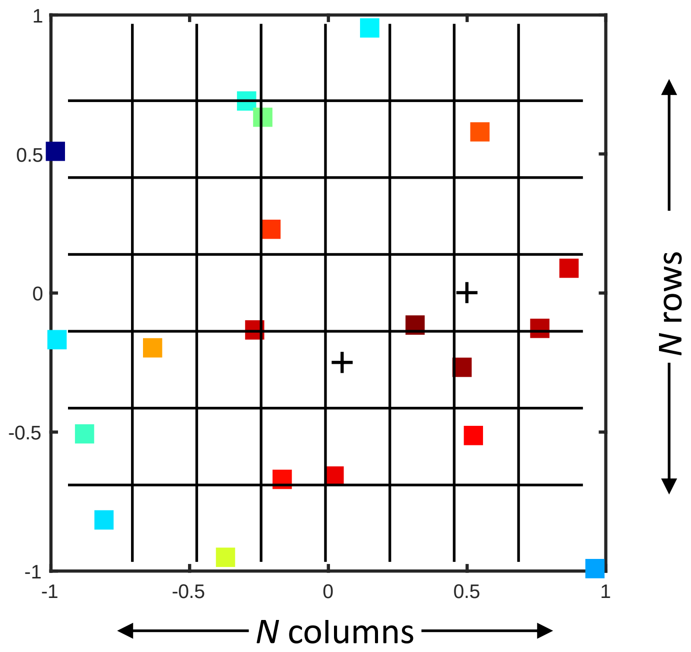


A sensor selection method improves the localization accuracy of WCL



Recall the non-parametric method based on matrix construction

Matrix observation H
on $N \times N$ grid points



Uniform construction

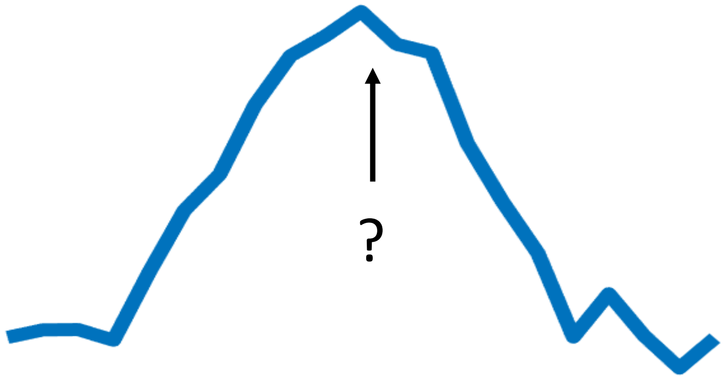
$$\underset{\boldsymbol{X} \in \mathbb{R}^{N \times N}}{\text{minimize}} \quad \|\boldsymbol{X}\|_*$$

$$\text{subject to} \quad X_{ij} = \mathsf{H}_{ij}, \qquad \forall (i,j) \in \Omega$$

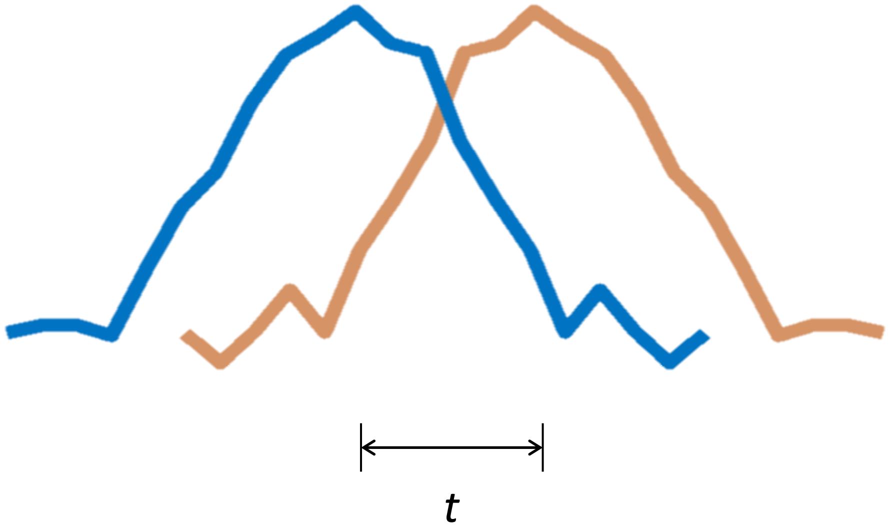
$$\boldsymbol{H} = \sigma_1 u_1 v_1 + \xi$$

