University of Moratuwa
Wisdom is all wealth

A Joint Convolutional and Spatial Quad-Directional LSTM Network for Phase Unwrapping

Malsha Perera and Ashwin De Silva

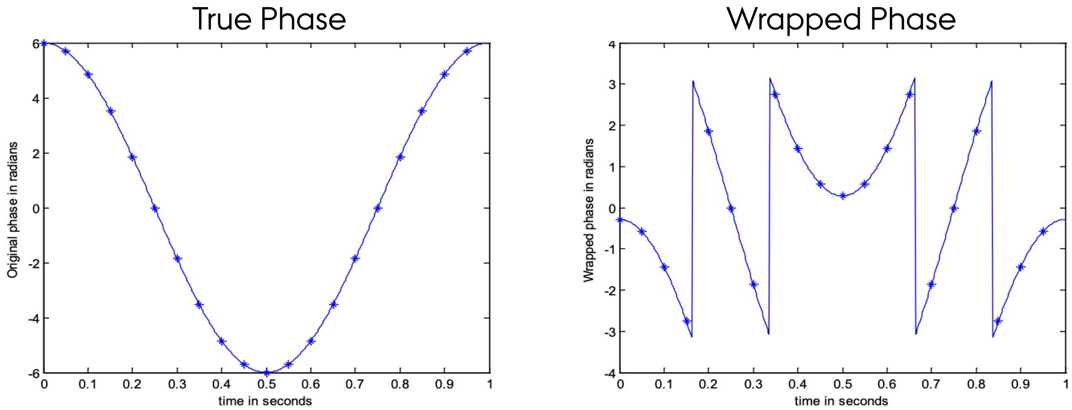
Dept. of Electronics and Telecommunication Engineering, University of Moratuwa, Sri Lanka

ICASSP2021
TORONTO
Canada
June 6-11, 2021
Metro Toronto Convention Centre

Malsha Perera, Ashwin De Silva
malshav@uom.lk, ashwind@uom.lk

Problem

- Objective : **Recover the true phase (ϕ) from the wrapped phase (ψ)**



Wrapping of phase $\rightarrow$ $\psi(t) = \angle\, e^{j\phi(t)}$

Unwrapping wrapped phase $\rightarrow$ $\phi(t) = \psi(t) + 2\pi\, k(t)\,;\; k(t) \in \mathbb{Z}$

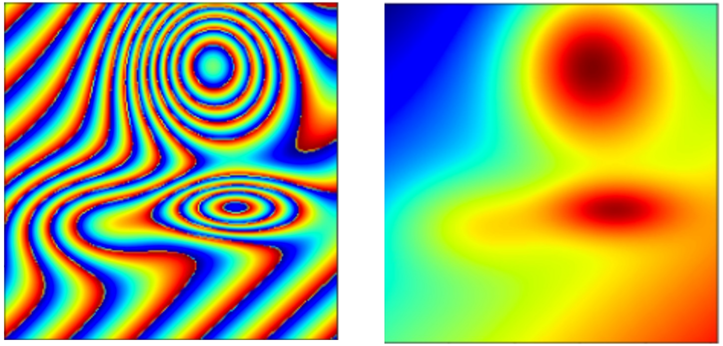
$k(t)$ is the wrap count

Malsha Perera, Ashwin De Silva
malshav@uom.lk, ashwind@uom.lk

Problem

- Objective : **Recover the true phase (ϕ) from the wrapped phase (ψ)**

Wrapped Phase   True Phase



Wrapping of phase $\rightarrow$ $\psi(x,y) = \angle\, e^{j\phi(x,y)}$

Unwrapping wrapped phase $\rightarrow$ $\phi(x,y) = \psi(x,y) + 2\pi\, k(x,y)\,;\; k(x,y) \in \mathbb{Z}$

$k(x,y)$ is the wrap count

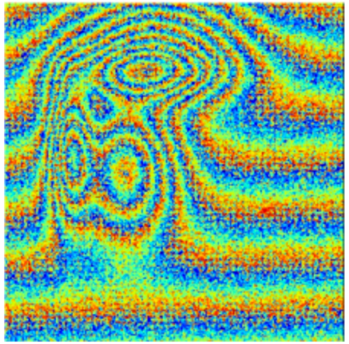
- Phase Unwrapping is convenient **under ideal noise-free conditions.**

