

A JOINT CONVOLUTIONAL AND SPATIAL QUAD-DIRECTIONAL LSTM NETWORK FOR PHASE UNWRAPPING

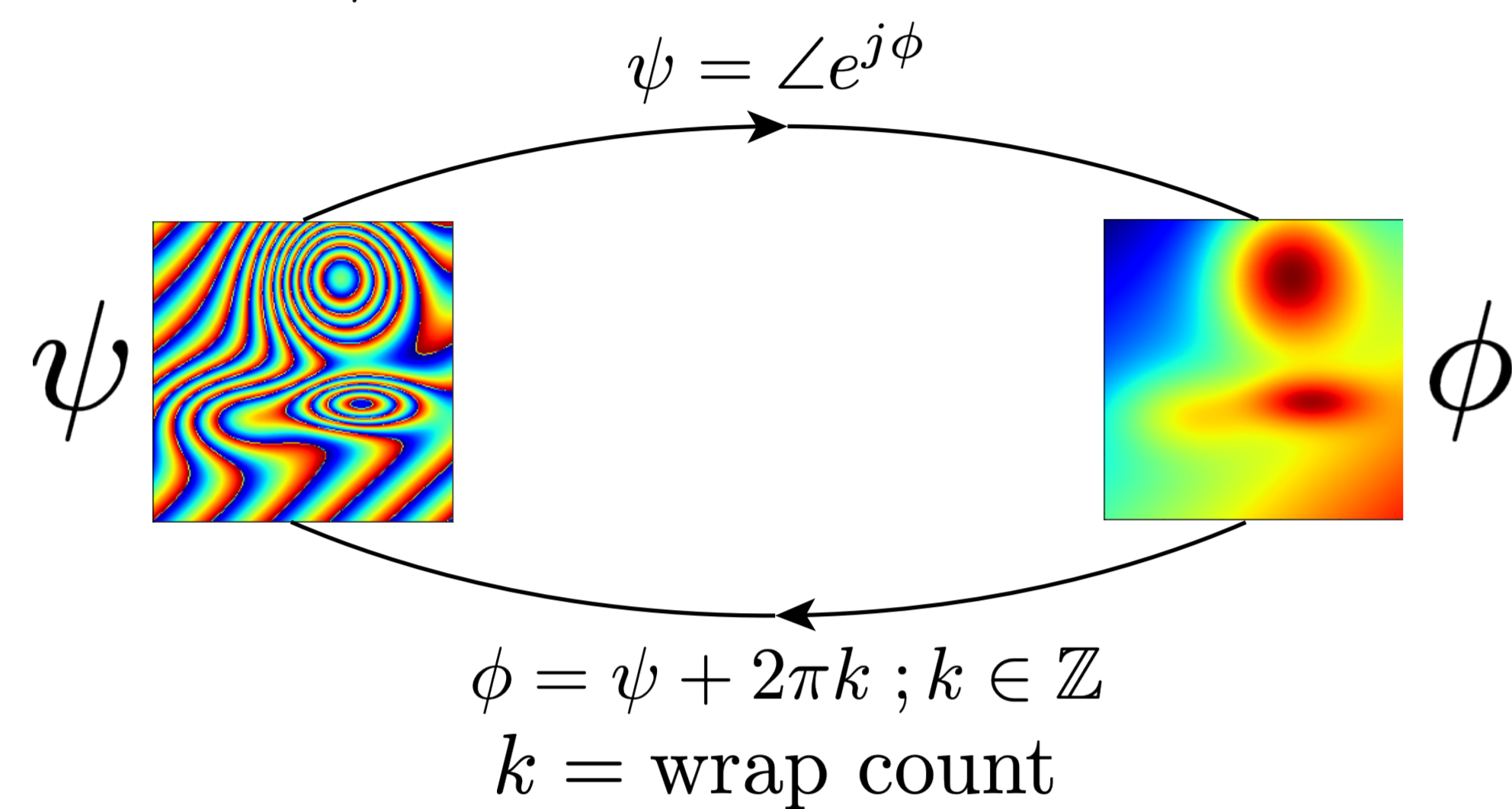
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Introduction

Objective of phase unwrapping : Recover the true phase ϕ from wrapped phase ψ



Since $\psi = \angle e^{j\phi}$, the family denoted by $\phi + 2\pi n; n \in \mathbb{Z}$ will give rise to the same wrapped phase ψ

Phase unwrapping becomes challenging at the presence of **noise, phase discontinuities, and rapid variation of phase.**

Phase unwrapping (PU) problem is prevalent in applications such as quantitative susceptibility mapping in MRI, synthetic aperture radar interferometry, and fringe projection techniques.

