TCLA Array: A New Sparse Array Design with Less Mutual Coupling

Ahmed M. A. Shaalan, Jun Du and Yan-Hui Tu

School of Information Science and Technology, University of Science and Technology of China, Hefei, Anhui, China
ShaalanAhmed@outlook.com, jundu@ustc.edu.cn, tuyanhui@ustc.edu.cn

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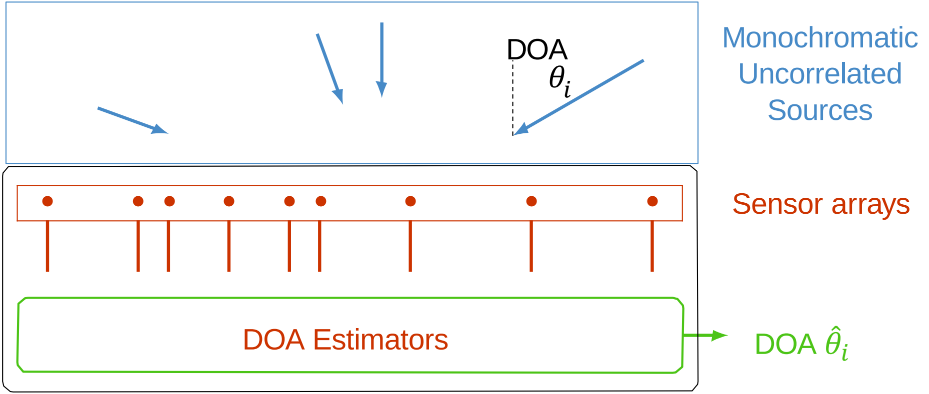
Outline

1. Introduction (DOA, Sensor Arrays, ...)
2. Mutual Coupling in Sensor Arrays
3. TCLA Arrays
4. Numerical Examples
5. Concluding Remarks

Outline

DOA estimation and sensor arrays[1]



Uniform Linear Array (ULA) was the common sensor array

[1] Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory, 2002.*

ULA and sparse arrays

ULA (not sparse)

- detect at most $N-1$ uncorrelated sources, given N sensors[1]
- Can only find fewer sources than sensors.

Traditional Sparse arrays

1. Minimum redundancy arrays (MRA)[2]
2. Minimum hole arrays (MHAs)[3]

- Identify $O(N^2)$ uncorrelated sources with $O(N)$ physical sensors.
- More sources than sensors
- No closed forms for sensor locations

[1] Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory*, .2002
[2] Moffet, *IEEE Trans. Antennas Propag.*,1968.
[3] H. Taylor, S. W. Golomb, *"Rulers, Part I", Tech. Rep. 85-05-01, Univ. Southern Calif., Los Angeles (1985)*

Array design criteria and new sparse arrays

Array design criteria

1. The array should have a closed form expression for its sensor locations.
2. The array should have a large central ULA segment in its co array.
3. …

New sparse arrays

1. Co-prime arrays (CPAs)[1]
2. Nested arrays (NAs)[2]

Novel arrays

- Criterion 1 ✓
- Criterion 2 ✓

3. Generalized co-prime arrays (CACIS & CADiS)[3]
4. Thinned co-prime arrays (TCAs)[4]
5. Optimized co-prime arrays (OpCA)[5]
6. Padded co-prime arrays (PCAs)[6]
7. Improved nested arrays (INAs)[7]
8. Generalized nested arrays (GNAs)[8]

[1]Vaidyanathan and Pal, *IEEE Trans. Sig. Proc., 2011.*

[2]Pal and Vaidyanathan, *IEEE Trans. Sig. Proc., 2010.*

[3]S. Qin Y. D. Zhang, and M. G. Amin, *IEEE Trans. Sig. Proc., 2015.*

[4]A. Raza, W. Liu and Q. Shen, *IEEE Trans. Sig. Proc., 2019.*

[5]Ahmed M. A. Shaalan and X. Yu, *IEEE Access, 2019.*

[6]Zheng, Zhang, Wang, Shen and Champagne, *IEEE Trans. Signal Proc.*, 2020.

[7]Yang, Sun, Yuan, and Chen, *Electron. Lett.*, 2016.

