SINGLE-POINT ARRAY RESPONSE CONTROL WITH MINIMUM PATTERN DEVIATION

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Introduction

Beampattern synthesis is one of the key techniques in the field of multi-antenna research, and has been applied to radar, sonar, and wireless communications. After determining a set of appropriate weight vectors, the controllable pattern is obtained to improve the system performance.

This paper presents a new beampattern synthesis algorithm so that the magnitude response at a given direction can be accurately adjusted. By analyzing the optimal weight vector in adaptive beamforming, it is found that the array response at one single direction is determined by a complex factor, and there are infinite complex factors that can achieve accurate magnitude response control. Moreover, the minimum pattern deviation criterion is established to optimize the complex factor. This results in the so-called single-point array response control with minimum pattern deviation (SPARC-MPD) method.

Background

The main task of beampattern synthesis is to design a complex weight vector $\mathbf{w}$ that the corresponding pattern satisfies some specific requirements, and the pattern of the designed weight is

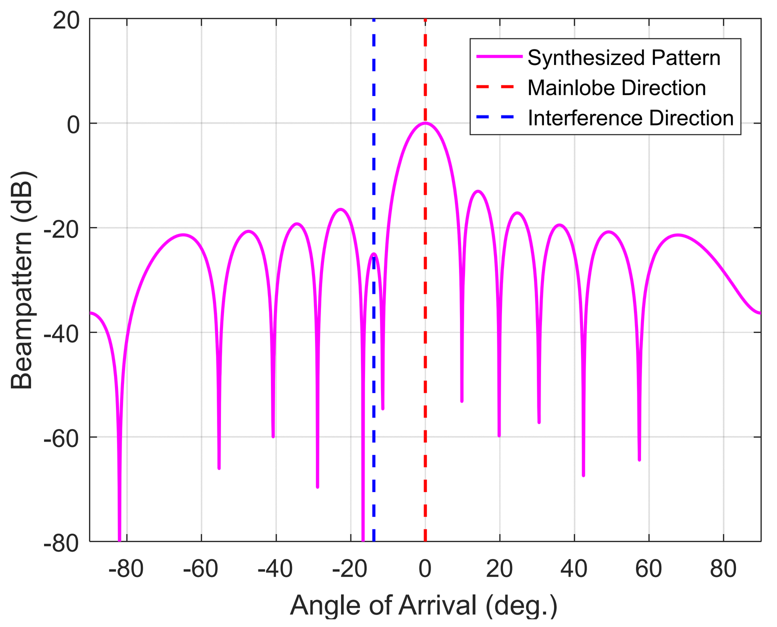
$$P_{\mathbf{w}}(\theta)=\mathbf{w}^H\mathbf{a}(\theta),\theta\in\Theta$$

In beampattern synthesis, it is desirable to design a pattern that has the minimum pattern error with the reference one in the mainlobe region Θ_M, and has a maximum ratio of the mainlobe power to the peak-sidelobe power on the sidelobe region Θ_S:

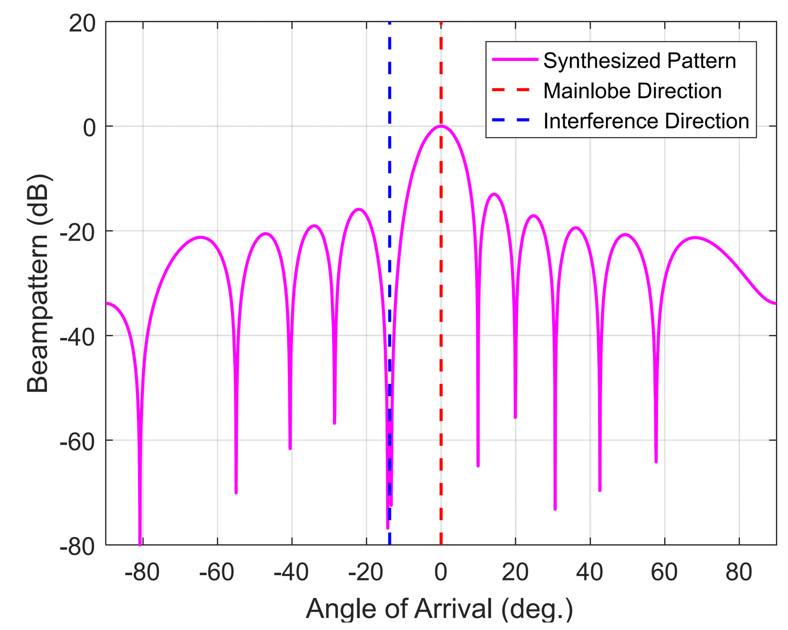
$$\left\||P_{\mathbf{w}}(\theta)|-|P_r(\theta)|\right\|\le\varepsilon,\forall\theta\in\Theta_M$$

$$|P_{\mathbf{w}}(\theta)|\le\eta,\forall\theta\in\Theta_S$$

The main idea of this paper comes from adaptive array theory. Specifically, in adaptive beamforming, the pattern is adaptively obtained based on the received data, and the array response at the interference direction would decrease with the increase of the interference power to suppress the interference component.



