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Joint Optimization of Spectrally Co-Existing Multi-Carrier Radar and Communication Systems in Cluttered Environment

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Background

- Spectrum sharing between radar and communication (comm) systems using overlapping frequency band has attracted much attention in recent years
  - There is an increasing pressure from wireless sector to release radar spectrum for shared access
  - Radar is used in increasingly more military and civilian applications, i.e., car cruise control and remote sensing
- Our previous work
  - Formulated a joint power allocation-based spectrum sharing framework by considering metrics for both systems
  - Proposed an alternating direction optimization approach
- Limitations of previous work
  - Multi-path and clutter were neglected to ease the development of proposed solutions
  - The alternating procedure is computationally intensive
- Main contribution of this work
  - Develop a general signal model for the coexistence of multicarrier radar and comm systems in cluttered environments
  - Propose a new non-alternating algorithm

F. Wang and H. Li, "Joint power allocation for multicarrier radar and communication coexistence," 2020 *IEEE International Radar Conference (RADAR), Washington, DC, USA, May 2020, pp. 141-145.*

Signal Model

- A multicarrier radar system with N subcarriers coexists with a comm system in the same frequency band
- Transmitted comm signal

$$x_c(t) = q_c(t)\sum_{n=1}^{N} d_n\sqrt{p_{c,n}}e^{j2\pi(f_c+n\Delta f_c)t} \triangleq \sum_{n=1}^{N} x_{c,n}(t)$$

$q_c(t)$: comm waveform
$p_{c,n}$: power allocated to n-th comm subcarrier
d_n : comm symbol carried by n-th subcarrier
f_c : carrier frequency; Δ_f : subcarrier spacing

- Transmitted radar signal

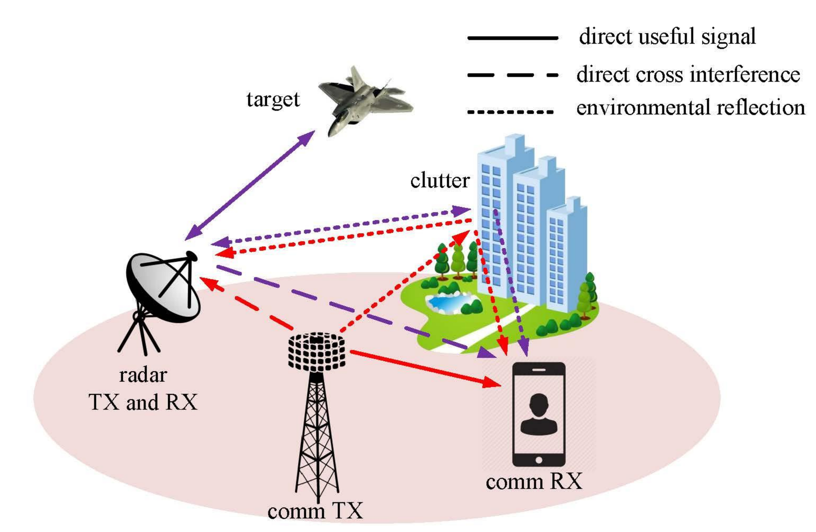
$$x_r(t) = q_r(t)\sum_{n=1}^{N}\sqrt{p_{r,n}}e^{j2\pi(f_c+n\Delta f)t} \triangleq \sum_{n=1}^{N} x_{r,n}(t)$$

$q_r(t)$: radar waveform
$p_{r,n}$: power allocated to n-th radar subcarrier

- Received signal at comm RX

$$y_{c,n}(t) = \sum_{k=1}^{K_{cc}}\alpha'_{cc,n,k}x_{c,n}(t-\tau_{cc,k}) + \sum_{k=1}^{K_{rc}}\beta'_{rc,n,k}x_{r,n}(t-\tau_{rc,k}) + w'_{c,n}(t)$$

$\alpha'_{cc,n,k},\tau_{cc,k}$: coefficient and delay of k-th comm path
K_{cc} : total number of comm paths
$\beta'_{rc,n,k},\tau_{rc,k}$: coefficient and delay from radar TX to comm RX due to k-th clutter scatterer
K_{rc} : total number of clutter scatterers



