# Exploiting The Dual-Tree Complex Wavelet Transform for Ship Wake Detection in SAR Imagery


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## Abstract

1. Detecting ships and their corresponding wakes has always been a significant topic in terms of marine applications.
2. In this study, the ship wake detection problem is addressed as a Radon transform based inverse problem with **Cauchy and DT-CWT** regularizations.
3. The results show a successful detection performance of up to **90.91**%


## The Proposed Method

Since ship wakes have linear features, SAR image formation can be expressed in terms of its Radon transform as

$$Y = CX + N$$

Where $Y$ is the noisy SAR image, $X$ represents the data in the Radon domain. Also, $N$ is additive noise, and the operator $C = R^{-1}$ is the inverse Radon transform. The problem is solved to obtain an estimation for $X$ via the minimization problem of

$$\hat{X} = \arg\min_{X}\{\|Y - CX\|_2^2 - \log(\gamma/(\gamma^2 + X^2)) - \lambda\|\mathcal{B}X\|_1\}$$

Where $\mathcal{B}$ is forward DT-CWT operator.

Algorithm 1 is based on forward-backward (FB) splitting algorithm to obtain the optimized $\hat{X}$.

**Algorithm 1:** *Algorithmic representation of Cauchy and DT-CWT Proximal Splitting*

1: Input: SAR imagery $Y$ and coefficients $\mu, \gamma, \lambda$

2: Output: Radon image $X$

3: Set: $i$ = 0 and $X^{(0)} = \{0\}$

4: do

5: $Z^{(i)} = X^{(i)} - \mu\left(C^T\left(C(X^{(i)}) - Y\right) - \nabla\left(log\left(\frac{\gamma}{\gamma^2 + X^{(i)2}}\right)\right)\right)$

6: $W = soft\{\mathcal{B}(Z^{(i)}),\ \mu * \lambda\}$

7: $X^{(i)} = \mathcal{B}^{-1}(W)$

8: $i++$

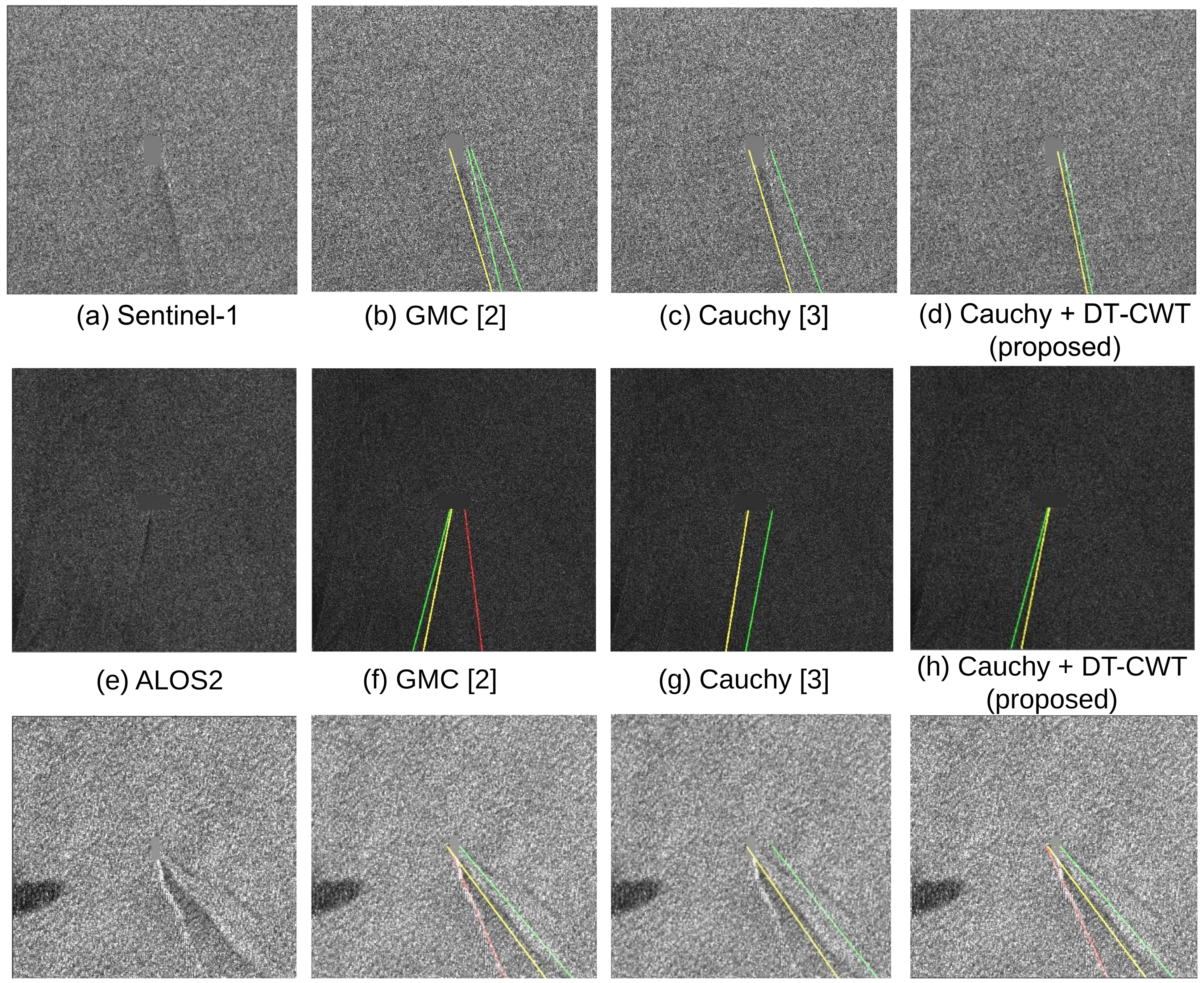
9: **while** $\varepsilon^{(i)} > 10^{-3}$ $or$ $i < maxIter$

