Real Number Signal Processing Can Detect Denial-of-Service Attacks

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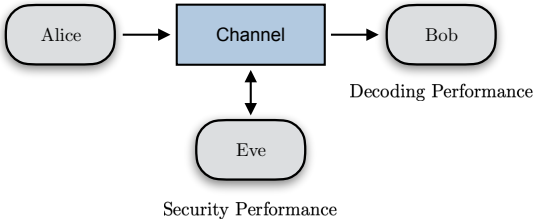
Motivation: Denial of Service as Security Attack



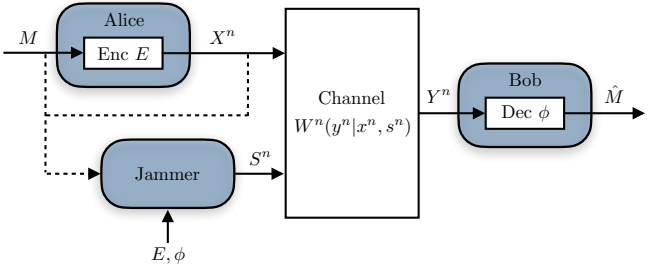
Attack strategies of Eve:

- Security $\Rightarrow$ quantum computer $\Rightarrow$ **very expensive**
- Communication / decoding performance – Denial of Service $\Rightarrow$ **inexpensive**

very expensive attacks on security mechanisms vs inexpensive jamming attacks

As a consequence, we need **resilience by design!**

Communication System with a Jammer



- Let $\mathcal{X}$, $\mathcal{Y}$, and $\mathcal{S}$ be finite input, output, state (jamming) alphabets
- For fixed $s^n \in \mathcal{S}^n$, the DMC is

$$W^n(y^n|x^n, s^n) = \prod_{i=1}^{n} W(y_i|x_i, s_i)$$

- *Jammer with Partial Knowledge* knows encoder E and decoder ϕ
- *Jammer with Full Knowledge* additionally knows actual message M (or $X^n = X^n(M)$)

Denial-of-Service (DoS) Attacks

- Goal of the Jammer: Successfully launch a DoS attack
- We are interested in studying DoS attacks, where the Jammer is able to completely disrupt the communication
  - ➠ Whatever decoding strategy the receiver may use and computational capabilities the receiver may have, it is **not** able to decode the transmitted message
- Such DoS attacks can be
  - unintentionally due to high interference coming from other (uncoordinated) transmitters or
  - intentionally due to jamming attacks from active adversaries
- The traditional approach of detecting such attacks and reacting to those is realized on higher layers based on channel state information (such as SINR, RSS, ...) and may further be integrated into the resource allocation

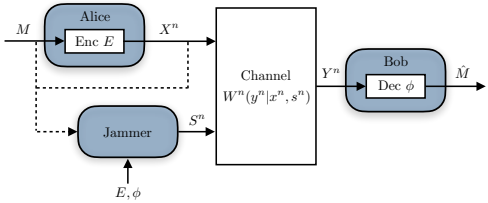
Denial-of-Service (DoS) Attacks (2)

Question: Is it possible at all to realize detection of (and subsequent reaction to) DoS attacks on higher layers?

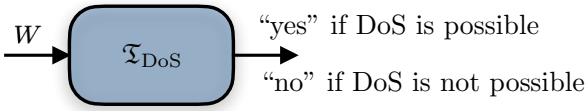
- Particularly relevant as there is a recent trend towards realizing functionalities in software only (such as *software-defined networking (SDN)* and *network function virtualization (NFV)*)

How can we formalize this in a precise and rigorous way?

Detection of DoS Attacks via Turing Machines



Problem formulation: Is the question *"Is the Jammer able to perform a DoS attack?"* decidable by a Turing machine? Is there a Turing machine $\mathfrak{T}_{\text{DoS}}$ such that



Properties of DoS Attacks

- Let $\mathcal{X}$, $\mathcal{Y}$, and $\mathcal{S}$ be the input, output and state sets
- Let $\mathcal{M}_{\text{DoS}}$ the set of all channels for which a DoS attack is possible

Theorem: [BSP 2020]

For all $|\mathcal{X}| \geq 2$, $|\mathcal{S}| \geq 2$, and $|\mathcal{Y}| \geq 2$, there is **no** Turing machine $\mathfrak{T}$ with $\mathfrak{T}(W) = 1$ if and only if $W \in \mathcal{M}_{\text{DoS}}$.

