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Extracting Knowledge from Information

Low-Complexity Parameter Learning for OTFS Modulation Based Automotive Radar

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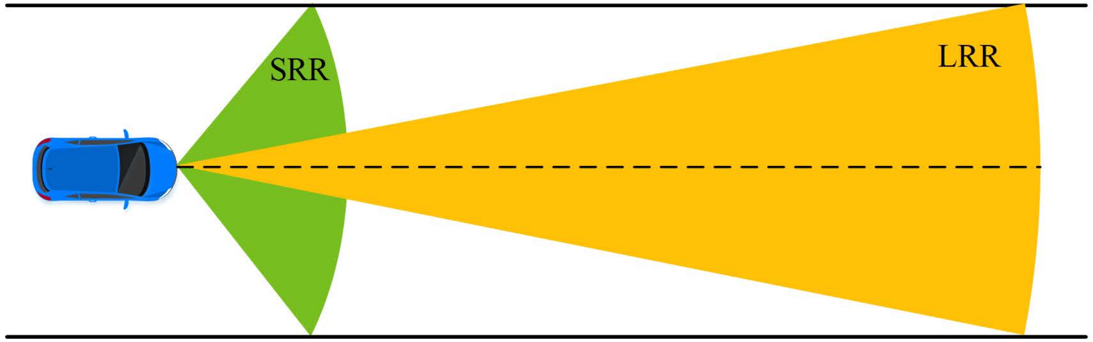
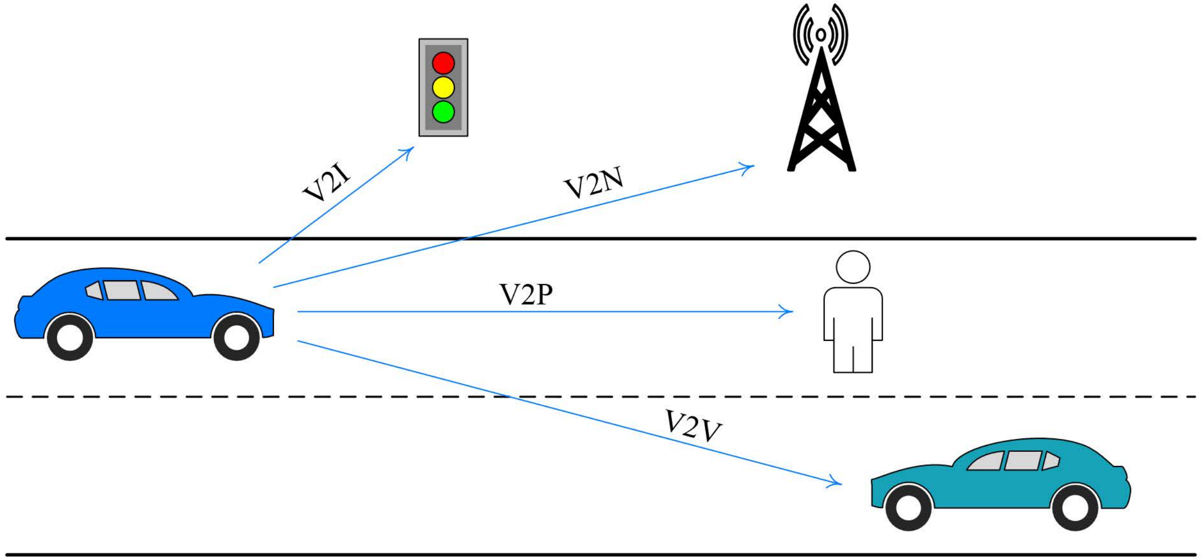
BACKGROUND





Radar + Communication

JRC

MOTIVATIONS

- **OTFS modulation** is more robust against doubly-selective channels in high mobility scenarios than OFDM modulation.

- A **low-complexity** target detection method is demanded for automotive radars to handle the large number of subcarriers and symbols.

