Globally Optimal Beamforming for Rate Splitting Multiple Access

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University of Bremen Imperial College London AALBORG UNIVERSITY

DFG Deutsche Forschungsgemeinschaft German Research Foundation

UKRI Engineering and Physical Sciences Research Council

Rate-Splitting Multiple Access (RSMA)

Spatial and power domains user multiplexing using linearly precoded rate splitting & SIC

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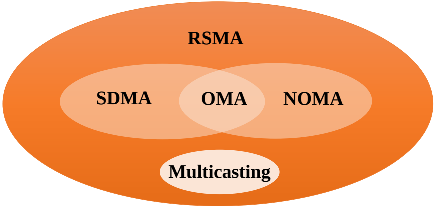
Benefits:

- Special cases: SDMA, NOMA, OMA and multicasting
- $\rightarrow$ Improved Spectral and Energy Efficiency
- Optimal DoF for perfect and imperfect CSIT
- Robust against
  - arbitrary user deployments
  - CSIT inaccuracy
  - network load



IEEE ComSoc Special Interest Group on RSMA:

https://sites.google.com/view/ieee-comsoc-wtc-sig-rsma

Upcoming tutorial:

- IEEE ICC 2021 Tutorial on Rate Splitting Multiple Access for Beyond 5G: Principles, Recent Advances, and Future Research Trends, Montreal, Canada
- Date: 14–18 June 2021
- Speakers: Prof. Bruno Clerckx, Dr. Yijie (Lina) Mao
- More info: https://icc2021.ieee-icc.org/program/tutorials#tut-03

System Model

- M transmit antennas and K single-antenna users: $\mathcal{K} = \{1, \dots, K\}$
- Message Splitting: $W_k \xrightarrow{\text{split}} \{W_{c,k}, W_{p,k}\}$

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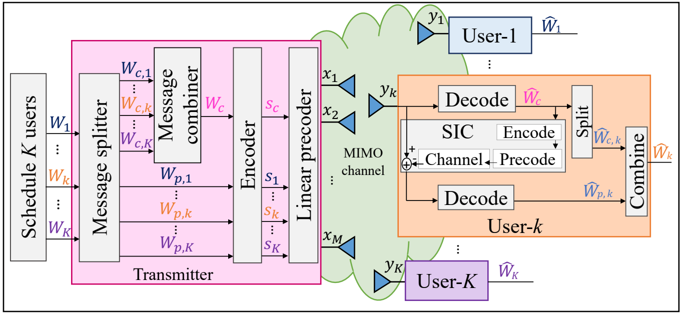
- Common stream: $\{W_{c,1}, \ldots, W_{c,K}\} \xrightarrow{\text{encode}} s_c$, Private streams: $W_{p,k} \xrightarrow{\text{encode}} s_k$
- Linear Precoding: $\mathbf{x} = \boldsymbol{p}_c s_c + \boldsymbol{p}_1 s_1 + \ldots + \boldsymbol{p}_K s_K$.
- Average power constraint: $\|\boldsymbol{p}_c\|^2 + \sum_{k \in \mathcal{K}} \|\boldsymbol{p}_k\|^2 \leq P$



All users decode s_c first, before decoding s_k (for user k)

Rate of user-k has been **split**: rate of s_k + part of the rate of s_c

Problem Statement

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$$\max_{\substack{\boldsymbol{p}_1,\dots,\boldsymbol{p}_K,\\ \boldsymbol{p}_c,\mathbf{c},\gamma_c,\gamma_p}} \quad \frac{\sum_{k\in\mathcal{K}} u_k \left(C_k + \log(1+\gamma_{p,k})\right)}{\mu\left(\|\boldsymbol{p}_c\|^2 + \sum_{k\in\mathcal{K}} \|\boldsymbol{p}_k\|^2\right) + P_c}$$

s.t.

