Optimal Questionnaires for Screening of Strategic Agents

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Background

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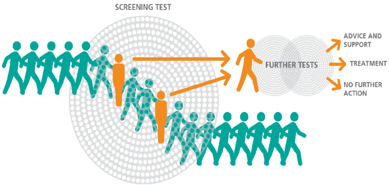

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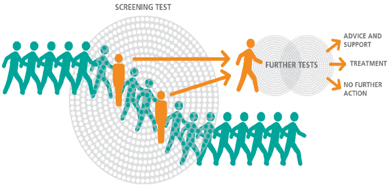

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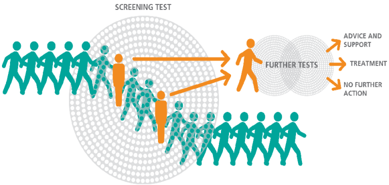

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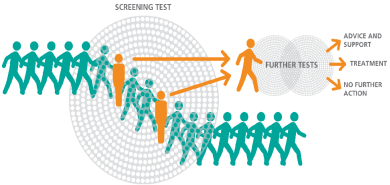

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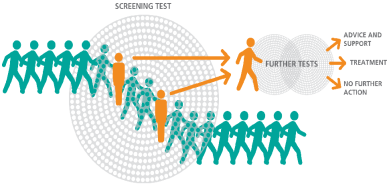

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- However, we still need to screen *all* the passengers
- Alternatively, one can possibly identify susceptible travellers from their travel history
- However, people have a tendency to misreport their true travel history, due to stigma, inconvenience due to testing and quarantine protocols

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- What should the health inspector do?
- How does this number grow with the length of travel history?

Model

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- Each of the travellers have a classification called the *type* which determines their degree of honesty. We denote the type as λ, $\lambda \in \Lambda$ and $|\Lambda| < \infty$.

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- We term the health inspector as the receiver and the travellers as senders

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- The receiver maps the response of the sender as $g_n : \mathcal{C}^n \to \mathcal{X}^n$
- We show that it is sufficient to choose a strategy g_n where $g_n(x) = x$ for all $x \in \mathcal{C}^n$.

Objectives of the senders and the receiver

- Let
$$\mathcal{D}(g_n, s_n^{\lambda}) := \left\{x \in \mathcal{X}^n \mid g_n \circ s_n^{\lambda}(x) = x\right\}$$
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- When the actual history is x and the history recovered by the receiver is $\widehat{x}$, the utility obtained by the sender λ is $\mathscr{U}_n(\widehat{x}, x, \lambda)$ where $\mathscr{U}_n : \mathcal{X}^n \times \mathcal{X}^n \times \Lambda \to \mathbb{R}$ is defined as
$$\mathscr{U}_n(\widehat{x}, x, \lambda) = \frac{1}{n}\sum_{i=1}^{n} \mathscr{U}(\widehat{x}_i, x_i, \lambda) \qquad \forall\ x, \widehat{x} \in \mathcal{X}^n,$$
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with $\mathscr{U} : \mathcal{X} \times \mathcal{X} \times \Lambda \to \mathbb{R}$ being the single letter utility.
- The receiver aims to maximize the average number of travel histories it can recover, while the senders try to maximize their respective utilities.

Formulation of problem as a Stackelberg game

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In a Stackelberg equilibrium, the strategy of the receiver is given as

$$g_n^* \in \arg\max_{g_n} \sum_{\lambda \in \Lambda} \mathbb{P}(\lambda) \left(\min_{s_n^\lambda \in \mathscr{B}(g_n, \lambda)} |\mathcal{D}(g_n, s_n^\lambda)| \right),$$

where the best response set of the sender λ is $\mathscr{B}(g_n, \lambda)$, where

$$\mathscr{B}(g_n, \lambda) = \Big\{ s_n^\lambda : \mathcal{X}^n \to \mathcal{X}^n \mid \mathscr{U}_n(g_n \circ s_n^\lambda(x), x, \lambda) \geq \mathscr{U}_n(g_n \circ s_n'^\lambda(x), x, \lambda) \quad \forall\, x \in \mathcal{X}^n, \forall\, s_n'^\lambda \Big\}.$$

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- The receiver does not have control over the choice of the best response.
- Assuming a pessimistic receiver, we incorporate the minimization over $\mathscr{B}(g_n, \lambda)$.