SIGNAL MODEL

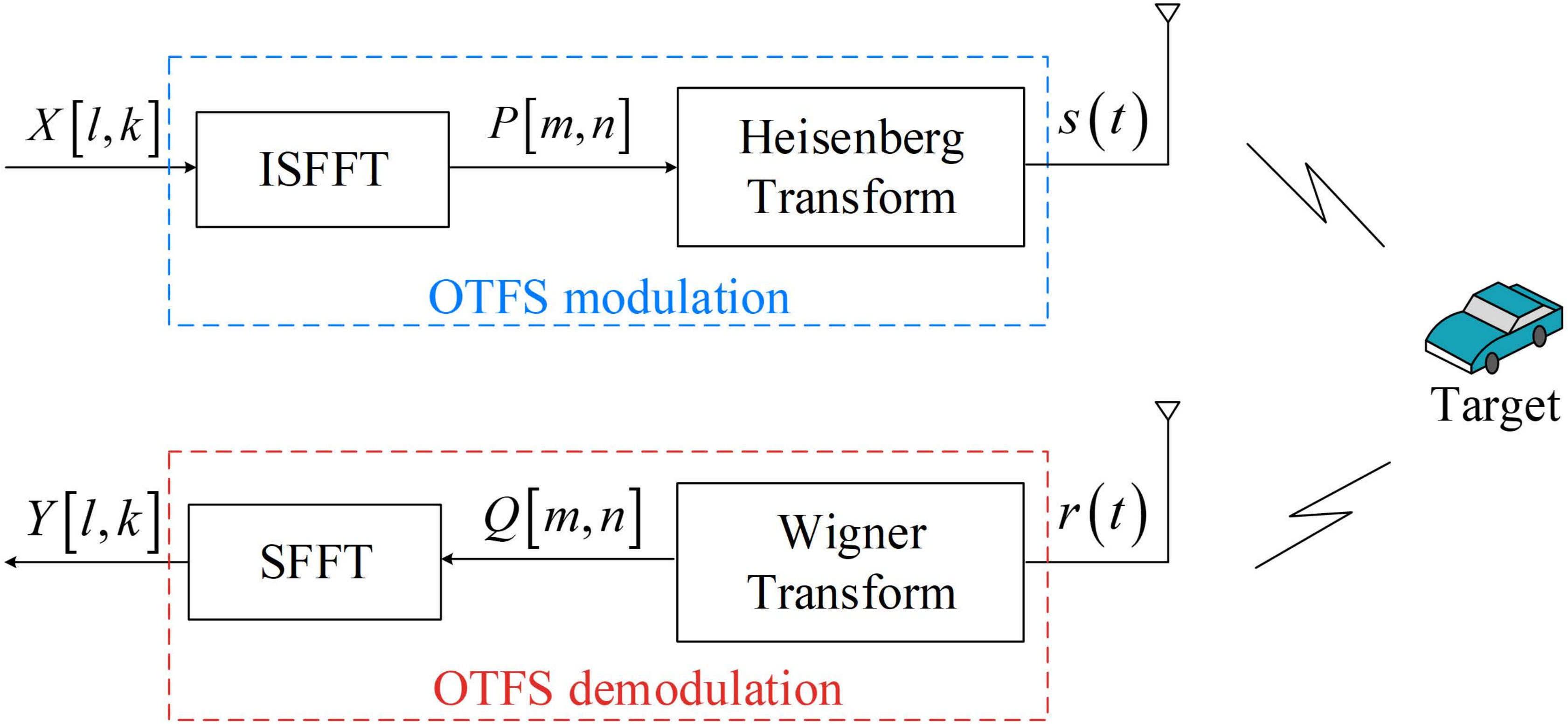
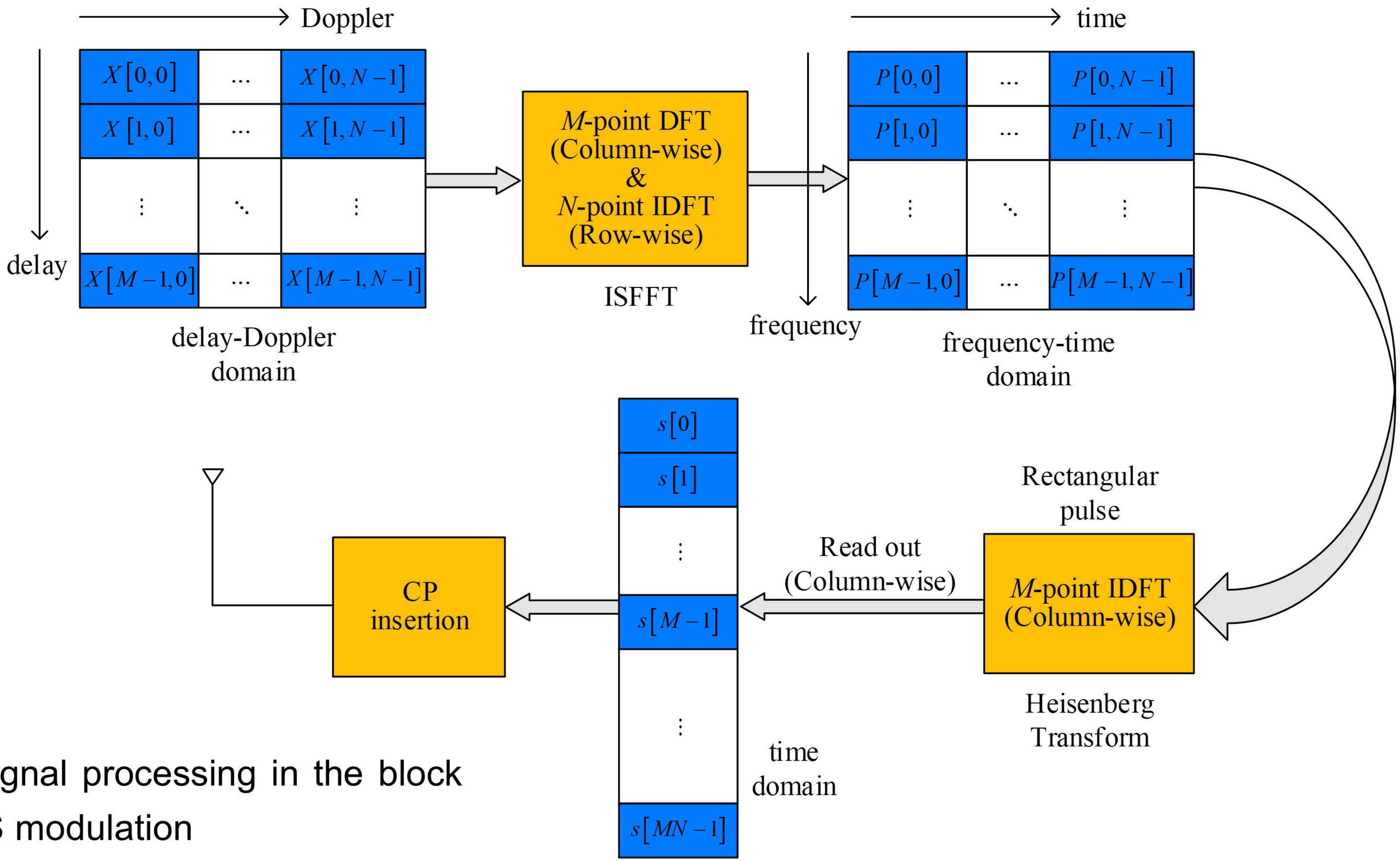


