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Globally Optimal Beamforming for Rate Splitting Multiple Access

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Summary

- Beamforming for rate splitting multiple access (RSMA): Globally optimal solution w.r.t. weighted sum rate and energy efficiency
- Also solves SDMA, OMA, 2-user NOMA, and multicast beamforming
- SoA is based on Branch-and-Bound (BB) + SOCP [1]
- Complexity is in feasible set → convergence issues with BB
- BB + Successive Incumbent Transcending Scheme [2] leads to improved numerical stability and faster convergence over state of the art

Contributions

- Successive incumbent transcending BB algorithm for joint unicast & multicast BF
  - Numerically stable & fast convergence (compared to SoA)
  - Special cases: 2-User NOMA, OMA, multicast beamforming
- 1st global optimization method for rate splitting multiple access
- 1st global optimization method for joint unicast & multicast BF for energy efficiency maximization

State of the Art

- Optimal Beamforming (e.g.): Unicast [3, 4] — Multicast [5]
- Joint Unicast & Multicast: Power minimization [6] — Weighted sum rate [7]

Rate Splitting Multiple Access [8]

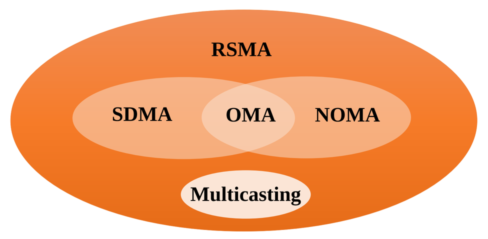
User Multiplexing

- Spatial and Power Domain
- Linearly Precoded Rate Splitting
- Successive Interference Cancellation (SIC)

Interference

- Partially treated as noise
- Partially decoded

} Arbitrary combinations



Relation to other Multiple Access Schemes

Benefits

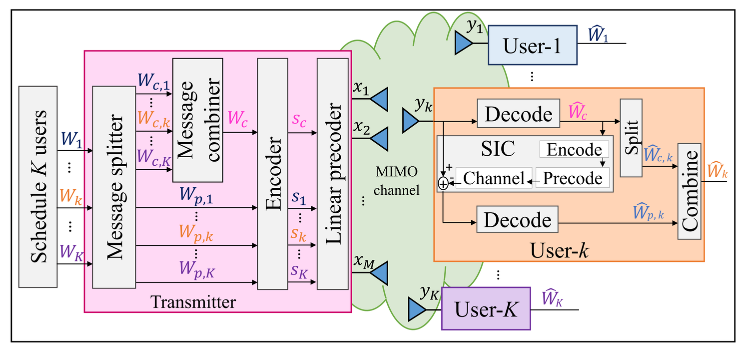
- Unifies SDMA, NOMA, OMA and multicasting into **one** scheme
- Improved Spectral and Energy Efficiency
- **DoF optimal** for perfect and imperfect CSI
- Robust against: Arbitrary user deployments — CSIT inaccuracy — Varying network load

Further Information

- IEEE ComSoc Special Interest Group on RSMA
- Tutorial @ IEEE ICC 2021 by Bruno Clerckx & Yijie (Lina) Mao
  RSMA for Beyond 5G: Principles, Recent Advances, and Future Research Trends

System Model: Downlink MISO Beamforming

- M transmit antennas, K single antenna receivers
- Each user: Split message W_k into $(W_{c,k}, W_{p,k})$
- Combine $W_c = (W_{c,k})_k$ into common stream s_c
- Create K private streams $W_{p,k} \mapsto s_k$
- Linear precoding: $\boldsymbol{x} = \boldsymbol{p}_c s_c + \sum_k \boldsymbol{p}_k s_k$
- Average power constraint P
- Received signal: $y_k = \boldsymbol{h}_k \boldsymbol{x} + n, \quad n \sim \mathcal{N}_{\mathbb{C}}$
- Decode $W_c \xrightarrow{\text{SIC}} W_{p,k}$



1-Layer RS for K-Users. The common stream s_c is shared by all users.

Problem Statement

Optimization Problem (R)

$$\max_{\substack{\boldsymbol{p}_1,\dots,\boldsymbol{p}_K,\\ \boldsymbol{p}_c,\mathbf{c},\gamma_c,\boldsymbol{\gamma}_p}} \quad \frac{\sum_{k\in\mathcal{K}} u_k \left(C_k + \log(1+\gamma_{p,k})\right)}{\mu\left(\|\boldsymbol{p}_c\|^2 + \sum_{k\in\mathcal{K}} \|\boldsymbol{p}_k\|^2\right) + P_c} \tag{1a}$$

$$\text{s.t.} \quad \gamma_c \le \min_k \left\{ \frac{|\boldsymbol{h}_k^H \boldsymbol{p}_c|^2}{\sum_{j\in\mathcal{K}} |\boldsymbol{h}_k^H \boldsymbol{p}_j|^2 + 1} \right\} \tag{1b}$$

$$\gamma_{p,k} \le \frac{|\boldsymbol{h}_k^H \boldsymbol{p}_k|^2}{\sum_{j\in\mathcal{K}\setminus k} |\boldsymbol{h}_k^H \boldsymbol{p}_j|^2 + 1} \tag{1c}$$

$$\sum_{k'\in\mathcal{K}} C_{k'} \le \log(1+\gamma_c) \tag{1d}$$

$$\forall k: C_k \ge \max\left\{0,\, R_k^{th} - \log(1+\gamma_{p,k})\right\} \tag{1e}$$

$$\|\boldsymbol{p}_c\|^2 + \sum_{k\in\mathcal{K}} \|\boldsymbol{p}_k\|^2 \le P \tag{1f}$$