Assume low rank, only first singular value and vectors matters and other term serves as noise.
The peak of the first singular vector represent the location with the unimodality and symmetry property

Estimation of location



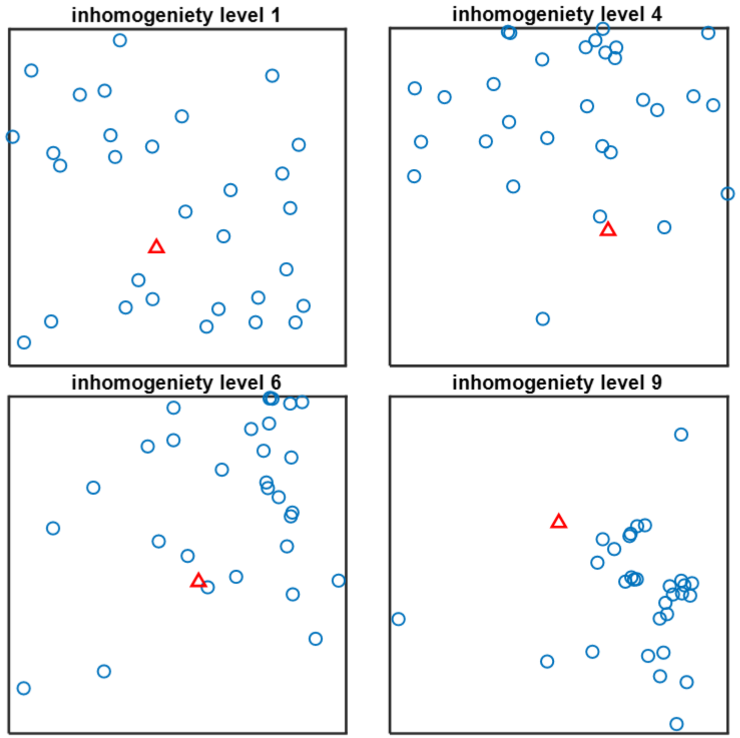
Noise corrupted vector $\hat{u}$



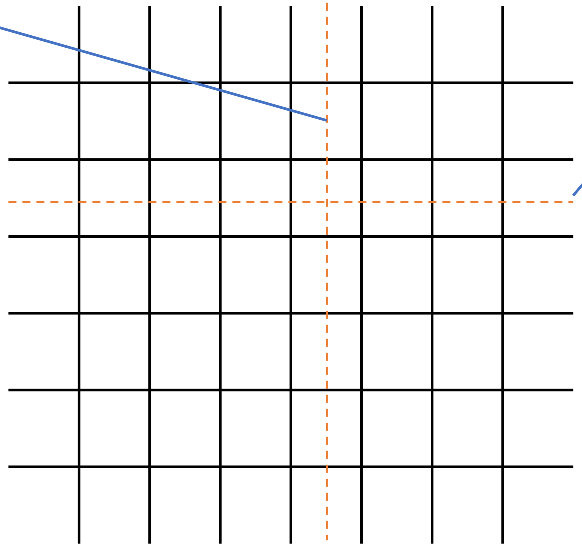
$$\hat{R}(t; \hat{\mathbf{u}}_1) = \int_{-\infty}^{\infty} \hat{u}(x)\hat{u}(-x+t)dx$$

$$\hat{\mathsf{s}}_1(\hat{\mathbf{u}}_1) = \frac{1}{2}\operatorname*{argmax}_{t\in\mathbb{R}} \hat{R}(t; \hat{\mathbf{u}}_1)$$

Unsolved issue: the formation of the grid



- Uniform construction:
  too few observation in one row/column → large matrix completion error
- Proposed: optimize the grid positions, i.e. x^c and y^c