$$\gamma_c \leq \min_k \left\{ \frac{|\boldsymbol{h}_k^H \boldsymbol{p}_c|^2}{\sum_{j\in\mathcal{K}} |\boldsymbol{h}_k^H \boldsymbol{p}_j|^2 + 1} \right\}$$

$$\gamma_{p,k} \leq \frac{|\boldsymbol{h}_k^H \boldsymbol{p}_k|^2}{\sum_{j\in\mathcal{K}\setminus k} |\boldsymbol{h}_k^H \boldsymbol{p}_j|^2 + 1}$$

$$\sum_{k'\in\mathcal{K}} C_{k'} \leq \log(1+\gamma_c)$$

$$\forall k : C_k \geq \max\left\{0,\ R_k^{th} - \log(1+\gamma_{p,k})\right\}$$

$$\|\boldsymbol{p}_c\|^2 + \sum_{k\in\mathcal{K}} \|\boldsymbol{p}_k\|^2 \leq P$$

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→ $\mu = 0, P_c = 1$: Weighted Sum Rate

→ $u_k = 1$: Energy Efficiency

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→ Common Msg SINRs

$$\gamma_{p,k} \leq \frac{|\boldsymbol{h}_k^H \boldsymbol{p}_k|^2}{\sum_{j\in\mathcal{K}\setminus k} |\boldsymbol{h}_k^H \boldsymbol{p}_j|^2 + 1}$$

→ Private Msg SINR

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Problem Types:

- Unicast Beamforming
- Multicast Beamforming

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NP-hard:

- Sum Rate Power Allocation [1]
- Multicast Beamforming [2]

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Observation:
- Optimal Beamformers are rotationally invariant [3]: $\boldsymbol{p}_k^\star = \boldsymbol{p}_k^\star e^{j\phi_k}$

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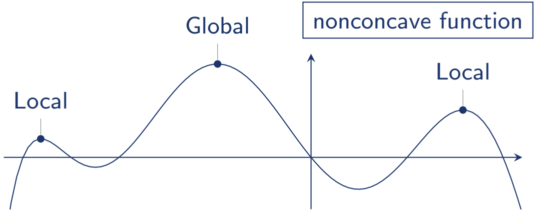
Global Solution



- Multiextremal problem

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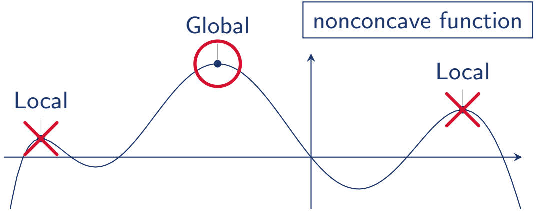

- Multiextremal problem
- **Goal:** Global maximum

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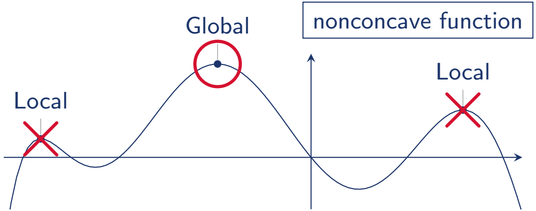

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- Polynomial time methods:
  Find local maximum at most
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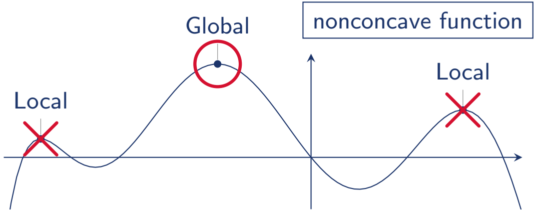

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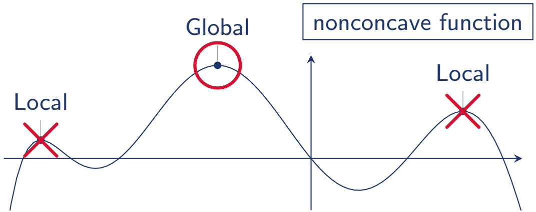