[8]Shi, Hu, Zhang and Zhou, *IEEE Com. Lett., 2018.*

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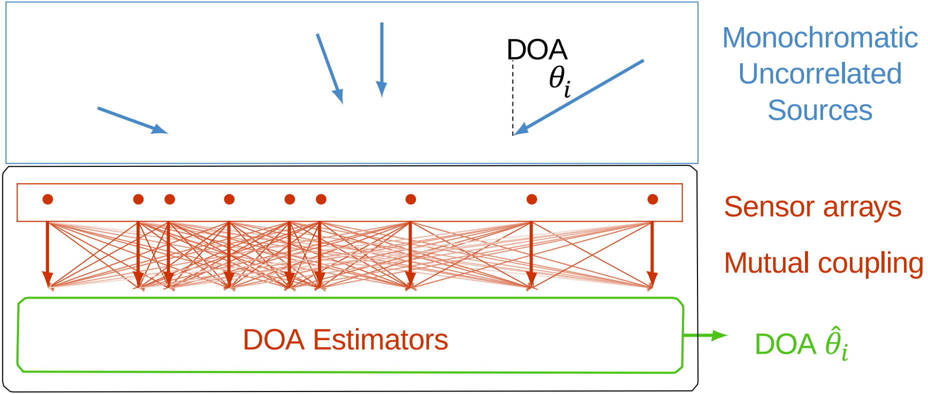
DOA estimation in the presence of mutual coupling[1]



So, mutual coupling is characterized by the space $\tilde{\rho}$ between two sensors and this space weight function $w(\tilde{\rho})$.

[1] Van Trees, *Optimum Array Processing: Part IV of Detection, Estimation, and Modulation Theory*, .2002

Array design criteria and recently proposed sparse arrays

Array design criteria

1. The array should have a closed form expression for its sensor locations.
2. The array should have a large central ULA segment in its co array.
3. The array should have small weight functions for small inter-sensor spacings/separations.

Recently proposed sparse arrays

1. Minimum inter-sensor spacing constraint (MISC) arrays[1] ★★
2. Augmented nested arrays (ANAs)[2]
3. Super nested arrays (SNA)[3]

- Criterion 1 ✓
- Criterion 2 ✓
- Criterion 3 ✓

- ANA ★★★
- SNA ★★★★

[1] Z. Zheng, W. Wang, Y. Kong and Y. D. Zhang, *IEEE Trans. Signal Proc., 2019.*
[2] J. Liu, Y. Zhang, Y. Lu, S. Ren, and S. Cao, *IEEE Trans. Signal Proc., 2017*.
[3] C. L. Liu and P. P. Vaidyanathan, *IEEE Trans. Signal Proc., 2016*.

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Goal: Desired properties of TCLA arrays

- Having closed-form sensor locations for any given sensor number N.
- Having a considerably large number of uniform Degrees Of Freedoms (DOFs), which should be, at least, no less than the uniform DOFs of nested and super arrays.
- Being sparser than the super nested array,

$$\omega_{TCLA}(1) \leq \omega_{Supernested}(1),$$
$$\omega_{TCLA}(2) \leq \omega_{Supernested}(2),$$
$$\omega_{TCLA}(3) \leq \omega_{Supernested}(3).$$

TCLA array geometry: Formal definition[1]

	Nested array		TCLA
N	N_1	N_2	N^o
Even	$N/2$ (odd)	$N/2$	$(N_1+1)/2$
Even	$N/2$ (even)	$N/2$	$N_1/2$
Odd	$(N_1-1)/2$ (odd)	$(N_1+1)/2$	$(N_1+1)/2$
Odd	$(N_1-1)/2$ (even)	$(N_1+1)/2$	$(N_1+1)/2$

Assume that N^o is obtained from the nested array optimal parameter N_1 as in Table 1, and N^t and N^e are afterwards determined as N^t-1, and $N-2N^o+1$ in sequential, TCAL arrays are, then, specified by the integer set $\mathbb{P}$, defined by

$$\mathbb{P} = \{\mathbb{P}_1 \cup \mathbb{P}_2 \cup \mathbb{P}_3\}, \quad \text{where}$$

[1] Ahmed M. A. Shaalan, J. Du and Y. Tu, *IEEE ICASSP*, 2021.