Cramér-Rao Bound Analysis for matrix completion

Cramér–Rao bound (CRB) :a lower bound on the variance of unbiased estimators of a deterministic (fixed, though unknown) parameter
MSE matrix defined as $\mathbb{E}\{\|\boldsymbol{H} - \bar{\mathbf{H}}(\gamma)\|_{\mathrm{F}}^2\}$ is lower bounded by

$$\Gamma(\boldsymbol{H}) \approx \max\Bigg\{\sum_{i=1}^{N}\Big(\sum_{j:(i,j)\in\Omega}\frac{u_{1j}}{\sigma^2_{M(i,j)}}\Big)^{-1}, \sum_{j=1}^{N}\Big(\sum_{i:(i,j)\in\Omega}\frac{v_{1i}}{\sigma^2_{M(i,j)}}\Big)^{-1}\Bigg\}$$

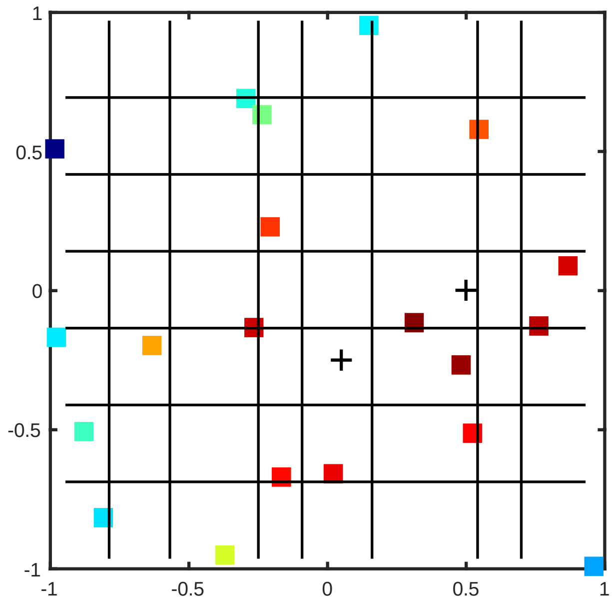
where u_{1j} is the jth element of the dominant singular vector $\boldsymbol{u}_1$ and v_{1i} is the ith element of $\boldsymbol{v}_1$. The (m,m)th element of $\boldsymbol{\Sigma}$ is $\sigma^2_{M(i,j)}$

Some insights getting from the derivation

- Each column and row of the observation matrix has at least one observation. → The chosen of N
- Reduce the noise term $\sigma^2_{M(i,j)}$

  (i) the measurement noise $\xi_{M(i,j)}$

  (ii) the discretization noise $f(d(\boldsymbol{s}, \boldsymbol{z}_{M(i,j)})) - H_{ij}$
- (ii) due to not measuring at the grid center
- Reduce the distance $\|\boldsymbol{z}_{M(i,j)} - \boldsymbol{c}_{ij}\|_2$ reduces the noise variance since $H_{ij} = f(d(\boldsymbol{s}, \boldsymbol{c}_{ij}))$



Challenge: Optimize the grid construction

Minimize the summation of senor-to-grid-center distance:

$$\underset{\{x_i^{\mathrm{c}}\},\{y_j^{\mathrm{c}}\}}{\text{minimize}} \quad \sum_{i,j} \sum_{m \in M(i,j)} \|\boldsymbol{z}_m - (x_i^{\mathrm{c}}, y_j^{\mathrm{c}})\|$$

Decomposed into an x-subproblem and a y-subproblem and L_1 -norm distance is considered:

$$\underset{\{x_i^{\mathrm{c}}\}}{\text{minimize}} \quad \sum_{i=1}^{N} \sum_{m \in \mathcal{R}_i} |z_{m,1} - x_i^{\mathrm{c}}|$$

$$\underset{\{y_j^{\mathrm{c}}\}}{\text{minimize}} \quad \sum_{i=1}^{N} \sum_{m \in \mathcal{C}_i} |z_{m,2} - y_j^{\mathrm{c}}|$$