Problem Types

- Unicast Beamforming
- Multicast Beamforming

NP-hard

- Sum Rate Power Allocation [9]
- Multicast Beamforming [10]

Details

- C_k: Common Msg Rate Allocation
- R_k^{th}: Per user minimum rate QoS
- User rate: $C_k + \log(1+\gamma_{p,k})$

Weighted Sum Rate

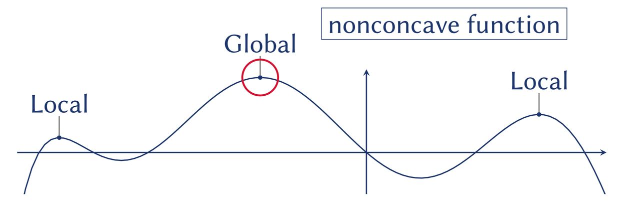
$$\mu = 0, P_c = 1: \sum_{k\in\mathcal{K}} u_k \left(C_k + \log(1+\gamma_{p,k})\right)$$

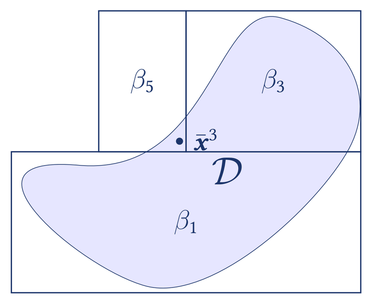
Energy Efficiency

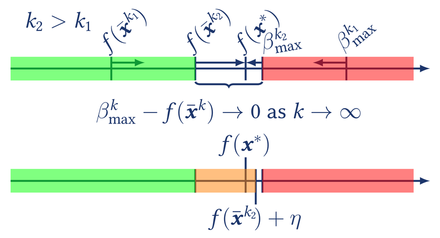
$$\forall k: u_k = 1: \frac{\sum_{k\in\mathcal{K}} \left(C_k + \log(1+\gamma_{p,k})\right)}{\mu\left(\|\boldsymbol{p}_c\|^2 + \sum_{k\in\mathcal{K}} \|\boldsymbol{p}_k\|^2\right) + P_c}$$

Global Optimization

- P-Time Algorithms: At most local maximum
- Convex Optimization: Local = Global
- Global Optimization: Solve multiextremal problems
- Branch-and-Bound: Structured search







- Partition feasible set systematically
- On each partition element: Compute upper and lower bound on feasible objective values
- Successively refine partition
- If convergence criteria met: Upper − Lower → 0 as size(partition elements) → 0

- Branch-and-Bound has convergence issues with complicated feasible sets
- Successive Incumbent Transcending (SIT) Scheme: Exchange objective and constraints

SIT Primal (P)

$$\max_{\boldsymbol{x}} \ f(\boldsymbol{x}) \quad \text{s.t.} \quad g_i(\boldsymbol{x}) \le -\varepsilon$$

SIT Dual (Q)

$$\min_{\boldsymbol{x}} \ \max_i g_i(\boldsymbol{x}) \quad \text{s.t.} \quad f(\boldsymbol{x}) \ge \gamma$$

Equivalence

$$v(P) < \gamma \iff v(Q) \ge -\varepsilon$$

- Solve (P) as a sequence of (Q)
- "Nice" Objective → (Q) is easy to solve by BB
- Leads to integrated SIT-BB procedure
- Finite convergence

Further Information

- Tutorial @ IEEE ICASSP 2021 & IEEE ICC 2021 by Bho Matthiesen & Eduard Jorswieck
  Efficient Global Optimization and its Application to Wireless Interference Networks

Solution Algorithm

Rotational Invariance [1]

$$\boldsymbol{p}_k^\star \text{ solves (R)} \iff \boldsymbol{p}_k^\star e^{j\phi_k} \text{ solves (R)} \quad (\phi_k \in \mathbb{R})$$

Unicast Beamforming

- Fix $\boldsymbol{h}_k^H \boldsymbol{p}_k \in \mathbb{R}_{\ge 0}$ for all k [1]
- $\Re\{\boldsymbol{h}_k^H \boldsymbol{p}_k\} \ge 0$ and $\Im\{\boldsymbol{h}_k^H \boldsymbol{p}_k\} = 0$
- $\boldsymbol{h}_k^H \boldsymbol{p}_k \ge \sqrt{\gamma_{p,k}\left(\sum_{j\in\mathcal{K}\setminus k} |\boldsymbol{h}_k^H \boldsymbol{p}_j|^2 + 1\right)}$