A radar and comm coexistence scenario in a cluttered environment

- After down-conversion, $y_{c,n}(t)$ passes through a matched filter (MF) matched to the LOS comm waveform $q_c(t-\tau_{cc,1})$ and is sampled at the symbol rate

$$y_{c,n} = \alpha_{cc,n}d_n\sqrt{p_{c,n}} + \beta_{rc,n}\sqrt{p_{r,n}} + w_{c,n}$$

$$\alpha_{cc,n} = \int_T \sum_{k=1}^{K_{cc}}\alpha'_{cc,n,k}q_c(t-\tau_{cc,k})q_c^*(t-\tau_{cc,1})dt$$

$$\beta_{rc,n} = \int_T \sum_{k=1}^{K_{rc}}\beta'_{rc,n,k}q_r(t-\tau_{rc,k})q_c^*(t-\tau_{cc,1})dt$$

- A moving target is located at rang R from radar with a roundtrip delay τ_{rr}

- Received signal at radar RX

$$y_{r,n}(t) = \bar{\alpha}\alpha'_{rr,n}x_{r,n}(\varepsilon(t-\tau_{rr})) + \sum_{k=1}^{K_{rr}}\beta'_{rr,n,k}x_{r,n}(t-\bar{\tau}_{rr,k}) + \sum_{k=1}^{K_{cr}}\beta'_{cr,n,k}x_{c,n}(t-\bar{\tau}_{cr,k}) + w'_{r,n}(t)$$

$\bar{\alpha}$: radar cross-section (RCS)
$\alpha'_{rr,n}$: channel coefficient of target path
ε : target Doppler scaling factor
$\beta'_{rr,n,k},\bar{\tau}_{rr,k},K_{rr}$: coefficient, delay, and number of clutter scatterer due to radar illumination
$\beta'_{cr,n,k},\bar{\tau}_{cr,k},K_{cr}$: parameters due to comm illumination
$w'_{r,n}(t)$: additive channel noise

- $y_{r,n}(t)$ is down-converted, Doppler compensated, filtered by a MF matched to the radar waveform $q_r(t-\tau_{rr})$ and sampled at pulse rate

$$y_{r,n} = \alpha_{rr,n}\sqrt{p_{r,n}} + \beta_{rr,n}\sqrt{p_{r,n}} + \beta_{cr,n}d_n\sqrt{p_{c,n}} + w_{r,n}$$

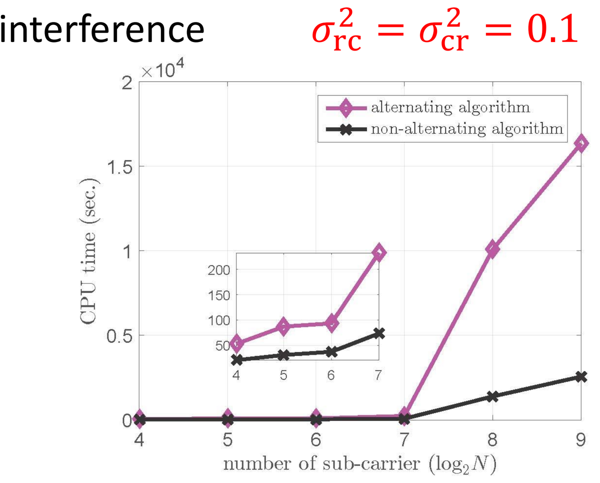
$$\alpha_{rr,n} = \bar{\alpha}\int_T \alpha'_{rr,n}q_r(\varepsilon(t-\tau_{rr}))q_r^*(\varepsilon(t-\tau_{rr}))dt$$

$$\beta_{rr,n} = \int_T \sum_{k=1}^{K_{rr}}\beta'_{rr,n,k}q_r(t-\bar{\tau}_{rr,k})q_r^*(\varepsilon(t-\tau_{rr}))dt$$