Previous Work

1. Path Following Algorithms (QGPU, Branch-Cut Algorithms, etc.)
2. Minimum Norm Approaches
3. CNN based Approaches
 - Formulating PU as a Semantic Segmentation Task (PhaseNet 2.0)
 $\phi^* = \psi + 2\pi f(\psi; \theta^*)$ where $\theta^* = \operatorname{argmin}_{\theta} \mathcal{L}(f(\psi; \theta), k)$
 - Formulating PU as a Regression Task (Ryu et al.)
 $\phi^* = g(\psi; \theta^*)$ where $\theta^* = \operatorname{argmin}_{\theta} \mathcal{L}(g(\psi; \theta), \phi)$

Shortcomings : **High Computational Cost (2), Low Noise Robustness (1), Difficulty of CNNs to model global spatial dependencies (3), High Data-intensiveness (3), Inappropriate Loss Functions (3)**

Methodology

Regression formulation : $\phi^* = g(\psi; \theta^*)$ where $\theta^* = \operatorname{argmin}_{\theta} \mathcal{L}_c(g(\psi; \theta), \phi)$

here, $\mathcal{L}_c = \lambda_1 \mathcal{L}_{var} + \lambda_2 \mathcal{L}_{tv}$ (problem-specific composite loss function)

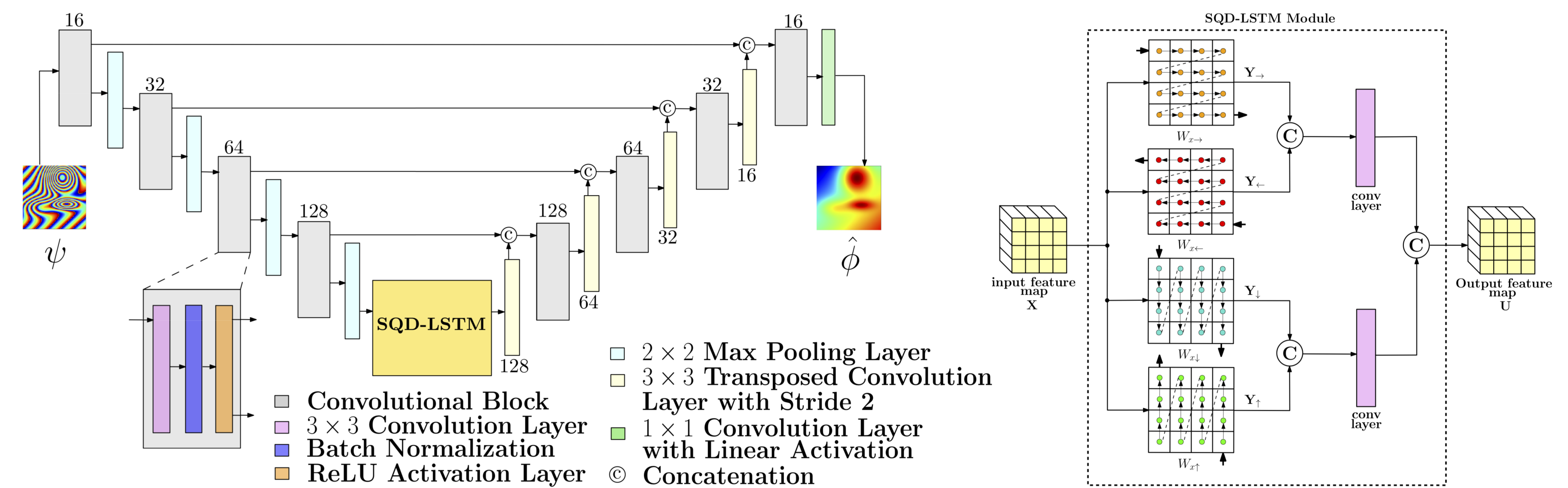
Variance of Error Loss $\rightarrow \mathcal{L}_{var} = \mathbb{E}[(\hat{\phi} - \phi)^2] - (\mathbb{E}[(\hat{\phi} - \phi)])^2$

Total Variation of Error Loss $\rightarrow \mathcal{L}_{tv} = \mathbb{E}[|\hat{\phi}_x - \phi_x| + |\hat{\phi}_y - \phi_y|]$

Here, ϕ is the true phase and $\hat{\phi} = g(\psi; \theta)$ is the predicted phase

The proposed \mathcal{L}_c allows for multiple solutions at convergence while enforcing the similarity between predicted phase and true phase.