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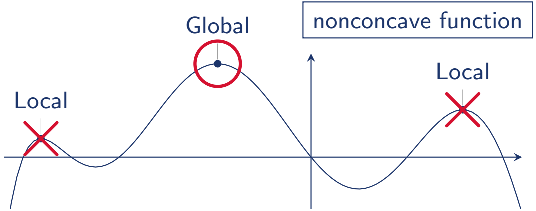

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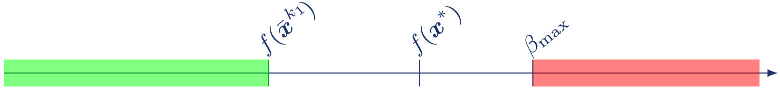

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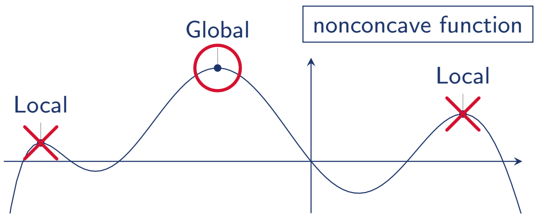

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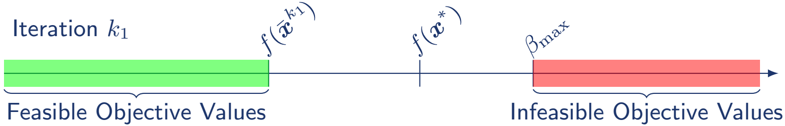

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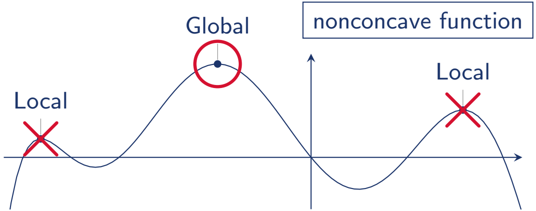

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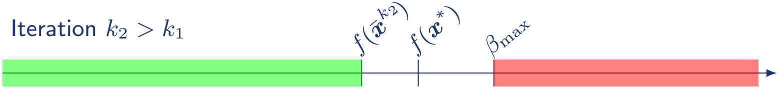
Iteration $k_2 > k_1$



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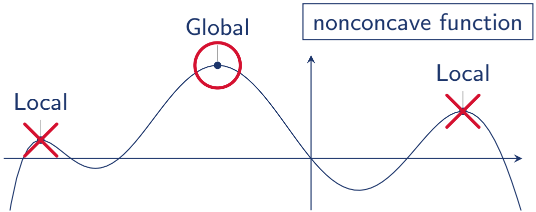

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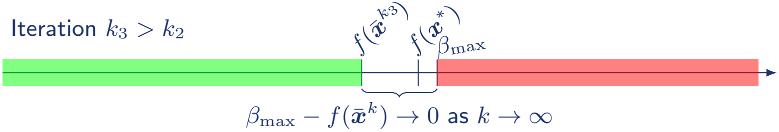

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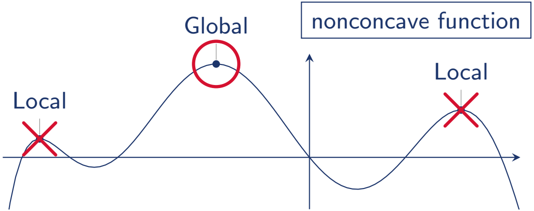

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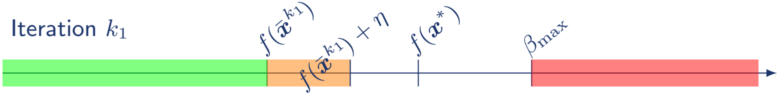
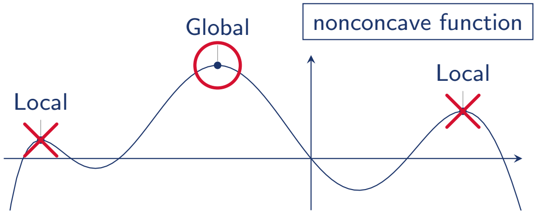