Malsha Perera, Ashwin De Silva
malshav@uom.lk, ashwind@uom.lk

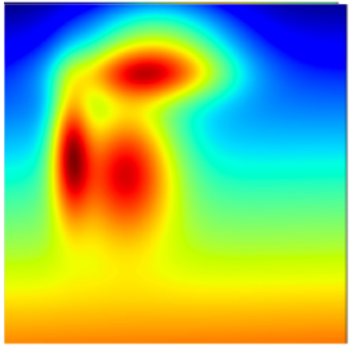
Problem

- Objective : **Recover the true phase (ϕ) from the wrapped phase (ψ)**

Wrapped Phase



True Phase



Wrapping of phase $\rightarrow$ $\psi(x,y) = \angle\, e^{j\phi(x,y)}$

Unwrapping wrapped phase $\rightarrow$ $\phi(x,y) = \psi(x,y) + 2\,\pi\, k(x,y)\,;\; k(x,y) \in \mathbb{Z}$

$k(x,y)$ is the wrap count

- At the **presence of noise, phase discontinuities, and rapid variation of phase**, the problem becomes challenging

Malsha Perera, Ashwin De Silva
malshav@uom.lk, ashwind@uom.lk

Previous Work (Traditional Approaches)

- Path following algorithms (e.g. Quality Guided Phase Unwrapping (QGPU), Branch cut algorithm, etc)
- Minimum norm based algorithms
- Drawbacks :
  - Computationally inefficient
  - Not robust to noise

Previous Work (Deep Learning Approaches)

- Recently **Convolutional Neural Networks (CNNs)** have attempted to solve the phase unwrapping problem.
- Two types of CNNs based phase unwrapping methods :
  - **Formulate the phase unwrapping problem as a semantic segmentation task** where each $k(x, y)$ is predicted (e.g PhaseNet, PhaseNet 2.0, etc)

    Unwrapping wrapped phase $\rightarrow$ $\phi(x, y) = \psi(x, y) + 2\pi\, k(x, y)\,;\; k(x, y) \in \mathbb{Z}$

  - **Formulate the phase unwrapping problem as a regression task** (e.g Ryu et al)

    Unwrapping wrapped phase $\rightarrow$ $\phi = f(\psi),\; f\,?$

Previous Work (Deep Learning Approaches)

Drawbacks :

- Less robust to noise
- CNNs ignore global spatial relationships in an image
- Highly data intensive (cannot be applied in real-world applications)
- Inappropriate loss functions