Fig.1 Block diagram of the signal flow in an OTFS radar

SIGNAL MODEL



Doppler

delay

$X[0,0]$ … $X[0,N-1]$

$X[1,0]$ … $X[1,N-1]$

⋮ ⋱ ⋮

$X[M-1,0]$ … $X[M-1,N-1]$

delay-Doppler domain

M-point DFT (Column-wise) & *N*-point IDFT (Row-wise)

ISFFT

time

frequency

$P[0,0]$ … $P[0,N-1]$

$P[1,0]$ … $P[1,N-1]$

⋮ ⋱ ⋮

$P[M-1,0]$ … $P[M-1,N-1]$

frequency-time domain

Rectangular pulse

M-point IDFT (Column-wise)

Heisenberg Transform

Read out (Column-wise)

$s[0]$

$s[1]$

⋮

$s[M-1]$

⋮

$s[MN-1]$

time domain

CP insertion

Fig.2 Signal processing in the block of OTFS modulation

SIGNAL MODEL

Discrete Radar Channel in the Delay-Doppler Domain

$$H(\tau,\nu)=\sum_{\breve{k}=0}^{N-1}\sum_{\breve{l}=0}^{M-1}H[\breve{l},\breve{k}]\delta\left(\tau-\frac{\breve{l}}{Mf_{\mathrm{s}}}\right)\delta\left(\nu-\frac{\left(\breve{k}\right)_{N}}{NT_{\mathrm{s}}}\right) \quad (1)$$

Doppler Tap of Target

$$(\breve{k})_N=\begin{cases}\breve{k}, & \breve{k}\leq N/2,\\ \breve{k}-N, & \text{otherwise.}\end{cases} \quad (2)$$

SIGNAL MODEL

Input-Output Relationship in the Delay-Doppler Domain

$$\underbrace{\boxed{\mathbf{y}}}_{\mathrm{vec}(\mathbf{Y})} = \widetilde{\mathbf{X}}\underbrace{\boxed{\mathbf{h}}}_{\mathrm{vec}(\mathbf{H})} + \mathbf{w} \tag{3}$$

Integrated Received Symbol Matrix

$$\widetilde{X}[\underbrace{p,q}_{\substack{p = kM + l \\ q = \breve{k}M + \breve{l}}}] = \alpha_{l,k}[\breve{l},\breve{k}] \cdot X\left[\underbrace{\langle l - \breve{l}\rangle_M}_{\text{modulo operation}}, \langle k - (\breve{k})_N\rangle_N\right], \tag{4}$$

Phase Shift Term

$$\alpha_{l,k}[\breve{l},\breve{k}] = \begin{cases} \exp\left\{\mathrm{j}\frac{2\pi}{MN}(\breve{k})_N\langle l - \breve{l}\rangle_M\right\}, & \breve{l} \leq l < M, \\ \exp\left\{\mathrm{j}\frac{2\pi}{MN}(\breve{k})_N\langle l - \breve{l}\rangle_M - \mathrm{j}\frac{2\pi}{N}k\right\}, & 0 \leq l < \breve{l}. \end{cases} \tag{5}$$

PRUNED MODEL

Effective Delay and Doppler Shift Bins

$$\begin{aligned} M_{\text{eff}} &= \lceil 2BR_{\max}/\mathrm{c} \rceil + 1, \\ N_{\text{eff}} &= \lceil 4f_{\mathrm{c}}MNV_{\max}/(\mathrm{c}B) \rceil + 1, \end{aligned} \quad (6)$$

Variation of Signal Model

$$\begin{aligned} \mathbf{h}: &\quad MN \times 1 \rightarrow M_{\text{eff}}N_{\text{eff}} \times 1 \\ \widetilde{\mathbf{X}}: &\quad MN \times MN \rightarrow MN \times M_{\text{eff}}N_{\text{eff}} \end{aligned} \quad (7)$$