TCLA array geometry: Depiction

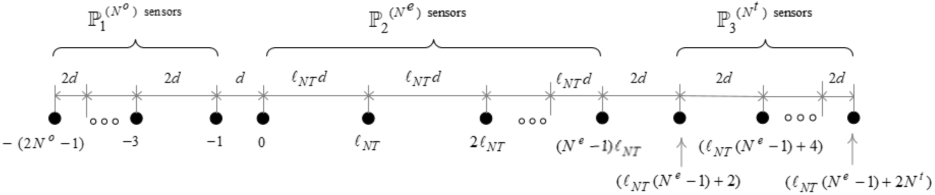
$$\mathbb{P}_1 = \{-(1+2\ell_o) | 0 \leq \ell_o \leq N^o - 1\}$$

$$\mathbb{P}_2 = \{(\ell_e \ell_{NT}) | 0 \leq \ell_e \leq N^e - 1\}$$

$$\mathbb{P}_3 = \{\ell_{NT}(N^e - 1) + 2\ell_t | 1 \leq \ell_t \leq N^t - 1\},$$

where $\ell_{NT} = 2N^o$.

three ULA

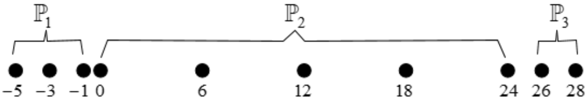


TCLA array geometry

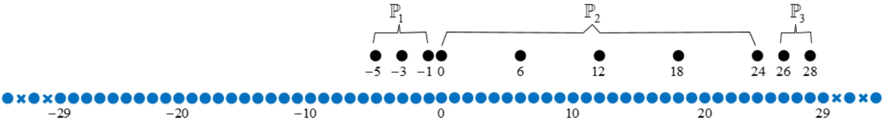
TCLA array geometry: A concrete example

The TCLA array with $N = 10$

$N^o = 3$ as $N_1 = 5$, and $N^t = 2$ with $N^e = 10 - (3+2) = 5$



Difference co-array $\mathbb{D} = \left\{\rho_i - \rho_j \middle| \rho_i, \rho_i \in \mathbb{P}\right\}$



Main properties of TCLA arrays: 1- Difference co-array[1]

Difference co-array of the TCLA array with N^o , N^e and N^t has a central ULA part within the range $[-(2N^oN^e-1), 2N^oN^e-1]^1$.

Difference co-array of the (super) Nested array with N_1 *and* N_2 *has a ULA part bounded by* $\pm N_2(N_1+1)-1$.

	NA/SNA			TCLA array			
N	N_1	N_2	uDOFs	N^o	N^t	N^e	uDOFs
10	5	5	29	3	2	5	29
12	6	6	41	3	2	7	41
13	6	7	48	4	3	6	47
15	7	8	63	4	3	8	63

[1] Ahmed M. A. Shaalan, J. Du and Y. Tu, *IEEE ICASSP*, 2021.

Main properties of TCLA arrays: 2- Weight functions[1]

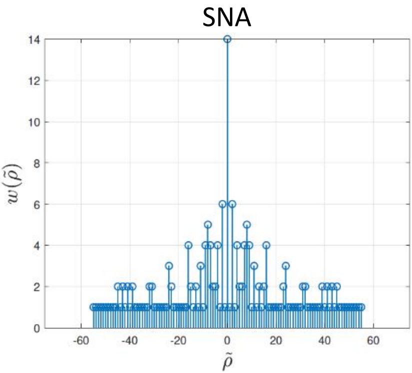
- The proposed geometry for TCLA arrays allows us to prove closed-form expressions for $\omega(1)$, $\omega(2)$, and $\omega(3)$.