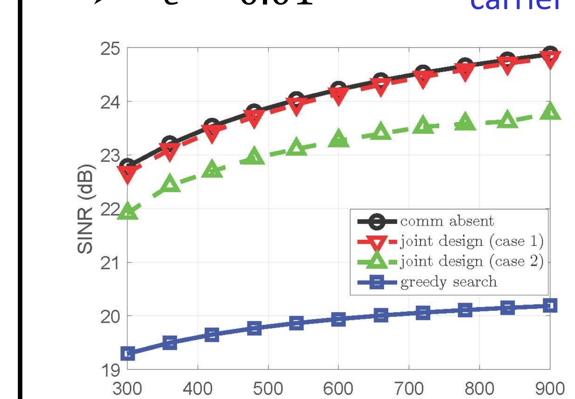
$$\beta_{cr,n} = \int_T \sum_{k=1}^{K_{cr}}\beta'_{cr,n,k}q_c(t-\bar{\tau}_{cr,k})q_r^*(\varepsilon(t-\tau_{rr}))dt$$

Numerical Results

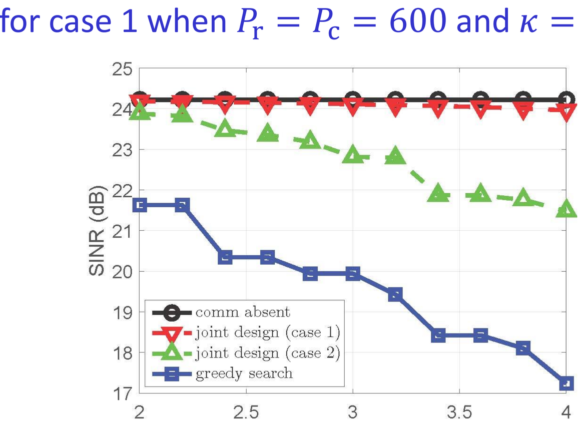
- Three system configurations are considered for comparison
  - **comm absent**: Maximize the radar SINR without any interference from the comm system
  - **joint design**: The proposed joint design and solved with the non-alternating algorithm
  - **greedy search**: Subcarrier allocation-based method that gives comm system its best subcarriers to meet throughput constraint and assigns the rest to radar
- The sub-carrier channel coefficients $\alpha_{rr,n}$, $\alpha_{cc,n}$, $\beta_{rc,n}$, $\beta_{rr,n}$, and $\beta_{cr,n}$ are generated with Gaussian distribution $\mathcal{CN}(0,\sigma_{rr}^2)$, $\mathcal{CN}(0,\sigma_{cc}^2)$, $\mathcal{CN}(0,\sigma_{rc}^2)$, $\mathcal{CN}(0,\sigma^2)$, and $\mathcal{CN}(0,\sigma_{cr}^2)$
- Two scenarios of cross channel strength
  - **case 1**: week cross interference $\sigma_{rc}^2 = \sigma_{cr}^2 = 0.01$
  - **case 2**: strong cross interference $\sigma_{rc}^2 = \sigma_{cr}^2 = 0.1$
- Noise variances
  - $\sigma_r^2 = \sigma_c^2 = 1$
- Other parameters
  - $\sigma_{rr}^2 = \sigma_{cc}^2 = 1$
  - $\sigma^2 = 0.05$
  - $N = 16$
  - $\epsilon = 0.01$



Computer simulation time versus number of sub-carrier for case 1 when $P_r = P_c = 600$ and $\kappa = 1.5$



Radar output SINR versus total radar power when $P_c = 600$ and $\kappa = 2.5$



Radar output SINR versus comm throughput when $P_r = P_c = 600$

Problem Formulation and Proposed Approach

- The problem of interest is to jointly optimize the radar and comm transmission power allocated to each sub-carrier by maximizing the radar output SINR while maintaining a minimum comm throughput constraint, along with total power constraints and sub-channel peak power constraints

$$\max_{p_{r,n},p_{c,n},n=1,\cdots,N}\sum_{n=1}^{N}\frac{\gamma_{rr,n}p_{r,n}}{\eta_{rr,n}p_{r,n}+\eta_{cr,n}p_{c,n}+1}$$