CPCSBL-GAMP

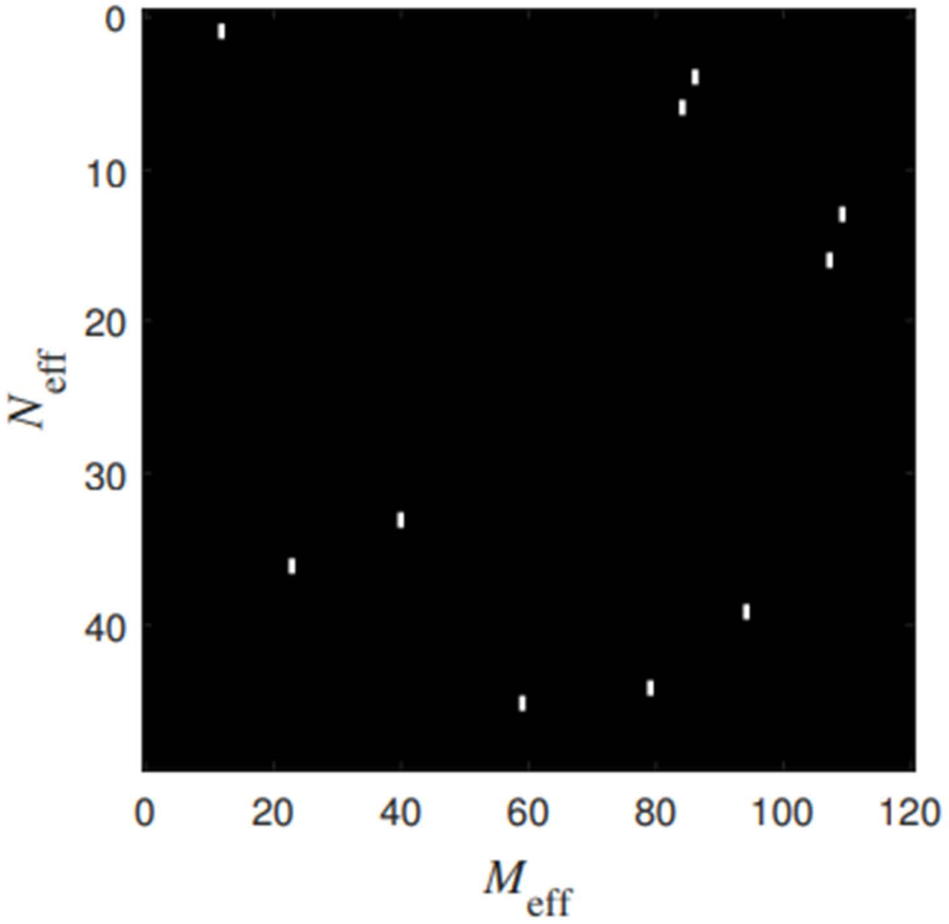


Fig.3 Visualization of the radar channel matrix with ten targets

Sparse!

CPCSBL-GAMP

Compressed Sensing Signal Model

$$\breve{\mathbf{y}} = \mathbf{A}\mathbf{h} + \mathbf{w} \tag{8}$$

where $\breve{\mathbf{y}} \in \mathbb{C}^{S\times 1}$ is a subvector of $\mathbf{y}$, $\mathbf{A} \in \mathbb{C}^{S\times M_{\text{eff}} N_{\text{eff}}}$ is a submatrix of $\tilde{\mathbf{X}}$. $\left(S \ll MN \right)$

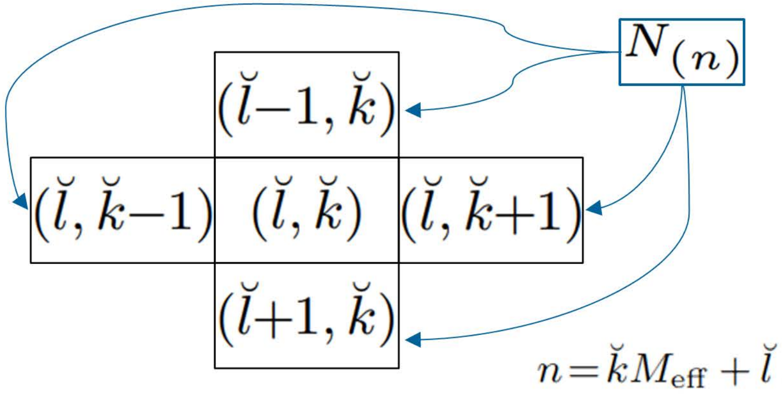
Pattern-Coupled Gaussian Prior Distribution

$$p(\mathbf{h}|\boldsymbol{\alpha}) = \prod_{n=0}^{M_{\text{eff}} N_{\text{eff}} - 1} \mathcal{CN}(h_n | 0, \eta_n^{-1}) \tag{9}$$