The pattern associated with the interference at −13.8° with mediate power.



The pattern associated with the interference at −13.8° with high power.

Proposed Approach

It is known that, in adaptive array theory, the optimal weight vector is a linear combination of the signal steering vector and the interference steering vector multiplied by a complex factor, the signal vector affects the beam axis, and the interference component determines the direction of null.

$$\mathbf{w}_{opt}=\mathbf{R}_{i+n}^{-1}\mathbf{a}_0\Rightarrow\mathbf{w}_{opt}=\mathbf{a}_0+\mu_{opt}\mathbf{a}_i\qquad\mu_{opt}=-\frac{\mathbf{a}_i^H\mathbf{a}_0}{INR+\|\mathbf{a}_i\|^2}$$

where $\mathbf{a}_0$ is the steering vector of mainbeam direction, $\mathbf{a}_i$ is the steering vector of interference direction, INR is the interference to noise ratio. Moreover, the complex factor determines the depth of null.

Inspired by adaptive array theory, the designed weights in this paper is constructed as an iterative form, and the desired pattern is obtained by successively adjusting the array response at the directions where the requirements do not meet. The designed weights in the k-th step is formulated as:

$$\mathbf{w}_k=\mathbf{w}_{k-1}+\mu_k\mathbf{a}_k$$

where $\mathbf{w}_{k-1}$ is the designed weights at the k-1-th step, $\mathbf{a}_k$ is the steering vector of response control direction θ_k, and the designed complex factor is μ_k

Accurate pattern control is the basis of desired beampattern synthesis. To this end, the normalized array response at direction θ_k is defined as:

$$L_{\mathbf{w}_k}(\theta_k,\theta_0)\triangleq\frac{P_{\mathbf{w}_k}(\theta_k)}{P_{\mathbf{w}_k}(\theta_0)}=\rho_k e^{j\phi_k}$$

The magnitude response is obtained based on the requirements of the reference pattern

$$\rho_k=\frac{P_r(\theta_k)}{P_r(\theta_0)}$$

The magnitude response deviation at the previous step is considered to determine the response control direction:

$$E_{k-1}(\theta)\triangleq\left||P_{\mathbf{w}_{k-1}}(\theta)|-|P_r(\theta)|\right|$$

Mainlobe array response control direction:

$$\theta_{M,k}=\arg\max_{\theta\in\Theta_M}\{|E_{k-1}(\theta)|\}$$

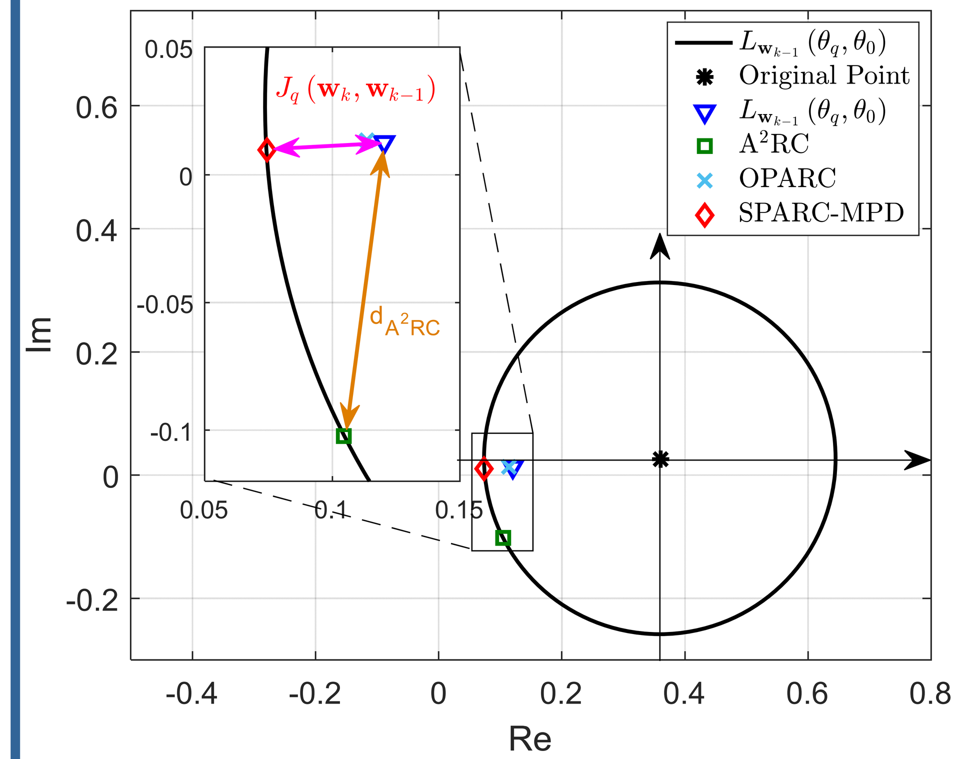
Sidelobe array response control direction:

$$\theta_{S,k}=\arg\max_{\theta\in\Theta_S}\{E_{k-1}(\theta)\}$$

For $\theta_{M,k}$ and $\theta_{S,k}$, if the mainlobe pattern is not satisfactorily synthesized, the angle where the magnitude response deviates most from the reference level is selected as θ_k. Otherwise, set $\theta_k=\theta_{S,k}$.

The phase ambiguity issue: there are infinite solutions to achieve the accurate magnitude response control, and the trajectory set of complex factor is a circle.

$$\mu_k=\frac{\rho_k e^{j\phi_k}\mathbf{a}_0^H\mathbf{w}_{k-1}-\mathbf{a}_k^H\mathbf{w}_{k-1}}{\|\mathbf{a}_k\|^2-\rho_k e^{j\phi_k}\mathbf{a}_0^H\mathbf{a}_k}$$



Realize accurate array response control at an arbitrary direction θ_k, the geometric illustration of the trajectory set of the array response at another direction θ_q with different φ_k.

For the phase response, the idea criterion is to obtain a desired array response in one direction while the responses in other directions remain unchanged. However, this criterion is invalid since the array response is a continuous function of the designed weights.

An appropriate phase response is the one that the corresponding weight vector is capable of achieving the accurate magnitude response at an intended direction and minimizing the pattern deviation at other directions.

Pattern deviation: $J_q(\mathbf{w}_k,\mathbf{w}_{k-1})=\left|L_{\mathbf{w}_k}(\theta_q,\theta_0)-L_{\mathbf{w}_{k-1}}(\theta_q,\theta_0)\right|$

Minimum pattern deviation problem:

$$\left.\begin{aligned}&\min_{\phi_k}\sum_{q=1}^{Q}J_q(\mathbf{w}_k,\mathbf{w}_{k-1}),\theta_q\in\Theta\\&s.t.\ \mathbf{w}_k=\mathbf{w}_{k-1}+\frac{\rho_k e^{j\phi_k}\mathbf{a}_0^H\mathbf{w}_{k-1}-\mathbf{a}_k^H\mathbf{w}_{k-1}}{\|\mathbf{a}_k\|^2-\rho_k e^{j\phi_k}\mathbf{a}_0^H\mathbf{a}_k}\mathbf{a}_k\end{aligned}\right\}\Rightarrow\phi_k=\angle\left(\frac{\mathbf{a}_k^H\mathbf{w}_{k-1}}{\mathbf{a}_0^H\mathbf{w}_{k-1}}\right)$$

Conclusion

This paper presents a single-point array response control with minimum pattern deviation (SPARC-MPD) approach to synthesize beampattern. Unlike previous contributions of other adaptive array theory-based methods, the SPARC-MPD approach can adjust the magnitude response at one preassigned direction and minimize the pattern deviations at other directions. Moreover, the SPARC-MPD approach obtains desired weights in a low-complexity manner. Numerical results under the linear and planar arrays are carried out to validate the effectiveness of our approach, and the reasons for the pattern distortion issue in A^2RC and OPARC are analyzed. In future work, we shall consider how to obtain desired patterns with multi-point array response control to improve the efficiency of beampattern synthesis.