$$\text{s.t.}\ \sum_{n=1}^{N}p_{r,n}\le P_r,\ \sum_{n=1}^{N}p_{c,n}\le P_c$$

$$0\le p_{r,n}\le\xi_r,\ 0\le p_{c,n}\le\xi_c,\ \forall n$$

$$\sum_{n=1}^{N}\log_2\left(1+\frac{\gamma_{cc,n}p_{c,n}}{\eta_{rc,n}p_{r,n}+1}\right)\ge\kappa$$

$$\gamma_{rr,n}=\frac{\mathbb{E}\{|\alpha_{rr,n}|^2\}}{\sigma_r^2},\quad \eta_{rr,n}=\frac{\mathbb{E}\{|\beta_{rr,n}|^2\}}{\sigma_r^2},\quad \eta_{cr,n}=\frac{\mathbb{E}\{|\beta_{cr,n}|^2\}}{\sigma_r^2},\quad \gamma_{cc,n}=\frac{\mathbb{E}\{|\alpha_{cc,n}|^2\}}{\sigma_c^2},\quad \eta_{rc,n}=\frac{\mathbb{E}\{|\alpha_{rc,n}|^2\}}{\sigma_c^2}$$

P_r, P_c : total transmission power budget
ξ_r, ξ_c : subchannel peak power threhold
κ : comm throughput threhold

- The above problem may be solved by employing an iteratively alternating optimization procedure, i.e., iteratively solve with respect to $p_{r,n}$ while keeping $p_{c,n}$ fixed, and vice versa, until convergence is reached
  - It is computationally intensive and does not guarantee convergence
  - Signal-dependent clutter makes the optimization problem significantly more challenging even with fixed $p_{c,n}$
- To address these challenges, we consider a different approach. Specifically, it can be reformulated by combining the radar and comm power variables into a single stacked variable

$$\max_{\mathbf{p}_n,n=1,\cdots,N}\sum_{n=1}^{N}\frac{\mathbf{r}_{rr,n}^T\mathbf{p}_n}{\mathbf{r}_{cr,n}^T\mathbf{p}_n+1}$$

$$\text{s.t.}\ \sum_{n=1}^{N}\log_2\left(1+\frac{\mathbf{r}_{cc,n}^T\mathbf{p}_n}{\mathbf{r}_{rc,n}^T\mathbf{p}_n+1}\right)\ge\kappa$$

$$\mathbf{r}_{rc,n}=[\eta_{rc,n},\ 0]^T,\quad \mathbf{r}_{cr,n}=[\eta_{rr,n},\ \eta_{cr,n}]^T,\quad \mathbf{r}_{rr,n}=[\gamma_{rr,n},\ 0]^T,\quad \mathbf{r}_{cc,n}=[0,\ \gamma_{cc,n}]^T,\quad \mathbf{p}_n=[p_{r,n},\ p_{c,n}]^T$$

linear power constraints are ignored here

- The above problem is non-convex. To solve it, we can reformulate the objective function and employ convex relaxation for the throughput constraint
- By introducing a set of slack variables λ_n and applying quadratic transform, the equivalent form of cost function

$$\sum_{n=1}^{N}\left(2\lambda_n\sqrt{\mathbf{r}_{rr,n}^T\mathbf{p}_n}-\lambda_n^2(\mathbf{r}_{cr,n}^T\mathbf{p}_n+1)\right)$$

- Let $\tilde{\boldsymbol{p}}_n^{(l-1)}$ denote solutions obtained from $(l-1)$-st iteration, $\lambda_n^{(l)}$ can be updated by solving

$$\max_{\lambda_n,N=1,\cdots,N}\sum_{n=1}^{N}\left(2\lambda_n\sqrt{\mathbf{r}_{rr,n}^T\tilde{\mathbf{p}}_n^{(\ell-1)}}-\lambda_n^2(\mathbf{r}_{cr,n}^T\tilde{\mathbf{p}}_n^{(\ell-1)}+1)\right)$$

- It has a closed-form solution

$$\lambda_n^{(\ell)}=\sqrt{\mathbf{r}_{rr,n}^T\tilde{\mathbf{p}}_n^{(\ell-1)}}/(\mathbf{r}_{cr,n}^T\tilde{\mathbf{p}}_n^{(\ell-1)}+1)$$