Hyperparameter and Its Distribution

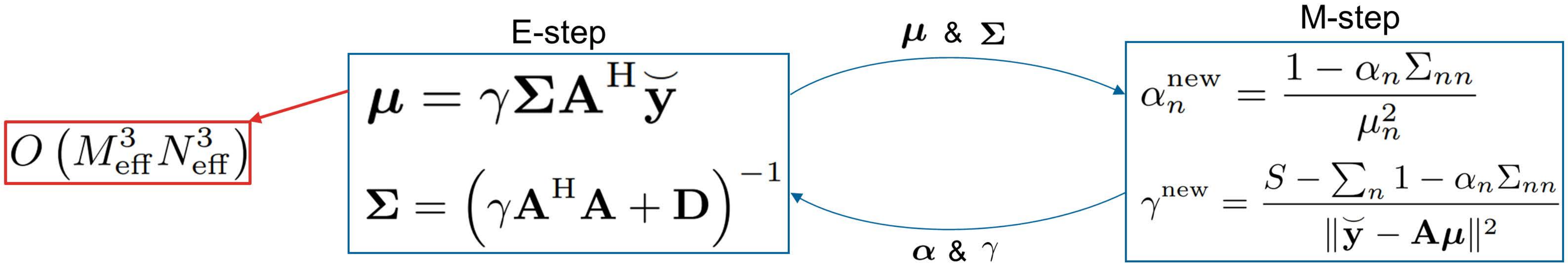
$$\eta_n = \alpha_n + \beta \sum_{i \in \boxed{N_{(n)}}} \alpha_i \tag{10}$$

$$p(\boldsymbol{\alpha}) = \prod_{n=0}^{M_{\text{eff}} N_{\text{eff}} - 1} \mathrm{Gamma}\left(\alpha_n | a, b\right) \tag{11}$$



CPCSBL-GAMP

- Expectation-maximization (EM) method



$O\left(M_{\text{eff}}^3 N_{\text{eff}}^3\right)$

E-step

$$\boldsymbol{\mu} = \gamma \boldsymbol{\Sigma} \mathbf{A}^{\mathrm{H}} \breve{\mathbf{y}}$$

$$\boldsymbol{\Sigma} = \left(\gamma \mathbf{A}^{\mathrm{H}} \mathbf{A} + \mathbf{D}\right)^{-1}$$

$\boldsymbol{\mu}$ & $\boldsymbol{\Sigma}$

$\boldsymbol{\alpha}$ & γ

M-step

$$\alpha_n^{\text{new}} = \frac{1 - \alpha_n \Sigma_{nn}}{\mu_n^2}$$

$$\gamma^{\text{new}} = \frac{S - \sum_n 1 - \alpha_n \Sigma_{nn}}{\|\breve{\mathbf{y}} - \mathbf{A}\boldsymbol{\mu}\|^2}$$

