Generalized Thinned Coprime Array for DOA Estimation (1710)

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Contribution: Generalized *and* flexible Coprime Array

We propose a generalized thinned coprime array by introducing the flexible inter-element spacings, where the conventional one can be seen as a special case. We derive closed-form expression for the range of consecutive lags, written as the functions of the antenna numbers and inter-element spacings. We show that, after optimization, the proposed array can achieve more consecutive lags than the other coprime arrays. In particular, the optimized results also provide the minimum number of antenna pairs with small separation. Simulation results demonstrate the superiority of the proposed GTCA using the subspace-based method.

Abstract: Inspired By TCA

Compared with nested arrays, coprime arrays are proposed with sparser subarrays. A prototype coprime array is made up of two ULAs, where one is M-antenna ULA with inter-element spacing Nd and the other one is N-antenna ULA with spacing Md. M, N are set as coprime integers and d represnets the unit spacing.

Then, generalized coprime arrays were developed with two operations. Specifically, one is designed by compressing the inter-element spacing of one subarray and the other is by connecting two displaced subarrays, referred to as coprime array with compressed inter-element spacing (CACIS) and coprime array with displaced subarrays (CADiS). It is shown that CACIS has some redundant antennas and CADiS cannot produce a higher number of consecutive lags. Further, a thinned coprime array (TCA) was proposed by removing the redundancy of the coprime array, which possesses the same number of consecutive lags with fewer physical antennas.

In this paper, inspired by the TCA, we propose a generalized TCA (GTCA) by introducing two coprime integers as the inter-element spacings of subarrays.

References

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Coarray Signal Model

Consider K far-field uncorrelated and narrow-band sources impinging on a nonuniform linear array (NLA) of N antennas from directions θ_k with powers σ_k^2. By introducing a mutual coupling matrix $\mathbf{C}$, the received signal model is given as

$$\mathbf{x}(t) = \mathbf{C}\mathbf{A}\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

The entries of $\mathbf{C}$ is often approximated by a B-band mode

$$\mathbf{C}_{ij} = \begin{cases} c_{|a_i - a_j|}, & \text{if } |a_i - a_j| \leq B \\ 0, & \text{otherwise} \end{cases}, \tag{2}$$

where $a_i, a_j \in \mathbb{A}$, $1 = c_0 > |c_1| > \cdots > |c_B| > |c_{B+1}| = 0$, $c_1 = c_0 e^{j\pi/3}, c_l = c_1 e^{-j(l-1)\pi/8}/l, l = 2, \cdots, B$. In fact, the coupling coefficient $\mathbf{C}_{ij}$ is inversely proportional to the antenna separations a_i, a_j.

By collecting L snapshots, the covariance matrix is approximated as

$$\begin{aligned}\mathbf{R} &= E\left[\mathbf{x}(t)\mathbf{x}^H(t)\right] = \mathbf{C}\mathbf{A}\mathbf{R}_s\mathbf{A}^H\mathbf{C}^H + \sigma_n^2\mathbf{I}_N \\ &\approx \frac{1}{L}\sum_{t=1}^{L}\mathbf{x}(t)\mathbf{x}^H(t),\end{aligned} \tag{3}$$

where $\mathbf{R}_s = \mathrm{E}\left[\mathbf{s}(t)\mathbf{s}^H(t)\right] = \mathrm{diag}\left[\sigma_1^2, \sigma_2^2, \cdots, \sigma_K^2\right]$ denotes the source covariance matrix. Then, vectorizing $\mathbf{R}$ yields

$$\begin{aligned}\mathbf{r} &= \mathrm{vec}(\mathbf{R}) = (\mathbf{C}\mathbf{A})^* \circ (\mathbf{C}\mathbf{A})\,\mathbf{p} + \sigma_n^2 \mathrm{vec}(\mathbf{I}_N) \\ &= \tilde{\mathbf{C}}(\mathbf{A}^* \circ \mathbf{A})\,\mathbf{p} + \sigma_n^2 \mathrm{vec}(\mathbf{I}_N),\end{aligned} \tag{4}$$

where $\tilde{\mathbf{C}} = \mathbf{C}^* \otimes \mathbf{C}$. From (4), the antenna positions in the virtual manifold matrix $\mathbf{A}^* \circ \mathbf{A}$ can be given as the following difference coarray