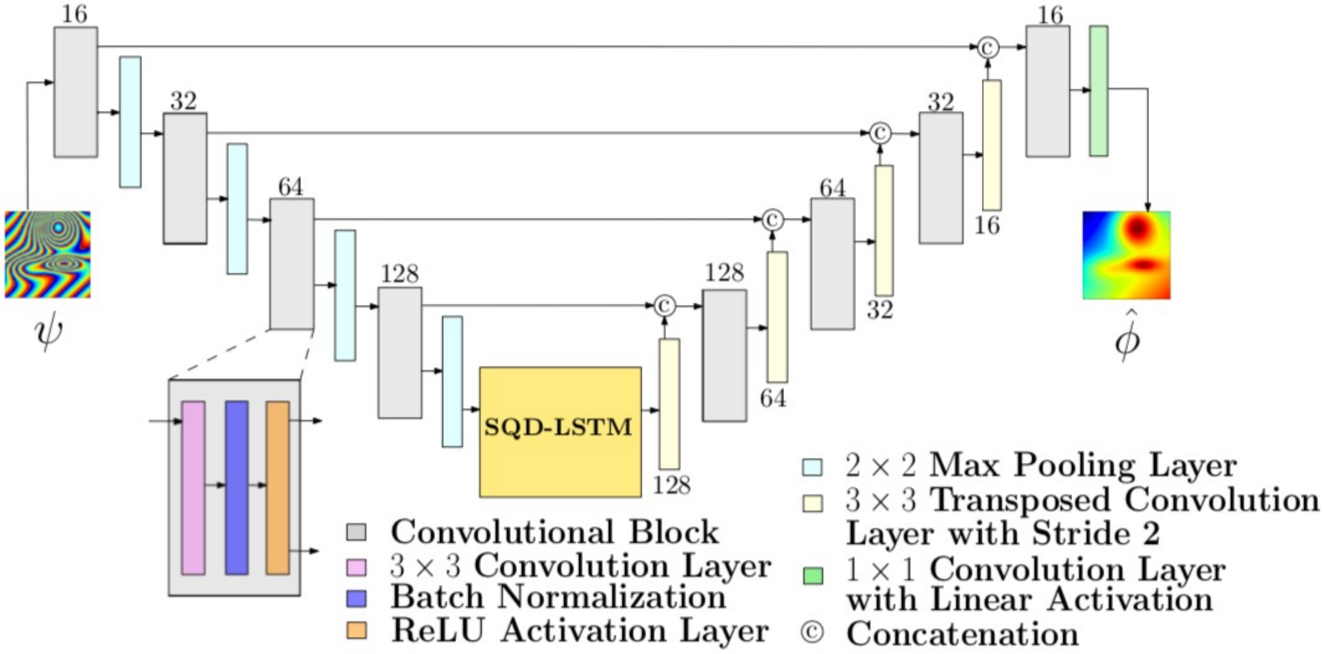
▶ Also, reframing the problem as a regression task has not been explored well

Malsha Perera, Ashwin De Silva
malshav@uom.lk, ashwind@uom.lk

Our Main Contributions

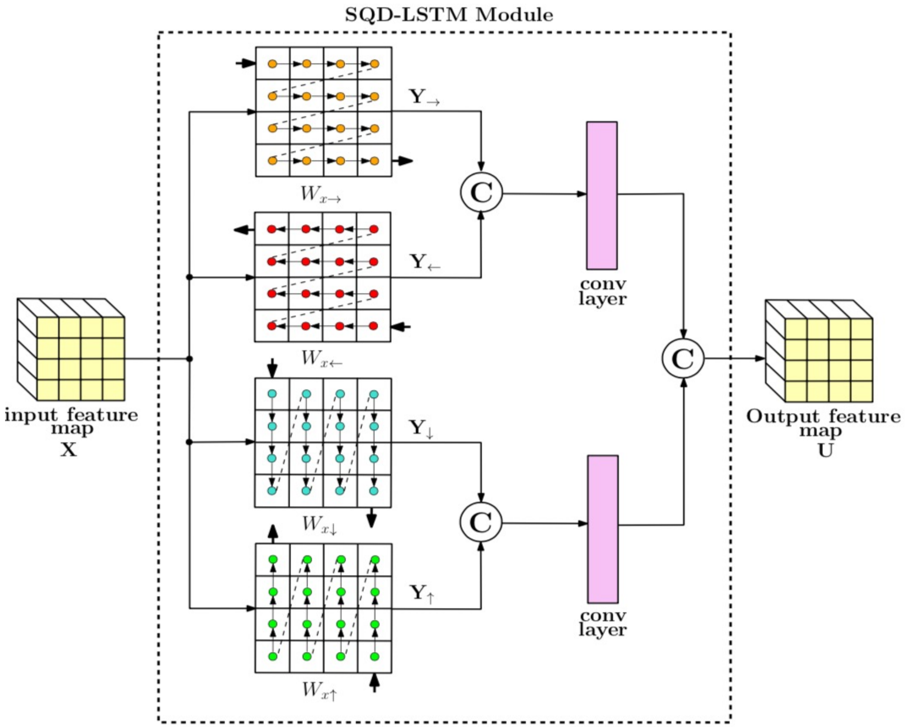
- A convolutional neural architecture modeled after the **U-Net**
- **Spatial Quad-Directional Long Short-Term Memory (SQD-LSTM) Block** to model global spatial dependencies
- Formulating the phase unwrapping problem as a **regression task**
- A problem-specific **composite loss function**