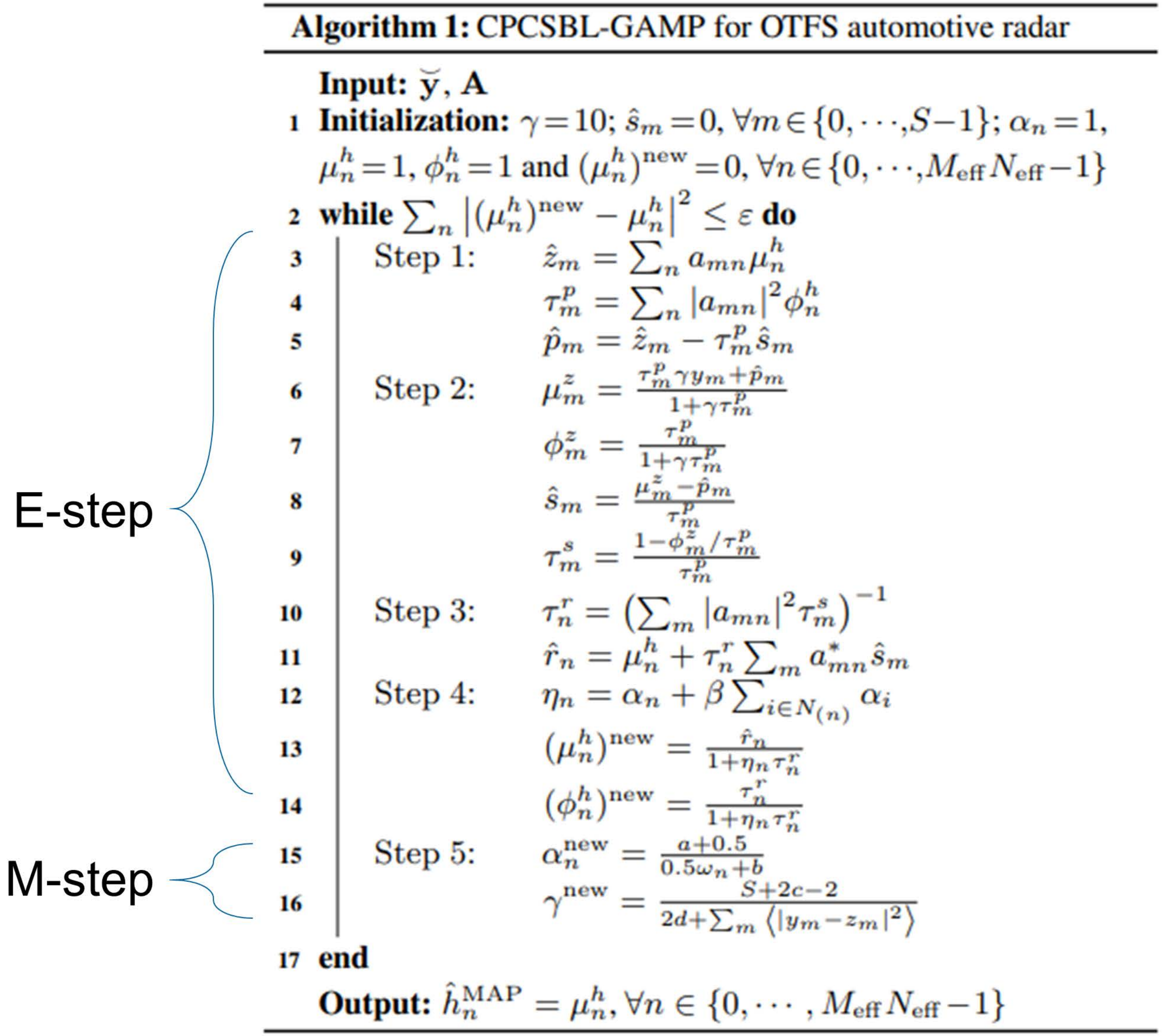
MAP Estimation of Radar Channel Vector

$$\hat{\mathbf{h}}_{\text{MAP}} = \boldsymbol{\mu}^* \tag{12}$$

the last iteration

- Low-complexity GAMP method
  - ✓ Approximation of the posterior distribution of $\mathbf{h}$
  - ✓ Extended to the complex-value domain