Global Solution



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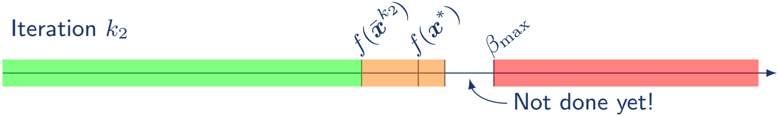
Iteration k_2



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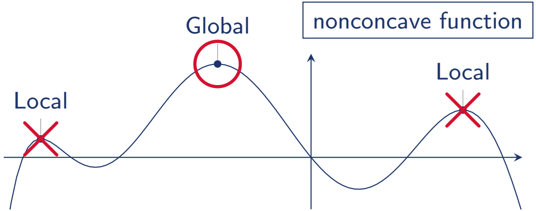

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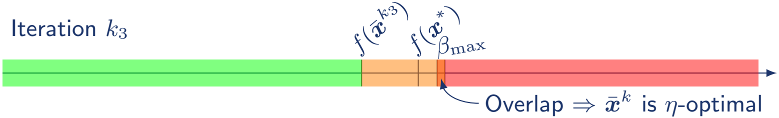

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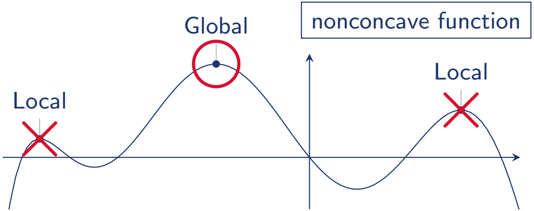

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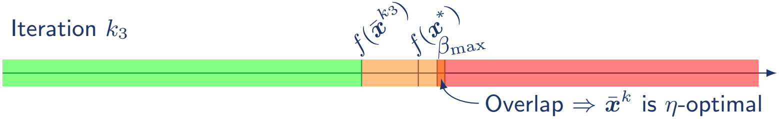

  - Finite Procedure (i.e., an algorithm) only if convergence provably after $< \infty$ iterations

Unicast Beamforming

- Challenging constraint: $$\gamma_{p,k} \leq \frac{|\boldsymbol{h}_k^H \boldsymbol{p}_k|^2}{\sum_{j \in \mathcal{K} \setminus k} |\boldsymbol{h}_k^H \boldsymbol{p}_j|^2 + 1}$$

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Multicast Beamforming

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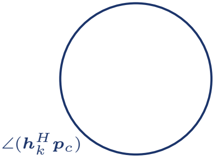

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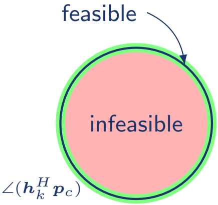

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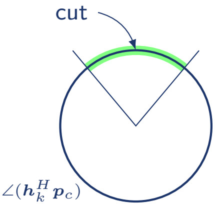

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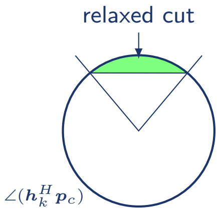

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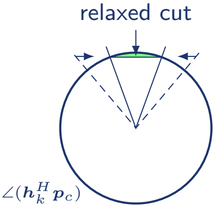

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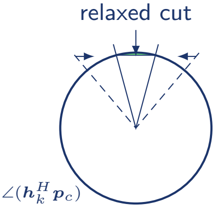

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Successive Incumbent Transcending Scheme:

- Modified BB procedure: numerically stable & guaranteed finite convergence
- Details: Paper & ICASSP Tutorial T-3

  Efficient Global Optimization and its Application to Wireless Interference Networks

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Contributions:

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