Network Architecture



Malsha Perera, Ashwin De Silva
malshav@uom.lk, ashwind@uom.lk

SQD-LSTM Block



$$x_{\rightarrow} = \{\mathbf{r}_i\}_{i=1\ldots h}\,; \mathbf{r}_i = (x_{i1}, x_{i2}, \ldots, x_{iw})$$
$$x_{\leftarrow} = \{\mathbf{r}_i\}_{i=h\ldots 1}\,; \mathbf{r}_i = (x_{iw}, \ldots, x_{i2}, x_{i1})$$
$$x_{\downarrow} = \{\mathbf{r}_i\}_{i=1\ldots w}\,; \mathbf{r}_i = (x_{1i}, x_{2i}, \ldots, x_{hi})$$
$$x_{\uparrow} = \{\mathbf{r}_i\}_{i=w\ldots 1}\,; \mathbf{r}_i = (x_{hi}, \ldots, x_{2i}, x_{1i})$$

$$y^{(s)} = \mathrm{LSTM}(x^{(s)}, y^{(s-1)}; W_x, u)$$

SQD-LSTM Block learns the global spatial dependencies

Loss Function

- **MSE Loss for regression??**
- Since $\psi = \angle e^{j\phi}$, it follows that $\phi + 2n\pi \, ; n \in \mathbb{Z}$ family give rise to the **same wrapped phase** ψ
- Thus, the loss function should allow for **multiple solutions at convergence** while **increasing the similarity** between the predicted phase $\hat{\phi}$ and true phase ϕ

$$\mathcal{L}_c = \lambda_1 \mathcal{L}_{var} + \lambda_2 \mathcal{L}_{tv}$$

$$\mathcal{L}_{var} = \mathbb{E}\left[(\hat{\phi} - \phi)^2\right] - \left(\mathbb{E}\left[(\hat{\phi} - \phi)\right]\right)^2$$

Variance of Error Loss

$$\mathcal{L}_{tv} = \mathbb{E}\left[|\hat{\phi}_x - \phi_x| + |\hat{\phi}_y - \phi_y|\right]$$

Total Variation of Error Loss

Malsha Perera, Ashwin De Silva
malshav@uom.lk, ashwind@uom.lk

Experiments

- 256 x 256 Synthetic phase images were created by randomly setting parameters of the equation :

$$\phi(x,y) = m_1 x + m_2 y + C + \sum_{n=1}^{M} A \exp\left[-\left(\frac{(x-\mu_{n,1})^2}{2\sigma_{n,1}^2} + \frac{(x-\mu_{n,2})^2}{2\sigma_{n,2}^2}\right)\right]; (x,y) \in [-128,128) \times [-128,128)$$

- Two Datasets : (1) **Noise Free** (2) **Noisy** (0, 5, 10, 20, and 60 dB noise levels)
- **Train-Test Split : 5000 - 1000**
- **Ryu et al's network, PhaseNet 2.0,** and **QGPU** were implemented and tested
- To assess the significance of SQD-LSTM block and $\mathcal{L}_c$:
  - **U-Net (MSE Loss)** and **U-Net ($\mathcal{L}_c$ loss)** were implemented and tested

