Millimeter Wave MIMO Channel Estimation with 1-bit Spatial Sigma-delta Analog-to-Digital Converters

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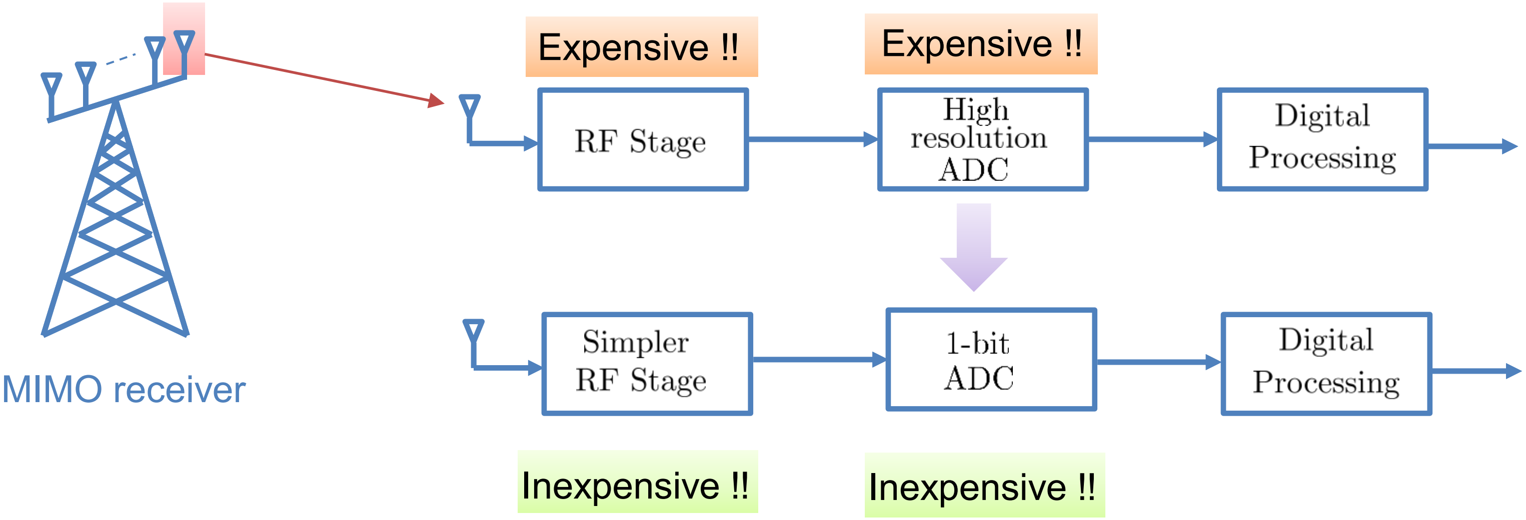
Millimeter-wave MIMO communication

- mmWave MIMO systems can provide cellular data rates of the order of Giga bit per second.



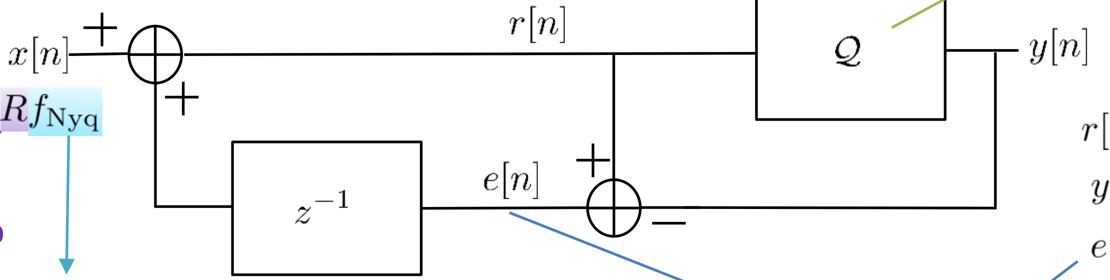
- Dedicated high-resolution quantizer per radio frequency (RF) chain is expensive.

- Solution : Use low-resolution/ coarse data converters. Issue : degradation in performance !

