Error estimate in second-order continuous-time sigma-delta modulators

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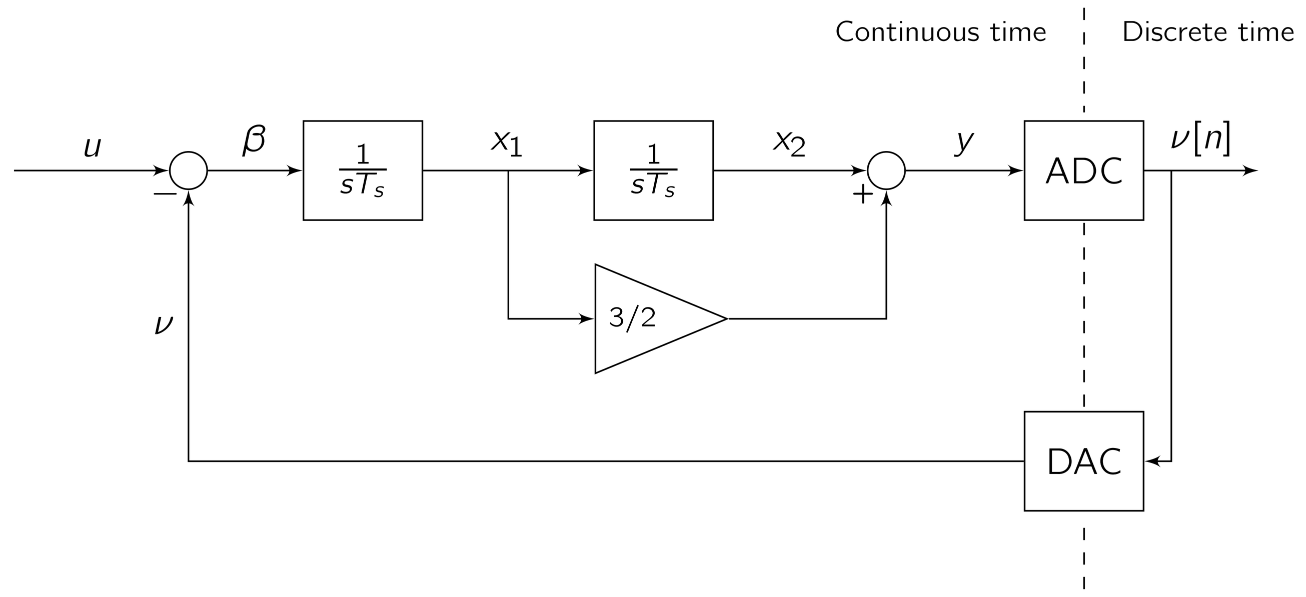
Context

Continuous-Time Sigma-Delta (CT-ΣΔ) modulators are oversampling 1-bit Analog-to-Digital converters that may provide higher sampling rates and lower power consumption than their discrete counterpart **[1]**. Whereas approximation errors are established for high-order discrete time ΣΔ modulators **[2]**, theoretical analysis of the error between the filtered output and the input remain scarce for CT-ΣΔ modulators. We developped a general framework to study this error: under regularity assumptions on the input and the filtering kernel, we prove for a second-order CT-ΣΔ that the **error estimate is in** $\mathbf{o(1/N^2)}$, where N is the oversampling ratio.

Second-order CT-ΣΔ modulator

Example of second-order CT-ΣΔ [3]

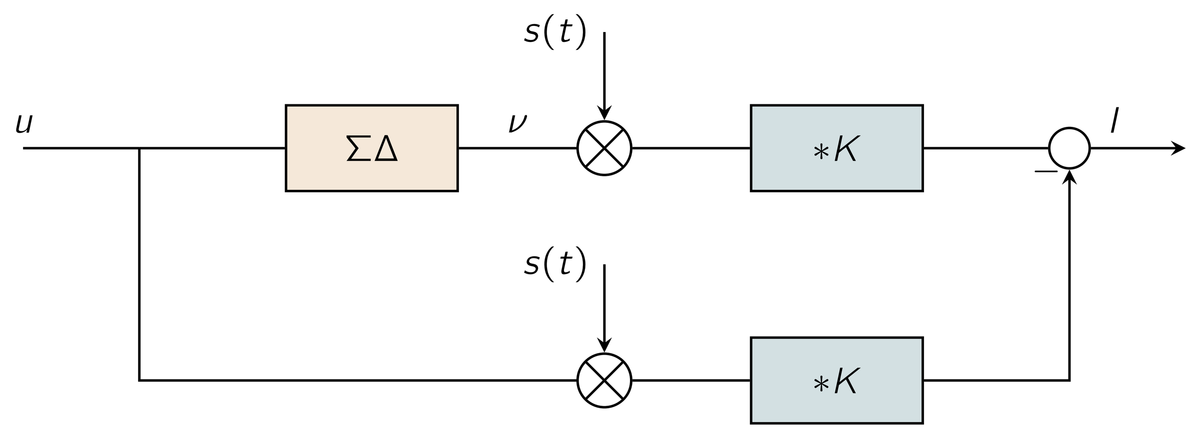


- u: input
- $x_{1,2}$: internal states
- $\nu[n]$: modulator output
- T_s: modulator sample time

Principle: The ΣΔ modulator produces a bitstream $\nu[n] \in \{0,1\}$ which is constant on every $[nT_s, (n+1)T_s)$, $k \in \mathbb{N}$. This bitstream digitally encodes the input u (varying in a timescale of ε), and, with an adequate filter K, u is estimated through $\nu * K$.