Resemble K-means problem

Evaluation: The Adaptive grid formation lower MSE

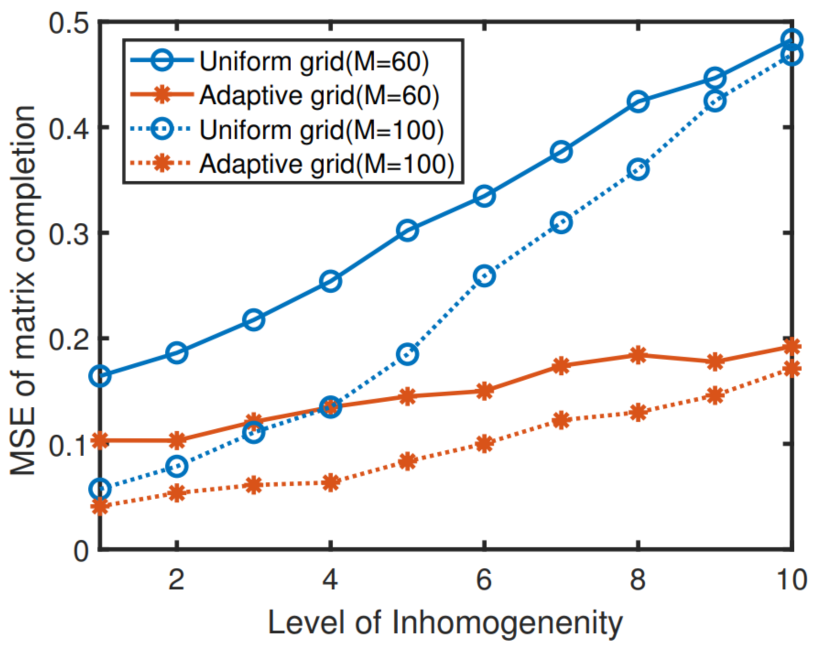
Model: $\gamma = (1 + d^{1.5} A(f)^d)^{-1} + \xi$

where $10\log_{10} A(f) = 0.11 f^2/(1+f^2)$
$+ 44 f^2/(4100 + f^2) + 2.75 \times 10^{-4} f^2 + 0.003$

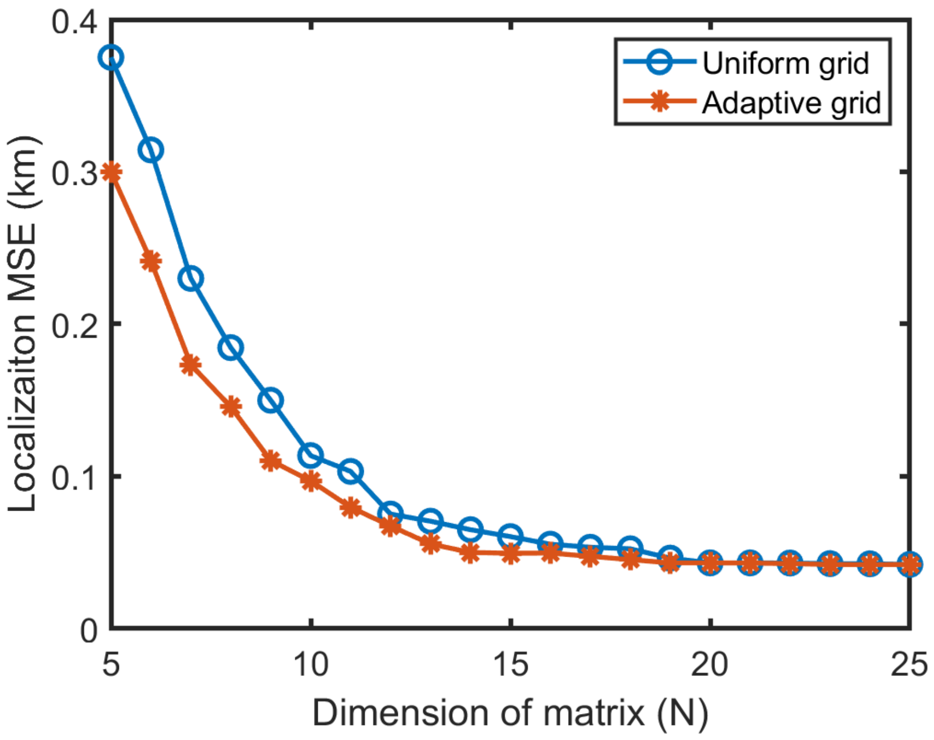
where f=5kHz, d is the distance, and $\xi \sim \mathcal{N}(0, \sigma^2)$ is to model the noise with $\sigma = 3$ dB.