$$\mathbb{D} = \left\{a_i - a_j, a_i, a_j \in \mathbb{A}\right\}. \tag{5}$$

Generalized Thinned Coprime Array for DOA Estimation

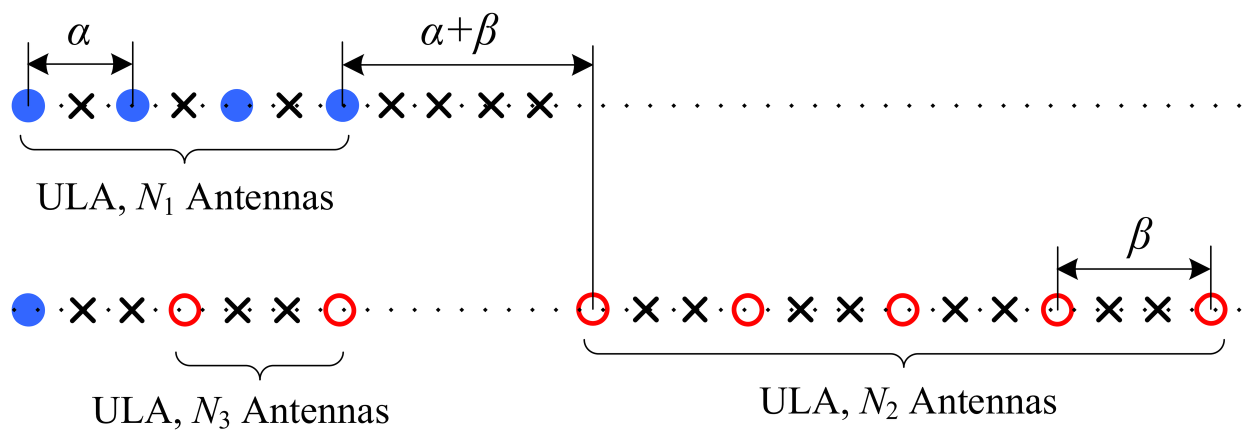

Fig.1:

As depicted in the above figure, GTCA consists of three ULAs, where the first ULA has the spacing α sharing the same first antenna with the third ULA, the second and third ULAs have the same inter-element spacing β and the spacing between the first and third ULAs is set as $\alpha + \beta$. In fact, the second and third ULAs can be regarded as a relocated sparse array. The numbers of antennas for three ULAs are given by N_1, N_2, N_3, and α, β are coprime integers. The total number of antennas equals $N = N_1 + N_2 + N_3$. It can be seen that TGCA is a generalized structure compared with the recently developed coprime arrays. To explore the structure features of GTCA, we first define the difference coarray as $\mathbb{F} = \left\{a_i - a_j, a_i, a_j \in \mathbb{A}_{\mathrm{F}}\right\}$, where $\mathbb{A}_{\mathrm{F}} = \left\{0, \alpha, 2\alpha, \cdots, (N_1 - 1)\alpha, N_1\alpha + \beta, \cdots, N_1\alpha + N_2\beta, \beta, \cdots, N_3\beta\right\}$ denotes the antenna positions of FCA.

Theorem 1: As shown, consider a GTCA with three ULAs, whose inter-element spacings are coprime integers. If $\alpha \in [1, \min(2N_3 + 1, N_2 + 1)]$, $\beta \in [1, N_1]$, the range of consecutive lags in the set $\mathbb{F}$ can be presented as $[-c, c]$, where $c = \alpha N_1 + \beta N_2 - (\alpha - 1)(\beta - 1)$, $N_2 \geq N_3$.

TCA is proposed by removing the redundant antennas in the conventional coprime array. In case of $N_2 = M - 1$, $N_3 = \lfloor M/2 \rfloor$, $\alpha = M$, $\beta = N_1$, TCA has $2M(N_1 + 1) - 1$ consecutive lags, where M, N are coprime integers. Since TCA is a special case of GTCA, the optimized results can always take the maximum number of consecutive lags, thus performing better performance. In particular, GTCA and TCA provide the least mutual coupling in case of $B = 3, N_1 > 3, N_3 \geq 2$. In addition, TCA also needs to satisfy the coprime condition of M, N, while the optimized GTCA just satisfy the coprime inter-element spacings, thus suitable for any number of physical antennas.