Sigma-delta quantization

- Classical technique to increase effective resolution of quantizers.

$$\mathcal{Q}(x) = b\text{sign}(\Re(x)) + jb\text{sign}(\Im(x))$$



sampled at $f_s = Rf_{\text{Nyq}}$

Oversampling ratio

Nyquist rate

$$\begin{aligned} r[n] &= x[n] + r[n-1] - y[n-1] \\ y[n] &= \mathcal{Q}(r[n]) \\ e[n] &= r[n] - y[n] \end{aligned}$$

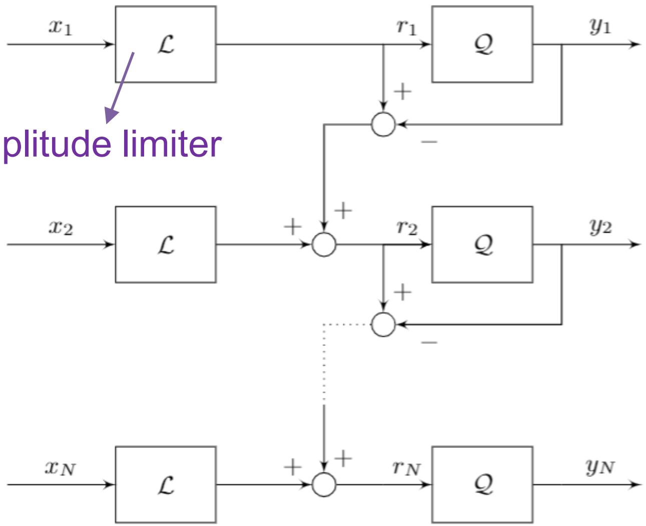
Quantization noise

$$\begin{aligned} R(z) &= X(z) + z^{-1}E(z) \\ Y(z) &= R(z) - E(z) \end{aligned} \quad \Longrightarrow \quad Y(z) = \underbrace{X(z)}_{\text{All pass filter}} - \underbrace{(1 - z^{-1})E(z)}_{\text{High pass filter}}$$

- Shape quantization noise to higher frequencies → Higher effective resolution for low-pass signals.
- Expensive when f_{Nyq} is high.

Spatial sigma-delta quantization

- Spatial oversampling followed by spatial feedback.



$$-b \le \Re(x_i), \Im(x_i) \le b \,, i = 1, 2, ..., N$$

$$\mathcal{Q}(x) = b\mathrm{sign}(\Re(x)) + jb\mathrm{sign}(\Im(x))$$

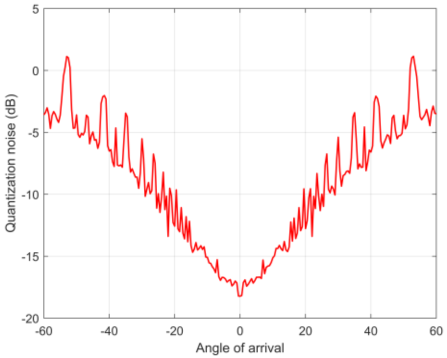
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} \quad \mathbf{U} = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & & 0 \\ 1 & 1 & \cdots & \cdots & 1 \end{bmatrix}$$

$$\mathbf{V} = \mathbf{U} - \mathbf{I}_N$$

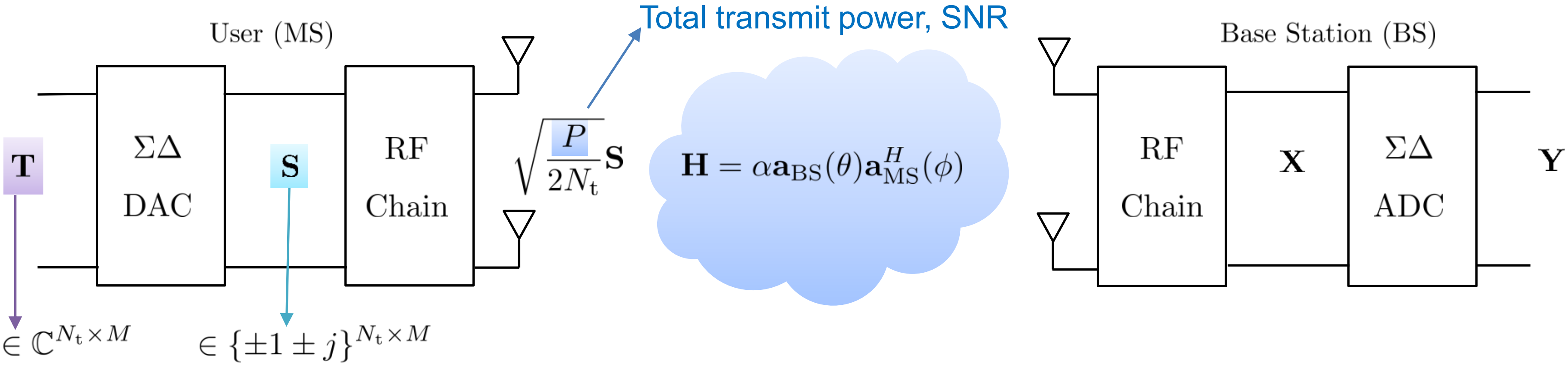
$$\mathbf{y} = \mathcal{Q}(\mathbf{U}\mathbf{x} - \mathbf{V}\mathbf{y})$$

- Noise is shaped towards higher spatial frequencies !