CPCSBL-GAMP



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Algorithm 1: CPCSBL-GAMP for OTFS automotive radar
Input: $\breve{\mathbf{y}}$, $\mathbf{A}$
Initialization: $\gamma=10$; $\hat{s}_m=0, \forall m\in\{0,\cdots,S-1\}$; $\alpha_n=1$, $\mu_n^h=1$, $\phi_n^h=1$ and $(\mu_n^h)^{\mathrm{new}}=0, \forall n\in\{0,\cdots,M_{\mathrm{eff}}N_{\mathrm{eff}}-1\}$
while $\sum_n \left|(\mu_n^h)^{\mathrm{new}}-\mu_n^h\right|^2 \le \varepsilon$ do
    Step 1:  $\hat{z}_m = \sum_n a_{mn}\mu_n^h$
             $\tau_m^p = \sum_n |a_{mn}|^2\phi_n^h$
             $\hat{p}_m = \hat{z}_m - \tau_m^p\hat{s}_m$
    Step 2:  $\mu_m^z = \frac{\tau_m^p\gamma y_m+\hat{p}_m}{1+\gamma\tau_m^p}$
             $\phi_m^z = \frac{\tau_m^p}{1+\gamma\tau_m^p}$
             $\hat{s}_m = \frac{\mu_m^z-\hat{p}_m}{\tau_m^p}$
             $\tau_m^s = \frac{1-\phi_m^z/\tau_m^p}{\tau_m^p}$
    Step 3:  $\tau_n^r = \left(\sum_m |a_{mn}|^2\tau_m^s\right)^{-1}$
             $\hat{r}_n = \mu_n^h + \tau_n^r\sum_m a_{mn}^*\hat{s}_m$
    Step 4:  $\eta_n = \alpha_n + \beta\sum_{i\in N_{(n)}}\alpha_i$
             $(\mu_n^h)^{\mathrm{new}} = \frac{\hat{r}_n}{1+\eta_n\tau_n^r}$
             $(\phi_n^h)^{\mathrm{new}} = \frac{\tau_n^r}{1+\eta_n\tau_n^r}$
    Step 5:  $\alpha_n^{\mathrm{new}} = \frac{a+0.5}{0.5\omega_n+b}$
             $\gamma^{\mathrm{new}} = \frac{S+2c-2}{2d+\sum_m\left\langle|y_m-z_m|^2\right\rangle}$
end
Output: $\hat{h}_n^{\mathrm{MAP}} = \mu_n^h, \forall n\in\{0,\cdots,M_{\mathrm{eff}}N_{\mathrm{eff}}-1\}$