Simulation Results: DOF, Mutual Coupling and Accuracy

Fig.2

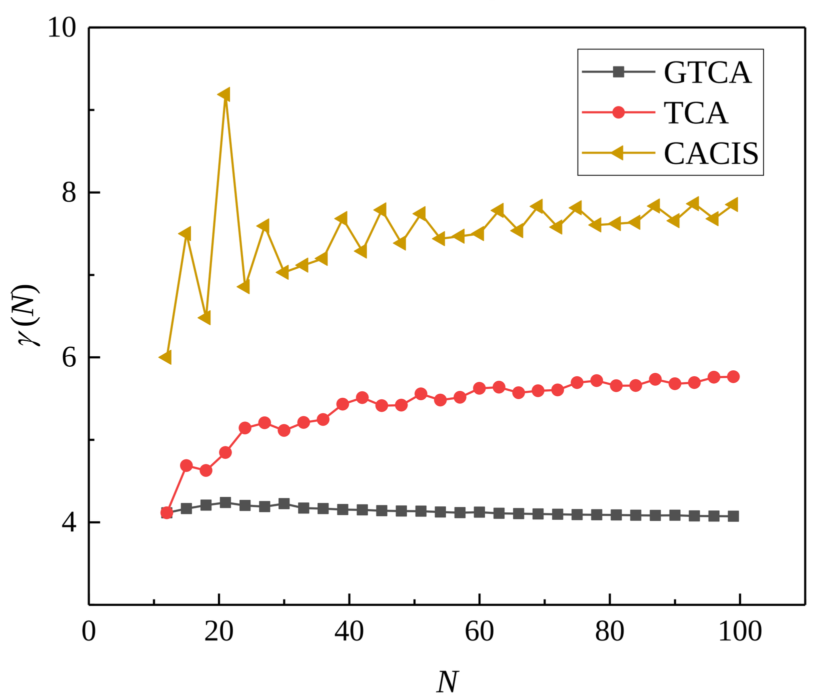

Fig.3

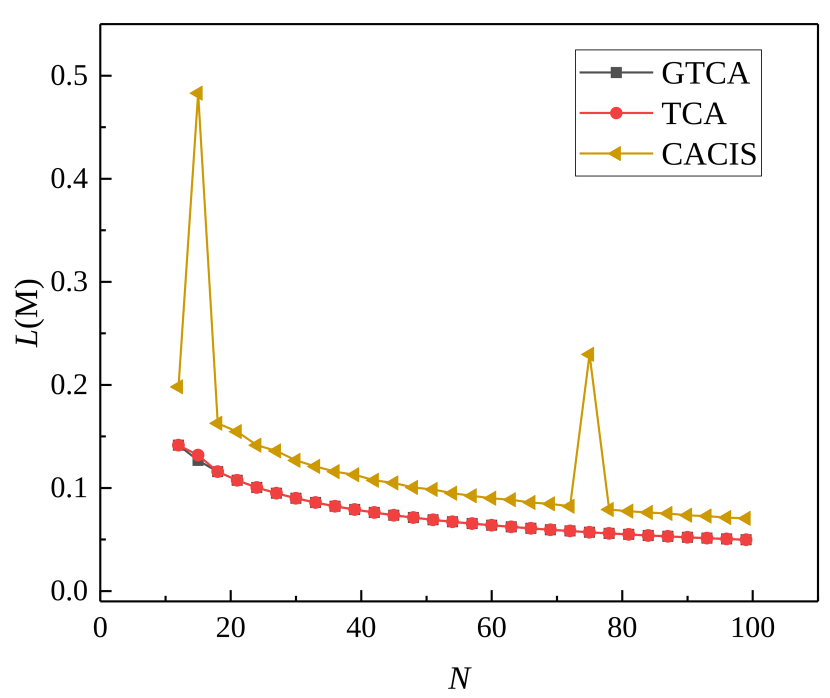

Fig.4

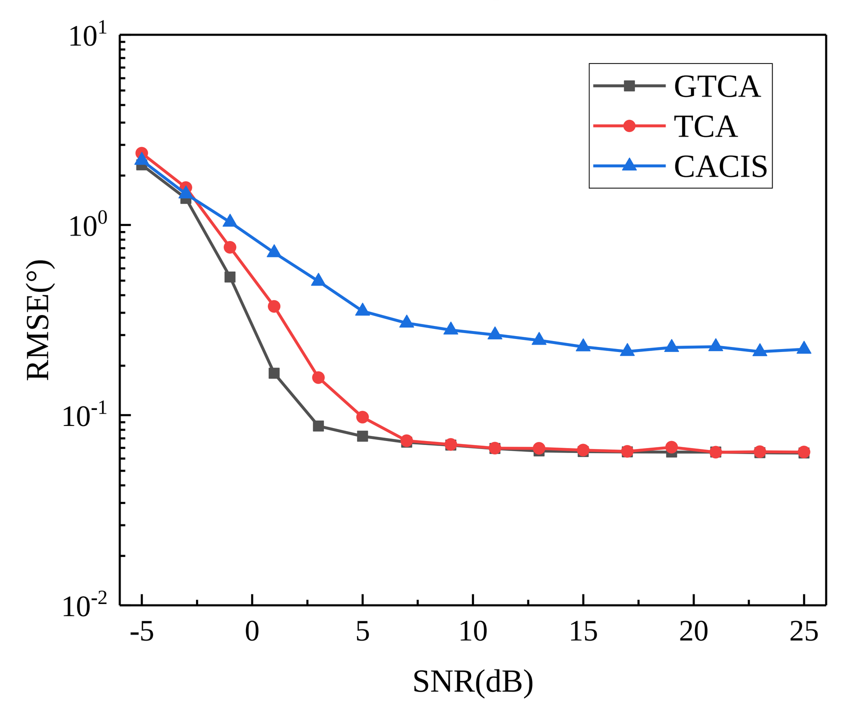

Fig.2 depicts the DOF ratio $\gamma(N)$ versus N for the number of consecutive lags. It is seen that, GTCA achieves higher consecutive lag capacity than the others. Fig.3 plots the mutual coupling leakage $L(\mathbf{M})$ versus N. GTCA and TCA can take the same mutual coupling with $w(1) = w(2) = w(3) = 1$, both of which possess a much lower $L(\mathbf{M})$ than CACIS. Fig.4 compares the RMSE as a function of the input SNR. The GTCA performs the best among all sparse arrays due to the increased consecutive lags and reduced mutual coupling.