Multicast Beamforming

$$\forall k: |\boldsymbol{h}_k^H \boldsymbol{p}_c|^2 \ge \gamma_c \left(\sum_{j\in\mathcal{K}} |\boldsymbol{h}_k^H \boldsymbol{p}_j|^2 + 1\right)$$

- Rotational invariance for $k = 1$
- Argument cuts [5] for $k > 1$

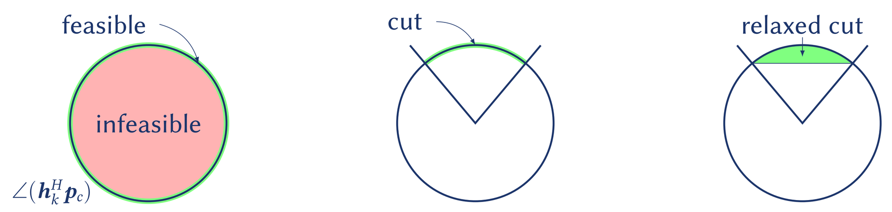
→ Branch-and-Bound over argument of $\boldsymbol{h}_k^H \boldsymbol{p}_c$

Algorithm

- Complexity in feasible set → SIT-BB
- Branch over $(\boldsymbol{\gamma}_p, \gamma_c, \angle \boldsymbol{h}_2^H \boldsymbol{p}_c, \dots, \angle \boldsymbol{h}_K^H \boldsymbol{p}_c)$
- SOCP bounding and feasibility problems

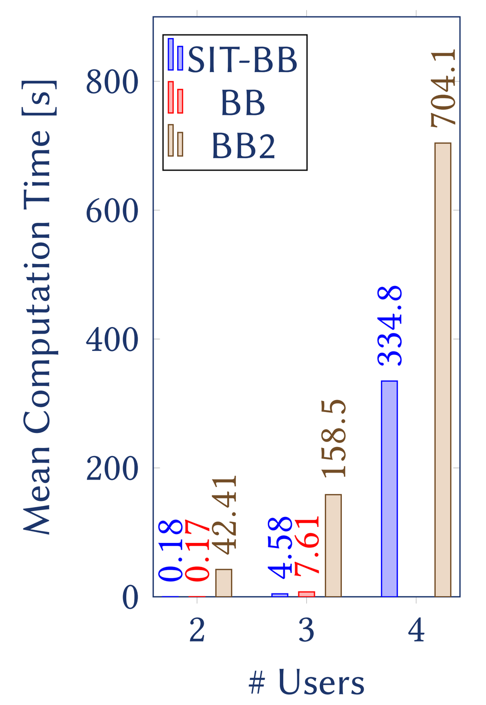
Non-SIT Algorithms for Unicast Beamforming

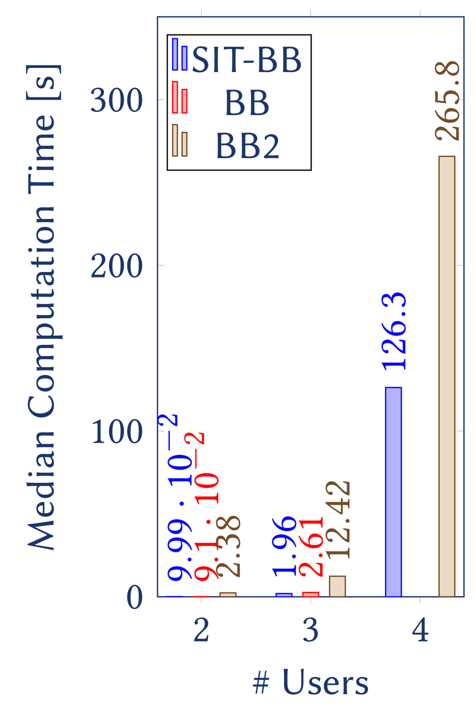
- Direct implementation
  - Infinite procedure
  - Numerical issues due to tiny feasible set
- Modified bounding problem [3, §2.2.2]
  - Fixes numerical problems
  - Slow convergence due to few feasible points
- Line search feasible point acquisition
  - Combined with previous approaches
  - Bisection to obtain feasible points
  - Finite algorithm
  - Increased computational complexity



Numerical Results

- Unicast only
- 100 channel realizations, $P \in [-10 : 5 : 20]$ dB
- Ignored instances w/o conergence for all Algorithms
- $K = M$ antennas
- Time limit 60 minutes
- Unsolved instances
  - SIT-BB: 4 users: 4 instances (OoT)
  - BB: 3 users: 13 instances (num)
  - BB2 (stall): 2: 364, 3: 146, 4: 27
  - BB2 (OoT): 4: 60
- SIT-BB with multicast ($K = 2$):
  - Mean: 942 s
  - Median: 2786 s





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DFG Deutsche Forschungsgemeinschaft