```

E-step

M-step

Complexity Comparison:

Proposed: $O(N_{\mathrm{iter}}SM_{\mathrm{eff}}N_{\mathrm{eff}})$

Matched filter: $O(M^2N^2)$

SIMULATION RESULTS

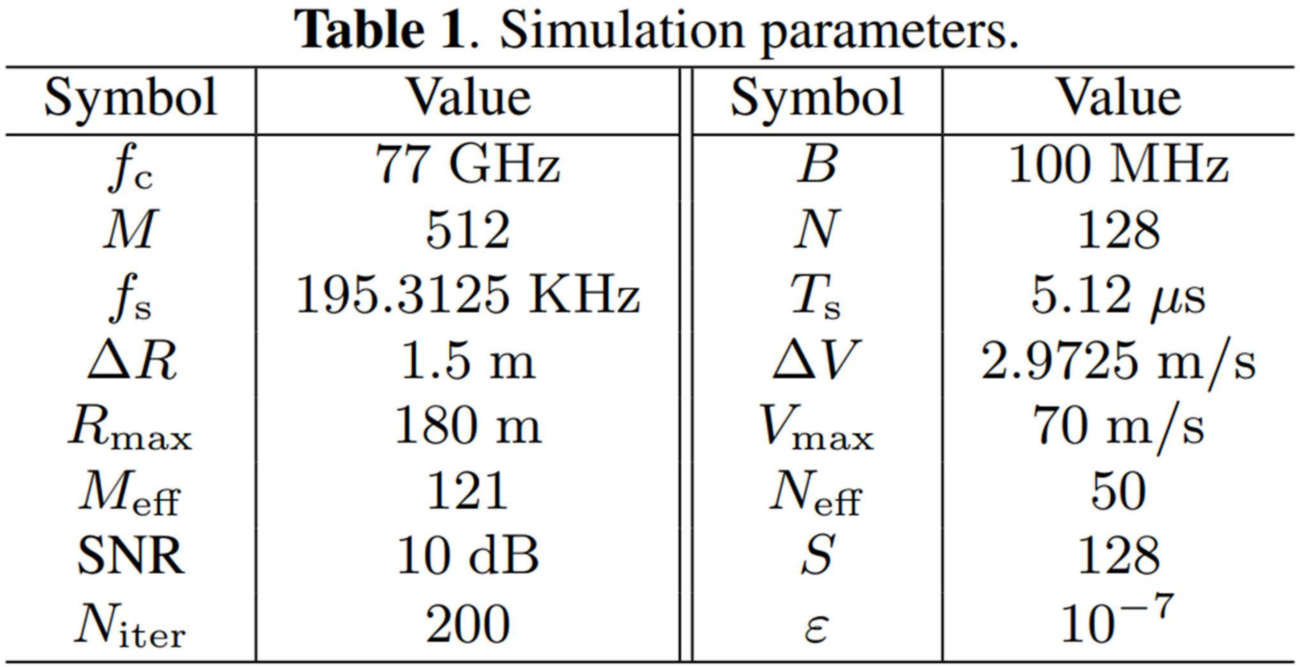

Table 1. Simulation parameters.

Symbol	Value	Symbol	Value
f_c	77 GHz	B	100 MHz
M	512	N	128
f_s	195.3125 KHz	T_s	5.12 μs
ΔR	1.5 m	ΔV	2.9725 m/s
$R_{\max}$	180 m	$V_{\max}$	70 m/s
M_{eff}	121	N_{eff}	50
SNR	10 dB	S	128
N_{iter}	200	ε	10^{-7}

$a = 0.1,\ b = 10^{-10}$

$c = 1,\ d = 10^{-10}$

$\beta = 1$

QPSK modulation

SIMULATION RESULTS

Target with $R = 90\ \mathrm{m}, V = 59.45\ \mathrm{m/s}$

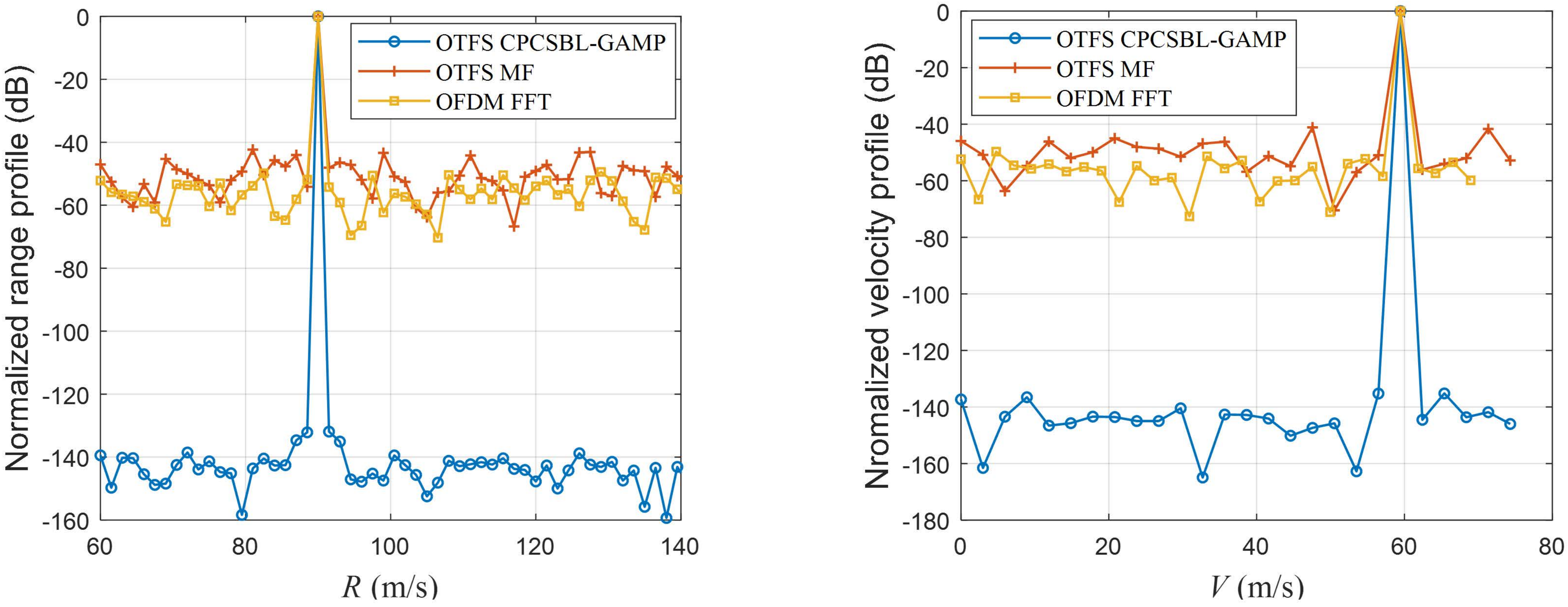

Fig. 4. Target detection results using different methods. (a) Range profile. (b) Relative velocity profile.

SIMULATION RESULTS

Target at $R = 90 \text{ m}$ but with varying relative velocity

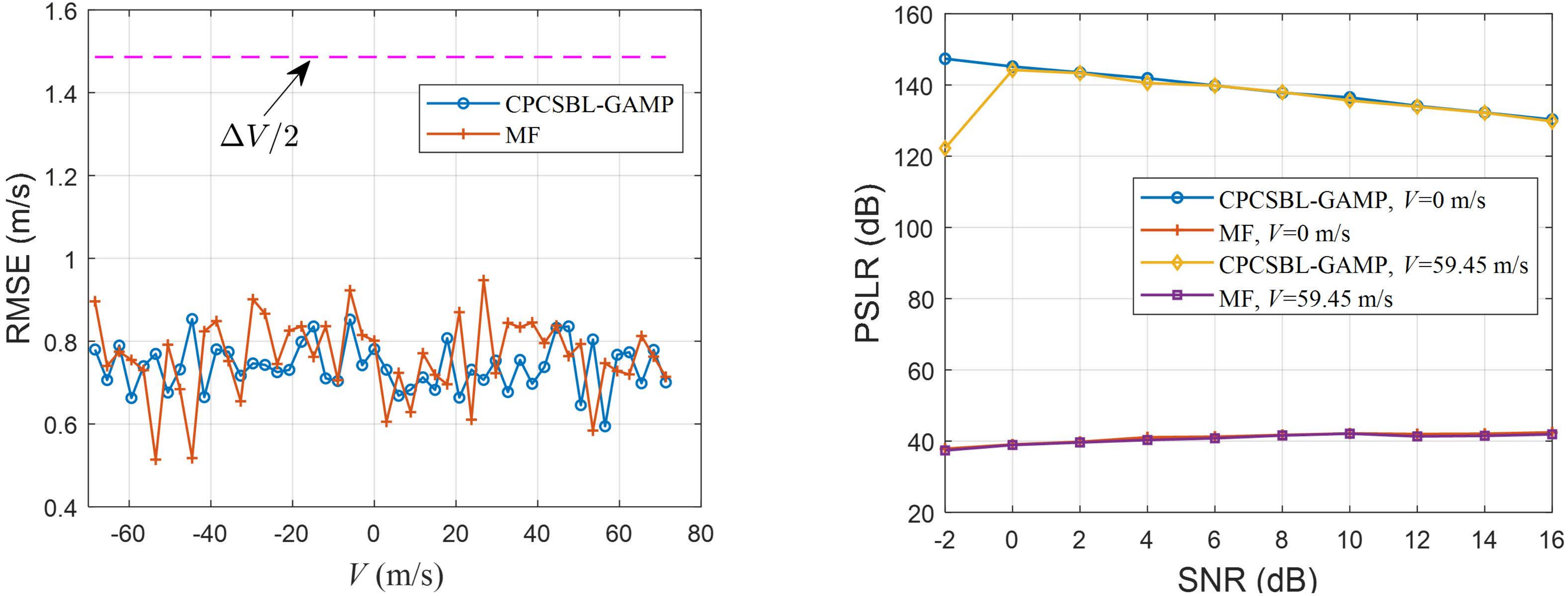

Fig. 5. Performance evaluation using objective metrics. (a) Performance evaluation using objective metrics. (b) PSLR versus SNR and relative velocity.

CONCLUSION

- Some prior information is utilized to reduce the dimension of radar channel vector.
- The structural sparsity of radar channel in delay-Doppler domain is observed.
- A low-complexity CPCSBL-GAMP algorithm is designed to obtain MAP estimation of radar channel vector.
- The algorithmic performance of the proposed scheme in joint range and velocity estimation is confirmed by simulation results.

Thank you!

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