Simulation Results

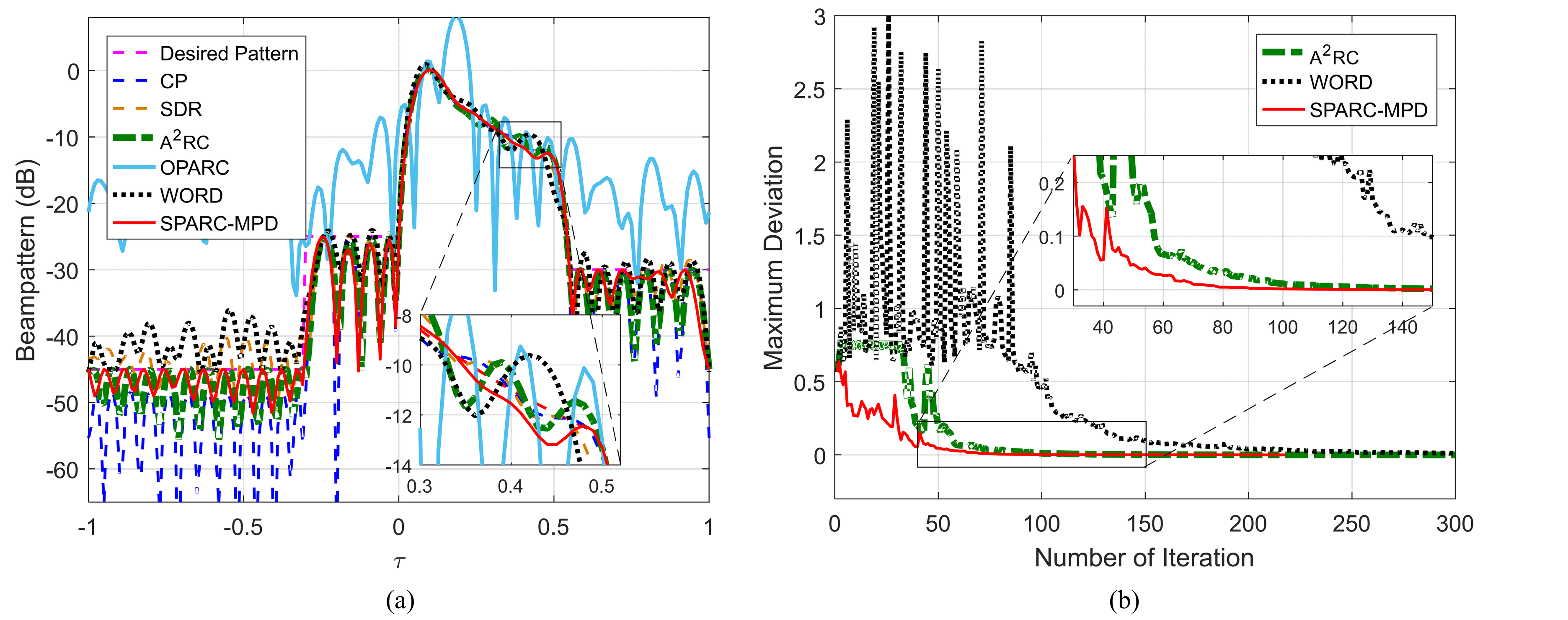
Cosecant pattern with nonuniform sidelobe:

Consider an array composed of 30 half-wavelength spaced isotropic elements. The reference pattern is a cosecant beam with

$$P_r(\tau)=\begin{cases}cosecant(\tau),0.1<\tau\le0.5\\-45dB,-1\le\tau\le-0.3\\-25dB,-0.3<\tau\le0\\-30dB,0.55\le\tau\le1\end{cases},\tau=sin(\theta),\theta\in\Theta$$

In this example, the maximum magnitude response deviation is introduced to intuitively explain the convergence speed of the tested methods. At the k-th step, the maximum magnitude response deviation is defined as:

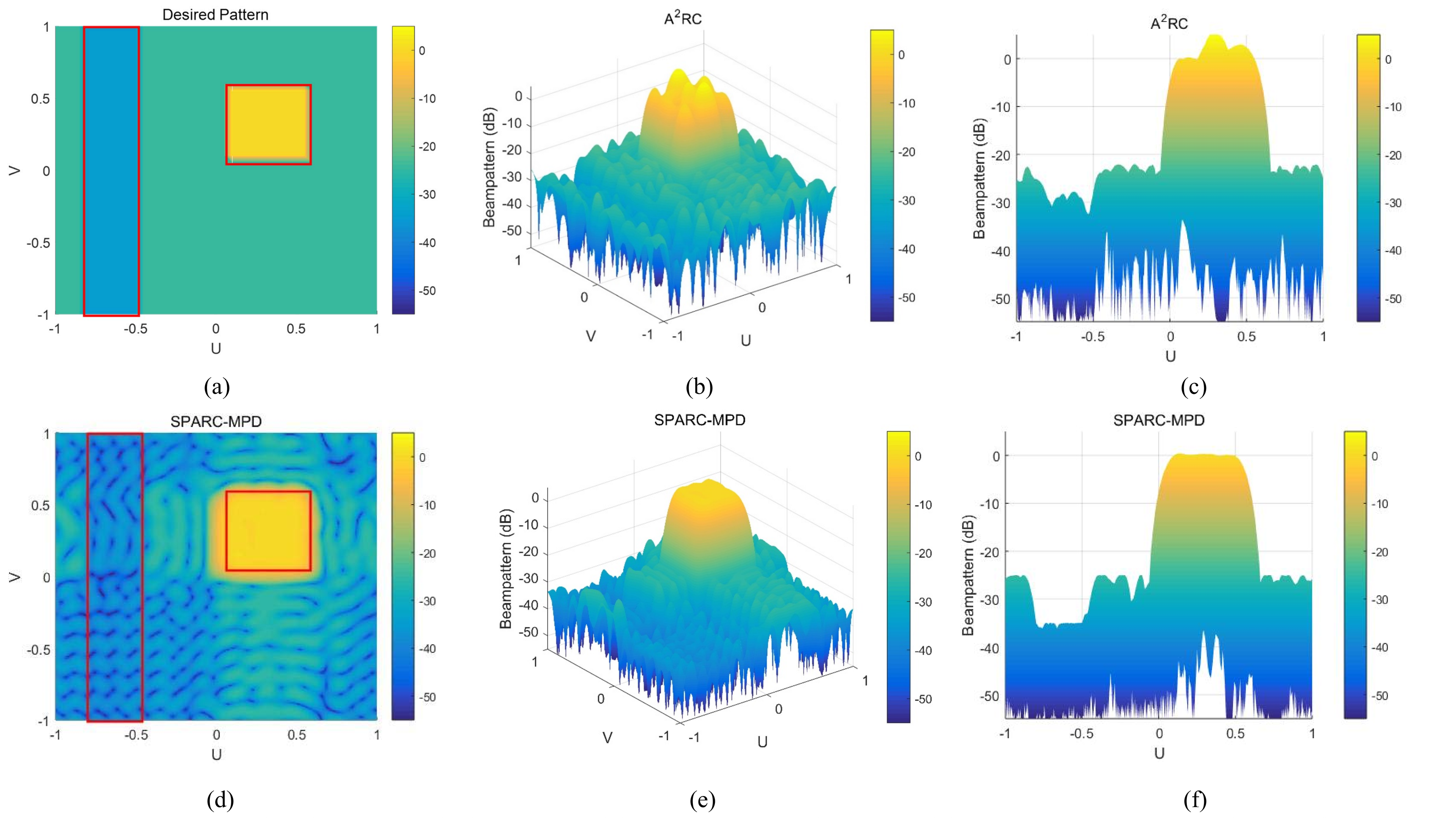
$$Max_k=\max_{\theta\in\Theta}\{E_k(\theta)\}$$



Synthesized cosecant pattern with nonuniform sidelobe. (a) Patterns associated with the tested approaches; (b) Maximum deviation versus the iteration number.

Pattern synthesis for two-dimensional arrays pattern with nonuniform sidelobe:

A rectangular array structure composed of 16×16 isotropic elements with half a wavelength is considered, and the steering vector becomes $\mathbf{a}(u,v)$ with $u=\sin(\theta)\cos(\varphi)$ and $v=\sin(\theta)\sin(\varphi)$, θ and φ are the elevation and azimuth angles, respectively. The reference pattern steers at $(u_0,v_0)=(0.3,0.3)$, the normalized magnitude array response at the mainlobe region $\Theta_M=\{(u,v)|0.1\le u\le0.5\cap0.1\le v\le0.5\}$ is 0dB. The notch area is $\Theta_N=\{(u,v)|-0.8\le u\le-0.5\}$ with the upper level -35 dB, and the sidelobe level is lower than -25 dB in the rest of the area.



2-D pattern synthesis with a planar array. (a) The top view of desired pattern; (b) The pattern of the A^2RC approach; (c) A side view of the pattern of the A^2RC approach; (d) The top view of the pattern of SPARC-MPD approach; (e) The pattern of the SPARC-MPD approach; (f) A side view of the pattern of the SPARC-MPD approach.