Related work

- Related to the general problem of communication between sender and receiver with misaligned objectives studied in
  - game theory [CS82, Bat02, SYG15]
  - control theory [SAB19, FTL16, ALB16],
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  - economics [KG11, BM19].
- They consider a neutral perspective or the viewpoint of the sender
- In [VK20a, VK20b], we studied a related information extraction problem where the receiver tried to achieve asymptotically vanishing probability of error.
- In [VK20d, VK20c] we studied an information extraction problem with a single sender and showed that the maximum rate is bounded above by the Shannon capacity of a certain graph.

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- In this paper, we characterize the set of sequences of histories that can be recovered perfectly by the receiver
- We characterize the optimal check-list to be chosen by the receiver
- We also give bounds on the rate of information extraction, for finite n as well as asymptotic

Definitions

Definition (*Sender graph*)

The sender graph for a sender of type λ, denoted as $G_\lambda^n = (\mathcal{X}^n, E)$, is a graph where $(x, y) \in E$ if either $\mathscr{U}_n(x, x, \lambda) \leq \mathscr{U}_n(y, x, \lambda)$ or $\mathscr{U}_n(y, y, \lambda) \leq \mathscr{U}_n(x, y, \lambda)$.

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Definition (λ-partition of a set)

Let $I^n \subseteq \mathcal{X}^n$ be any set. For $\lambda \in \Lambda$, the λ-partition of the set I^n is defined as

$$\bar{I}^n_\lambda := \{x \in I^n : \mathscr{U}_n(x, x, \lambda) > \mathscr{U}_n(y, x, \lambda) \ \forall \ y \in I^n, y \neq x\}.$$

The collection of all λ-partitions of the set I^n is denoted as $\mathcal{F}(I^n) = \{\bar{I}^n_\lambda\}_{\lambda \in \Lambda}$.

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- Thus, the set $\bar{I}^n_\lambda$ is the largest subset of I^n which is an independent set in G^n_λ.

Rate of information extraction

- We now define the notion of rate which determines the growth of the perfectly recovered sequences with n.

Definition (Rate of information extraction)

For a strategy g_n of the receiver, define $\mathcal{D}^*(g_n)$

$$\mathcal{D}^*(g_n) = \sum_{\lambda \in \Lambda} \mathbb{P}(\lambda) \left(\min_{s_n^\lambda \in \mathscr{B}(g_n, \lambda)} |\mathcal{D}(g_n, s_n^\lambda)| \right).$$

Then, the rate of information extraction is defined as

$$R(g_n) = (\mathcal{D}^*(g_n))^{1/n}.$$

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Theorem

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Corollary

Let $n \in \mathbb{N}$ be fixed. For a sender type $\lambda \in \Lambda$, let $\alpha(G_\lambda^n)$ be the independence number of the graph G_λ^n. Then, for all Stackelberg equilibrium strategies g_n^*,

$$\alpha\left(\cup_\lambda G_\lambda^n\right)^{1/n} \leq R(g_n^*) \leq \left(\sum_{\lambda \in \Lambda} \mathbb{P}_\Lambda(\lambda) \alpha(G_\lambda^n) \right)^{1/n}.$$

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$$\alpha\left(\cup_\lambda G_\lambda\right) \leq \limsup_{n\to\infty} R(g_n^*) \leq \Xi(\mathscr{U},\lambda^*),$$

where $\lambda^* = \max_{\lambda\in\Lambda} \Xi(\mathscr{U},\lambda)$, $\Xi(\mathscr{U},\lambda) := \lim_{n\to\infty} \alpha(G_\lambda^n)^{1/n}$.

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- The proof of existence of $\Xi(\mathscr{U},\lambda^*)$ can be found in [VK20c].

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- Interesting observations even for a simple type of questionnaire
- We characterized the optimal questionnaires for the receiver
- We also derived bounds on the rate of information extraction, for finite n and for n growing to infinity

References I

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Thank You

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