The function g is estimated by the following joint convolutional and spatial quad-directional LSTM (SQD-LSTM) network.



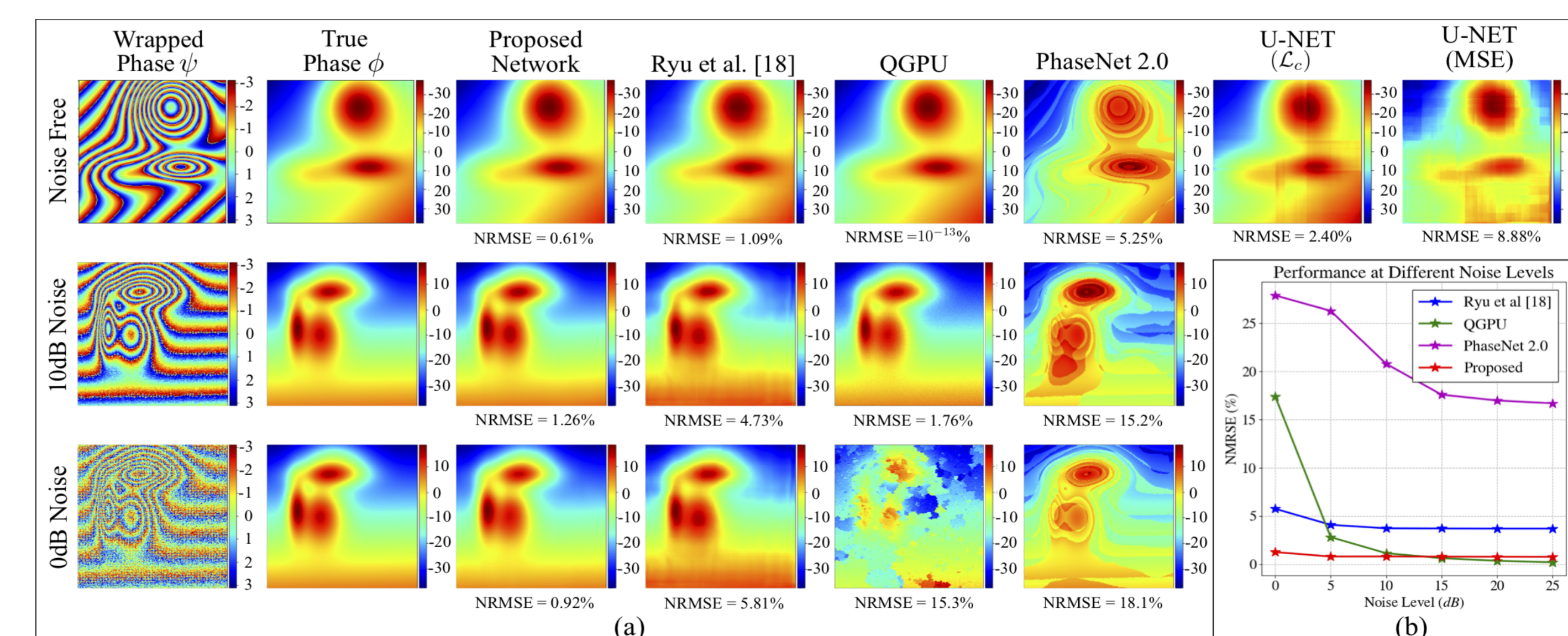
The SQD-LSTM block learns the global spatial dependencies within the phase images.

Experiments and Results

Phase images were simulated by adding gaussians of random shapes and positions, together with ramp phase.

Datasets : (1) Noise Free (2) Noisy (0, 5, 10, 20, and 60 dB noise levels)

Train-Test Split = 5000 : 1000



PU Performance Metric \rightarrow

$$\text{NRMSE} = \frac{\sqrt{\mathbb{E}[(\phi^* - \phi)^2]}}{\phi_{max} - \phi_{min}}$$

Table 1. Results

Method	Noise Free NRMSE	Noisy NRMSE	Computational Time (s)
UNET (MSE)	14.24%	-	0.234
UNET (\mathcal{L}_c)	2.75%	-	0.262
Ryu et al.[18]	2.23%	3.84%	0.687
PhaseNet 2.0 [8]	9.41%	17.53%	0.234
QGPU [5]	10 ⁻¹³ %	5.04%	35.42
Proposed Method	0.84%	0.90%	0.054

Conclusions

- Achieved state-of-the-art PU performance under severe noise levels
- Expends a significantly less computational time during inference