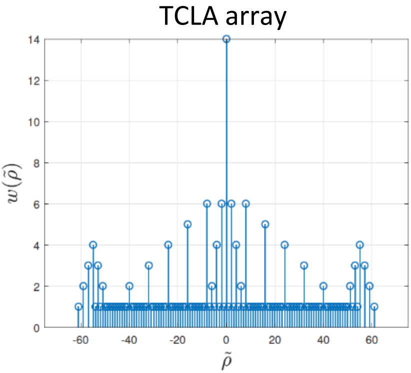
	NA	SNA	TCLA array
$\omega(1)$	N_1	$\begin{cases} 2, if\, N_1 \text{ is even}, \\ 1, if\, N_1 \text{ is odd}. \end{cases}$	1 ✓
$\omega(2)$	$N_1 - 1$	$\begin{cases} N_1 - 3, if\, N_1 \text{ is even}, \\ N_1 - 1, if\, N_1 \text{ is odd}. \end{cases}$	$N^o + N^t - 1$
$\omega(3)$	$N_1 - 2$	$\begin{cases} 3, if\, N_1 = 4, 6, \\ 4, if\, N_1 \text{ is even}, \\ 1, if\, N_1 \text{ is odd}. \end{cases}$	1 ✓

[1] Ahmed M. A. Shaalan, J. Du and Y. Tu, *IEEE ICASSP*, 2021.

Main properties of TCLA arrays: 2- The weight function distribution

- The proposed geometry for TCLA arrays has smaller weight functions for separations larger than 3.





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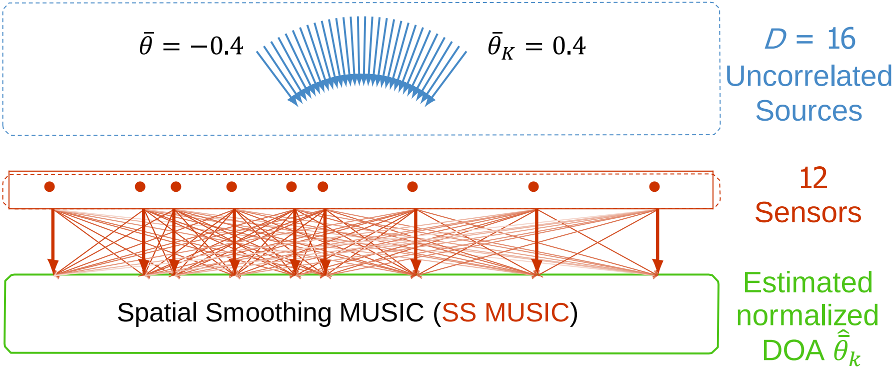
Simulation procedure[1]

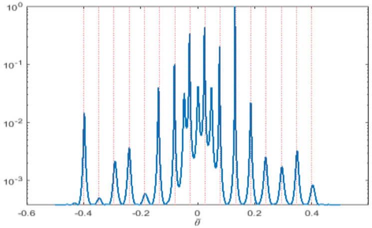


0 dB SNR, 1000 snapshots, $c_1 = 0.3e^{j\pi/3}$, $B = 50$; and
$c_i = c_1 e^{-j(i-1)\pi/8}/i$.

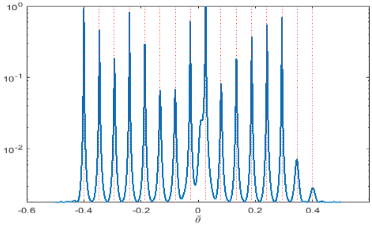
[1]Ahmed M. A. Shaalan, J. Du and Y. Tu, *IEEE ICASSP*, 2021.

MUSIC spectra and RMSE under mutual coupling

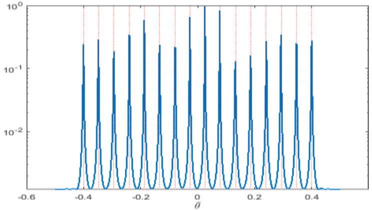
Nested Array (RMSE=0.0368)



SNA (RMSE=0.0075)



TCLA array (RMSE=0.0013)



The TCLA array is much better in the estimation accuracy.

Outline

Concluding remarks

- TCLA arrays
  - They have the same number of sensors, and the same uniform detection capacity as super nested arrays.
  - They have reduced mutual coupling than super nested arrays.

  .
- In the future, the high order extensions of the proposed TCLA array are introduced.
- For more information on TCLA arrays, Please refer to [1].

Thank you!

[1] Ahmed M. A. Shaalan, J. Du and Y. Tu, *IEEE Trans. Sig. Proc., under review*.