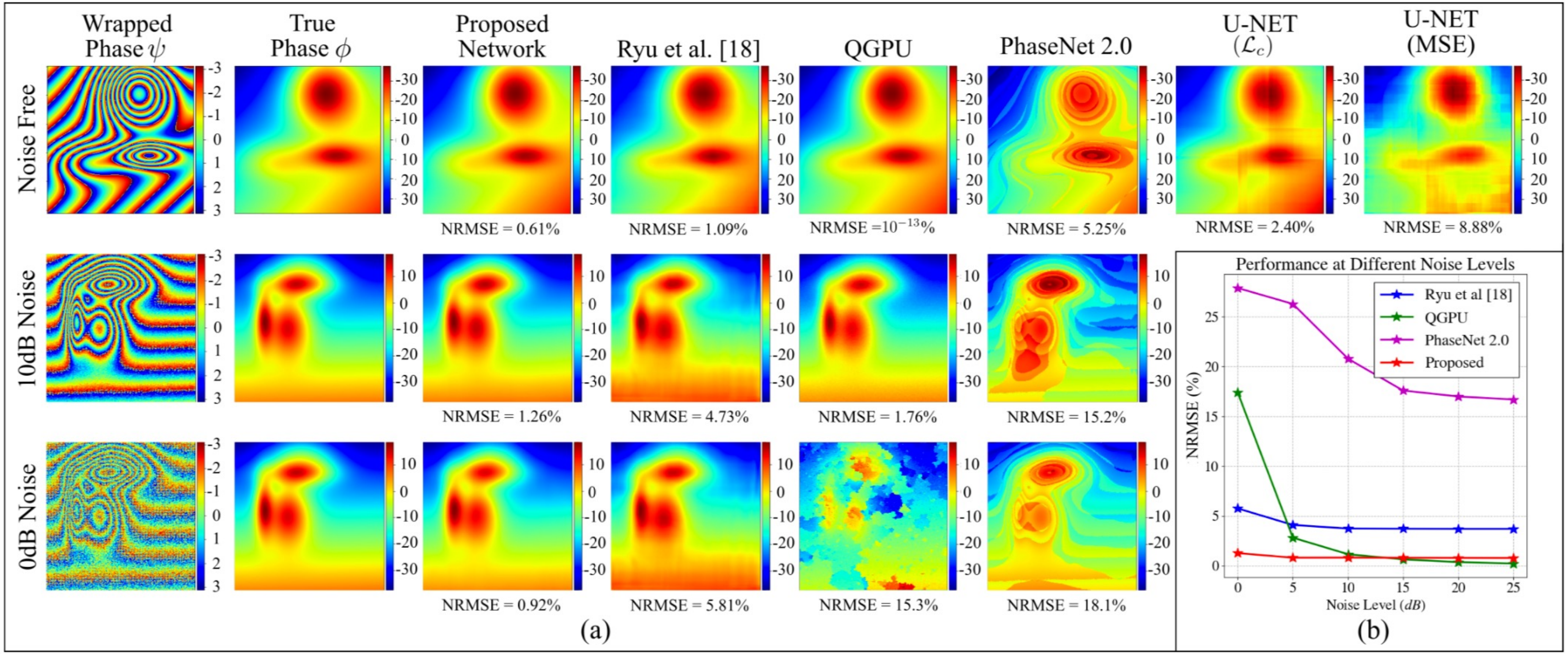
- **Normalized RMSE** was used as the performance metric $NRMSE = \frac{\sqrt{\mathbb{E}[(\phi - \hat{\phi})^2]}}{\phi_{max} - \phi_{min}}$

Results

Table 1. Results

Method	Noise Free NRMSE	Noisy NRMSE	Computational Time (s)
UNET (MSE)	14.24%	-	0.234
UNET ($\mathcal{L}_c$)	2.75%	-	0.262
Ryu et al.[18]	2.23%	3.84%	0.687
PhaseNet 2.0 [8]	9.41%	17.53%	0.234
QGPU [5]	$\mathbf{10^{-13}\%}$	5.04%	35.42
Proposed Method	0.84%	**0.90%**	**0.054**

Results



Malsha Perera, Ashwin De Silva
malshav@uom.lk, ashwind@uom.lk

Conclusions

- Formulated phase unwrapping as a **regression problem**
- Proposed a **novel convolutional architecture encompassing a SQD-LSTM Block**
- Introduced a **problem specific loss function**
- Achieved **state-of-the-art unwrapping performance** under severe **noise levels**
- Expends a signficantly **less computational time** during inference

- Future Work :
  - Applying the network to tackle phase unwrapping in QSM using MRI.
  - Experiments involving formulating the problem as a classification task
  - Experiments involving phase discontinuities

Acknowledgements

- Dr. Steffen Bollmann (Center for Advanced Imaging, Australia) for introducing us to this exciting problem.
- Dr. Kanghyun Ryu (Stanford University) for sharing the specifics of his work with us which later inspired this work.
- ICASSP 2021 Organizing Committee for organizing the virtual conference during these tough times.

Thank You!