B-S-P, "Denial-of-service attacks on communication systems: Detectability and jammer knowledge," *IEEE Trans. Signal Process.*, vol. 68, pp. 3754–3768, 2020

Feedback does **not** help – detection problem remains undecidable on Turing machines

B-S-P, "On the algorithmic solvability of channel dependent classification problems in communication systems," *IEEE/ACM Trans. Netw.*, 2021

Blum-Shub-Smale (BSS) Machines

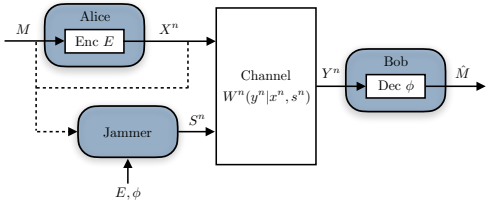
- It can store arbitrary real numbers, can compute all field operations on $\mathbb{R}$, i.e., "$+$" and "$\cdot$", and can compare real numbers according to the relations "$<$", "$>$", and "$=$"
- A BSS machine is similar to a Turing machine in the sense that it operates on an infinite strip of tape according to a so-called program. This is a finite directed graph with five types of nodes associated with different operations: input node, computation node, branch node, shift node, and output node

BSS-computable functions are input-output maps Φ of the BSS machine $\mathfrak{B}$, i.e., for every input $\boldsymbol{x}$, the output $\Phi_{\mathfrak{B}}(\boldsymbol{x})$ is defined if the ouput is reachable by the program of $\mathfrak{B}$.

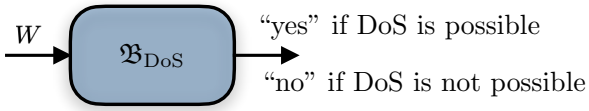
A set $\mathcal{A} \subset \mathbb{R}^N$ is *BSS-decidable* if there is a BSS machine $\mathfrak{B}_{\mathcal{A}}$ such that for all $\boldsymbol{x} \in \mathbb{R}^N$ we have $\mathfrak{B}_{\mathcal{A}}(\boldsymbol{x}) = \chi_{\mathcal{A}}(\boldsymbol{x})$, i.e., the characteristic function $\chi_{\mathcal{A}}$ of the set $\mathcal{A}$ is BSS-computable.

L. Blum, M. Shub, and S. Smale, "On a theory of computation and complexity over the real numbers: NP-completeness, recursive functions and universal machines," *Bull. Amer. Math. Soc.*, vol. 21, no. 1, pp. 1–46, Jul. 1989

Detection of DoS Attacks via BSS Machines



Problem formulation: Is the question *"Is the Jammer able to perform a DoS attack?"* decidable by a BSS machine? Is there a BSS machine $\mathfrak{B}_{\mathsf{DoS}}$ such that



DoS Attacks with Blum-Shub-Smale Machines

Theorem:

Let $\mathcal{X}$, $\mathcal{Y}$, and $\mathcal{S}$ be arbitrary finite alphabets. Then there exists a BSS machine $\mathfrak{B}$ that outputs $\mathfrak{B}(W) =$ "yes" if and only if $W \in \mathcal{M}_{\mathsf{DoS}}$, i.e., the DoS detection problem is BSS-decidable.

Main proof ingredient:

- Exploit connections to the theory of semialgebraic sets
- Show that both sets $\mathcal{M}_{\mathsf{DoS}}$ and $\mathcal{M}^{c}_{\mathsf{DoS}}$ are semialgebraic

➠ The result remains **true** also in case where the Jammer also knows the transmitted message, i.e., the most powerful jammer

Conclusions

- *Detection framework based on Turing machines*
  - Turing machines provide fundamental performance limits for today's digital computers and therewith of traditional signal processing
  - Turing machines are **not** capable of detecting DoS attacks!
  - Feedback does **not** help – detection problem remains undecidable

- *Detection framework based on BSS machines*
  - Allows the processing and storage of arbitrary reals
  - BSS machines are capable of detecting DoS attacks!
  - Real number signal processing enables the detection of DoS attacks

Solution to the DoS detectability problem: **Computing model** is very important!

Thank you for your attention!

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