- Next, $\tilde{\boldsymbol{p}}_n^{(l)}$ can be obtained by solving

$$\max_{\mathbf{p}_n,n=1,\cdots,N}\sum_{n=1}^{N}\left(2\lambda_n\sqrt{\mathbf{r}_{rr,n}^T\mathbf{p}_n}-\lambda_n^2(\mathbf{r}_{cr,n}^T\mathbf{p}_n+1)\right)$$

$$\text{s.t.}\ \sum_{n=1}^{N}\log_2\left(1+\frac{\mathbf{r}_{cc,n}^T\mathbf{p}_n}{\mathbf{r}_{rc,n}^T\mathbf{p}_n+1}\right)\ge\kappa$$

- The above problem is non-convex due to the non-convex throughput constraint, which is relaxed with an inner iteration of sequential convex programming (SCP) procedure
- Let $\hat{\boldsymbol{P}}_n^{(l_s-1)}$ denote the power vector from the (l_s-1)-st SCP inner iteration, the above constraint can be relaxed to a convex set as

$$\sum_{n=1}^{N}\log_2(\mathbf{r}_{cc,n}^T\mathbf{p}_n+\mathbf{r}_{rc,n}^T\mathbf{p}_n+1)-G(\mathbf{p}_n,\hat{\mathbf{p}}_n^{(\ell_s-1)})\ge\kappa$$

$$G(\mathbf{p}_n,\hat{\mathbf{p}}_n^{(\ell_s-1)})\triangleq\log_2(\mathbf{r}_{rc,n}^T\hat{\mathbf{p}}_n^{(\ell_s-1)}+1)+\frac{\mathbf{r}_{rc,n}^T(\mathbf{p}_n-\hat{\mathbf{p}}_n^{(\ell_s-1)})}{\ln 2(\mathbf{r}_{rc,n}^T\hat{\mathbf{p}}_n^{(\ell_s-1)}+1)}$$

- Thus, inside each inner iteration, the following convex problem is solved to obtain $\hat{\boldsymbol{p}}_n^{(l_s)}$

$$\max_{\mathbf{p}_n,n=1,\cdots,N}\sum_{n=1}^{N}\left(2\lambda_n\sqrt{\mathbf{r}_{rr,n}^T\mathbf{p}_n}-\lambda_n^2(\mathbf{r}_{cr,n}^T\mathbf{p}_n+1)\right)$$

$$\text{s.t.}\ \sum_{n=1}^{N}\log_2(\mathbf{r}_{cc,n}^T\mathbf{p}_n+\mathbf{r}_{rc,n}^T\mathbf{p}_n+1)-G(\mathbf{p}_n,\hat{\mathbf{p}}_n^{(\ell_s-1)})\ge\kappa$$

- The iteration ends when the difference of the cost function over two adjacent iterations is smaller than a tolerance ϵ

- Computational complexity
  - Depend on the number of quadratic transform iterations L as well as the number of SCP iterations L_s
  - Overall complexity is $\mathcal{O}(LL_sN^{3.5})$ when an interior-point method is employed to solve a convex problem
- Feasibility
  - The feasibility of the problem depends on if the maximum achievable throughput $C_{\max}$ under power constraints is larger than the throughput threshold κ
  - $C_{\max}$ is achieved when the radar system is absent while the comm system uses all subcarriers to maximize its throughput
- Initialization
  - A simple method of initialization is to use power constraints to generate a set of initial powers
  - A better way that considers the throughput constraint is an allocation-based greedy search method

Concluding Remarks

- Derived a general signal model for the coexistence of multi-carrier radar and comm systems with spectrum sharing in cluttered environments
- Introduced a joint design for the coexistence of multi-carrier radar and comm systems
  - Maximize the radar output SINR while meeting a comm throughput requirement along with power constraints
  - Proposed a non-alternating optimization algorithm along with SCP technique to solve the highly non-convex problem
- Numerical results show that
  - The proposed non-alternating algorithm is computationally more efficient
  - The joint design framework outperforms other heuristic solutions, i.e., greedy search