Objective: Finding an estimate of the difference $(u-\nu)*K$ as a function of the **oversampling ratio** $N := \varepsilon/T_s$. Also, for an exogenous signal s, deducing an estimate for $(u-\nu)s*K$.

Main result



- ΣΔ: Sigma-Delta modulator
- K: convolution kernel
- s: exogenous signal

Theorem: Under some regularity assumptions on s and K, the filtered input filtered output difference $I(t)$ (see diagram above) satisfies

$$I(t) := \int_{-\infty}^{+\infty} \beta(N\sigma)s(\sigma)K^k(t-\sigma)\,d\sigma = o(1/N^2)$$

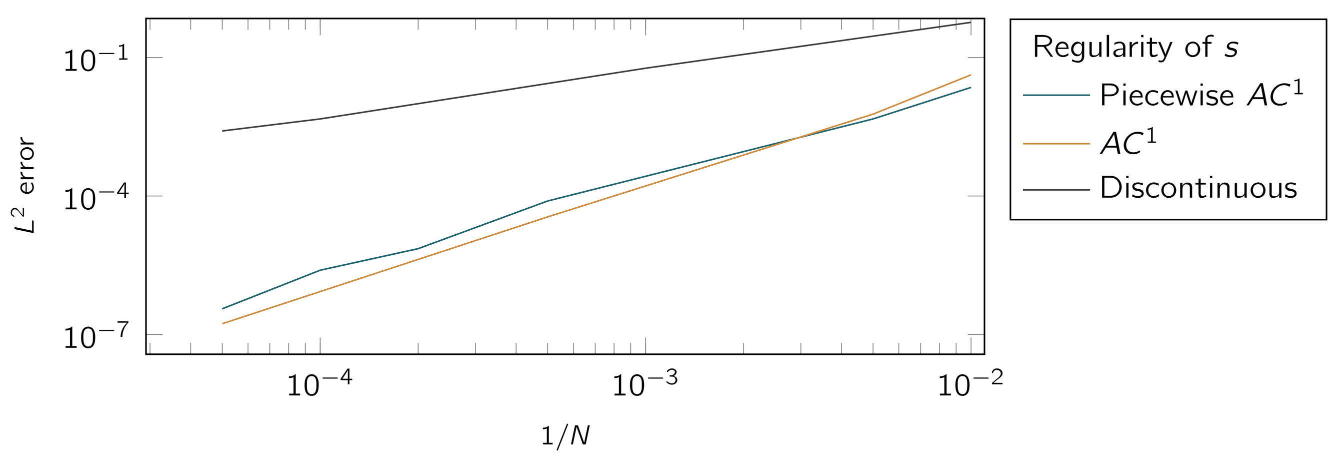
More precisely: if s is differentiable and its derivative s' is absolutely continuous (resp. piecewise absolutely continuous), then s is said to be AC^1 (resp. piecewise AC^1), and $I(t) = o(1/N^2)$ (resp. $O(1/N^2)$).

The *outline of the proof* is twofold: first we prove that the input-output difference β has a zero-mean primitive $\beta^{(-1)}$, which has itself a zero-mean primitive $\beta^{(-2)}$; then we show that for this very specific type of function β, based on a **generalized Riemann-Lebesgue lemma**, the estimate on $I(t)$ holds.

Numerical results

Asymptotic behavior of the L2-norm $\|I\|_2$ of the error $I(t)$ with respect to the oversampling ratio N for various signals s (resp. piecewise AC^1, AC^1 and discontinuous signals).

Selected kernel: third-order B-spline $K = \varepsilon^{-3} \times 1_{[0,\varepsilon]} * 1_{[0,\varepsilon]} * 1_{[0,\varepsilon]}$



Slope = approximation order: 1 (discontinuous), 2 (piecewise AC^1), 2.3 (AC^1)

What next?

- Extension to **various typologies** of ΣΔ modulator (MASH, CRFB, CIFB, etc.)
- Generalization to **nth-order CT-ΣΔ modulator**

Selected bibliography

[1] Maurits Ortmanns and Friedel Gerfers, Continuous- time sigma-delta A/D conversion, fundamentals, performance limits and robust implementations. Berlin: Springer, vol. 21, Springer, 01 2006.

[2] Ingrid Daubechies and Ron DeVore, "Approximating a bandlimited function using very coarsely quantized data: A family of stable sigma-delta modulators of arbitrary order," Annals of Mathematics, vol. 158, no. 2, pp. 679–710, 2003.

[3] Richard Schreier and Gabor C Temes, Understanding delta-sigma data converters, Wiley, NY, 2005.