## Why DT-CWT?

DT-CWT can significantly reduce the translation sensitivity and improve the direction selectivity of Discrete Wavelet Transform. In this case, implementing DT-CWT in Radon domain of SAR imagery would be useful to enhance linear features of ship wakes.

### Detection Performance for All SAR Images

| | TP | TN | FP | FN | Accuracy |
|---|---|---|---|---|---|
| Graziano et. al. [1] | 32.73% | 38.18% | 25.45% | 3.64% | 70.91% |
| GMC [2] | **43.64%** | 34.55% | 18.18% | 3.64% | 77.27% |
| TV | 36.18% | 33.64% | 26.36% | **1.82%** | 70.91% |
| Cauchy [3] | 37.27% | **47.27%** | 10.00% | 6.36% | 84.55% |
| Proposed Cauchy + DTCWT | 41.82% | **48.18%** | **5.45%** | 5.45% | **90.91%** |



(a) Sentinel-1 (b) GMC [2] (c) Cauchy [3] (d) Cauchy + DT-CWT (proposed)

(e) ALOS2 (f) GMC [2] (g) Cauchy [3] (h) Cauchy + DT-CWT (proposed)

(i) TerraSAR-X (j) GMC [2] (k) Cauchy [3] (l) Cauchy + DT-CWT (proposed)

## Detection Processing

| Step 1: | Step 2: |
|---|---|
| Pre-processing of SAR images: Obtain centered and masked image **Y** in the spatial domain | Use inverse problem Y = CX+N to reconstruct the Radon image $\hat{X}_{\mathrm{Cauchy-DTCWT}}$ by the Algorithm 1. |
| **Step 3:** | **Step 4:** |
| 3.1 Restriction of reconstructed Radon images to obtain restricted images $\hat{X}_{\mathrm{res}}$ | 4.1 Inverse Radon transform of detected bright(dark) points |
| 3.2 Detect Turbulent and first Narrow-V wake in the Radon domain | 4.2 Retain half lines and estimate right indexes |
| 3.3 Detect second Narrow-V wake and Kelvin arms in the Radon domain | 4.3 Validate and discard false wake detection |

### SAR Images and Visible Wakes

| Images | Turbulent | 1st Narrow | 2nd Narrow | 1st Kelvin | 2nd Kelvin |
|---|---|---|---|---|---|
| Sentinel-1 | √ | √ | × | × | × |
| ALOS2 | √ | √ | × | × | × |
| TerraSAR-X | √ | √ | × | √ | × |


## Reference

[1] M. Graziano, M. D'Errico and G. Rufino, "*Wake Component Detection in X-Band SAR Images for Ship Heading and Velocity Estimation*", Remote Sensing, vol. 8, no. 6, p. 498, 2016.

[2] O. Karakus, I. Rizaev, and A. Achim, "*Ship Wake Detection in SAR Images via Sparse Regularization*," IEEE Transactions on Geoscience and Remote Sensing, vol. 58, no. 3, pp. 1665–1677, 2020.

[3] O. Karakus, and A. Achim, "*On solving SAR imaging inverse problems using non-convex regularization with a Cauchy-based penalty*, "IEEE Transactions on Geoscience and Remote Sensing, 2020.