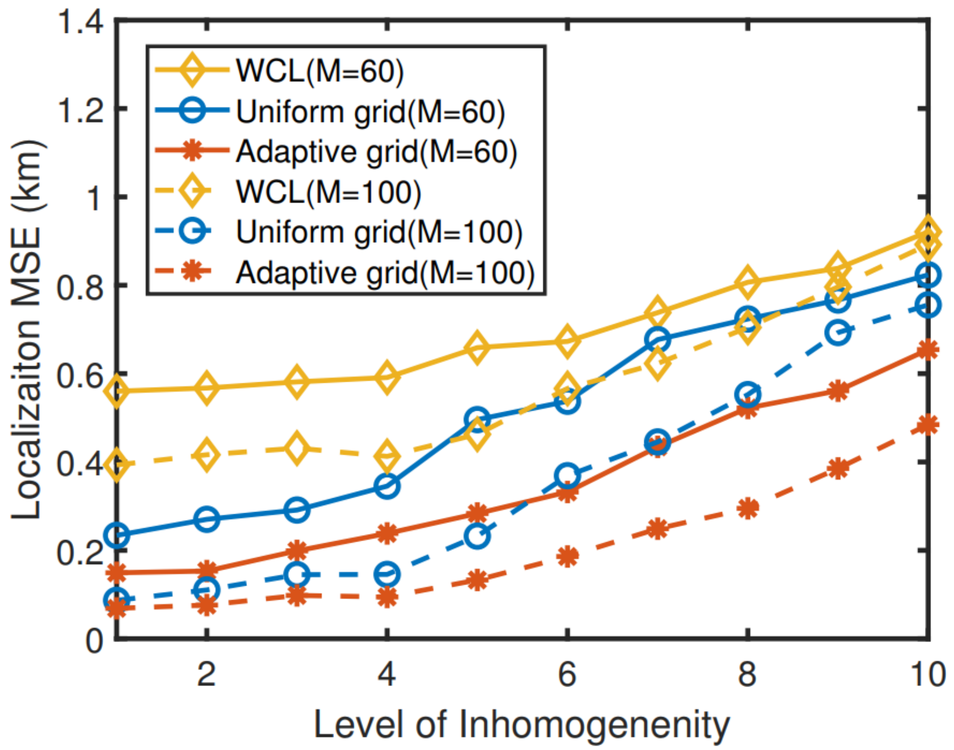
Baseline 1. Weighted centroid localization
$\hat{\boldsymbol{s}}_{\mathrm{WCL}} = \sum_{m=1}^{M} w_m \boldsymbol{z}_m / \sum_{m=1}^{M} w_m$, where
$w_m = \gamma_m$ serves as the weight.

Baseline 2. uniform grid method



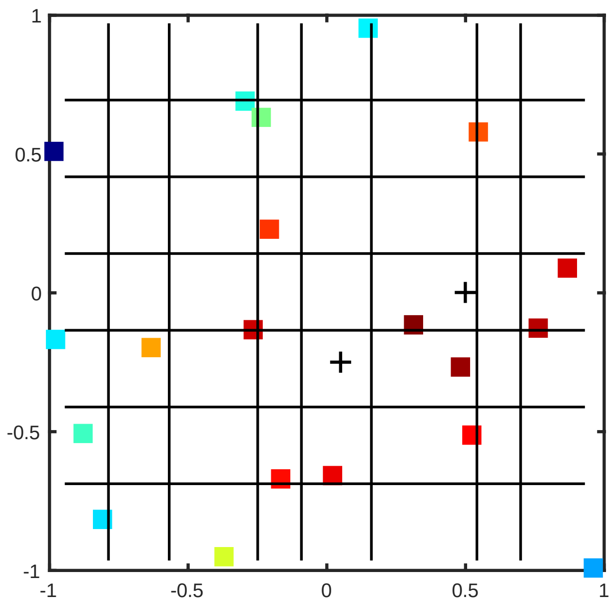
Evaluation: improves the localization accuracy





Conclusion: grid optimization improves localization accuracy

- Proposed method improves the localization accuracy under different inhomogeneity levels of sensor topologies and the proposed method significantly outperforms WCL schemes.
- Key idea: minimize the senor-to-grid-center distance→ optimize the grid
- Key techniques:
  - CRB analysis for matrix completion
  - adaptive grid formation method based on K-means method



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Thank you & Questions

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