- Higher effective resolution for spatial low-pass signals.



System and channel model



Total transmit power, SNR

User (MS)

$\mathbf{T}$

$\Sigma\Delta$ DAC

$\mathbf{S}$

RF Chain

$\sqrt{\frac{P}{2N_\mathrm{t}}}\mathbf{S}$

$\mathbf{H} = \alpha \mathbf{a}_\mathrm{BS}(\theta)\mathbf{a}_\mathrm{MS}^H(\phi)$

Base Station (BS)

RF Chain

$\mathbf{X}$

$\Sigma\Delta$ ADC

$\mathbf{Y}$

$\in \mathbb{C}^{N_\mathrm{t}\times M}$

$\in \{\pm 1 \pm j\}^{N_\mathrm{t}\times M}$

$$\mathbf{X} = \sqrt{\frac{P}{2N_\mathrm{t}}}\mathbf{H}\mathbf{S} + \mathbf{W}$$

$\mathbf{X} \in \mathbb{C}^{N_\mathrm{r}\times M}$

$\mathbf{W} \in \mathbb{C}^{N_\mathrm{r}\times M}$

$w_{ij} \sim \mathcal{CN}(0,1)$

$$\mathbf{Y} = \mathcal{Q}(\mathbf{U}\mathbf{X} - \mathbf{V}\mathbf{Y}) \in \{\pm b \pm jb\}^{N_\mathrm{r}\times N_\mathrm{t}}$$

Voltage level of ADC at BS

$$\mathbf{a}_\mathrm{MS}(\phi) = \left[1, \quad e^{j2\pi d\sin(\phi)/\lambda}, \quad \cdots, \quad e^{j2\pi d(N_\mathrm{t}-1)\sin(\phi)/\lambda}\right]^T$$

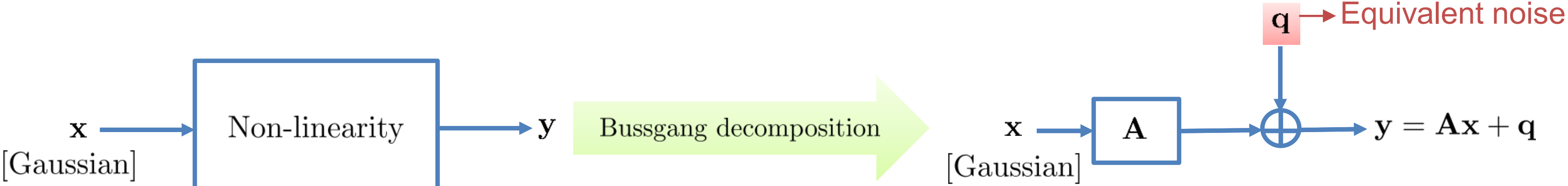
$$\mathbf{a}_\mathrm{BS}(\theta) = \left[1, \quad e^{j2\pi d\sin(\theta)/\lambda}, \quad \cdots, \quad e^{j2\pi d(N_\mathrm{r}-1)\sin(\theta)/\lambda}\right]^T$$

- We need to estimate $\{\alpha, \theta, \phi\}$ using the knowledge of P and $\mathbf{T}$

Prior art

- LMMSE channel estimator using Bussgang Decomposition and elementwise Bussgang Decomposition.

[S. Rao, et al. 2020]



$$\mathbf{A} = \mathbf{C}_{yx}\mathbf{C}_{x}^{-1}$$

$$\mathbf{C}_{yx} = \mathbb{E}[\mathbf{y}\mathbf{x}^{H}] \quad \mathbf{C}_{x} = \mathbb{E}[\mathbf{x}\mathbf{x}^{H}]$$

- Amplitude retrieval (AR): one-bit MIMO channel estimation using alternating optimization. *[C. Qian, et al. 2019]*

S. Rao, G. Seco-Granados, H. Pirzadeh, and A. L. Swindlehurst, “Massive MIMO channel estimation with low-resolution spatial sigma-delta ADCs,” ar preprint arXiv:2005.07752, May 2020.

C. Qian, X. Fu, and N. D. Sidiropoulos, “Amplitude retrieval for channel estimation of MIMO systems with one-bit ADCs,” IEEE Signal Process. Lett., vol. 26, no. 11, pp. 1698–1702, Nov. 2019.

Limitations in prior art

- Needs the knowledge of channel correlation to compute the Bussgang decomposition ($\mathbf{C}_x,\ \mathbf{C}_{yx}$) and to set voltage levels.

- In mmWave, knowing channel correlation amounts to knowing angles. Not practical.

- Existing method directly estimate $\mathbf{H}$ instead of estimating $\{\alpha, \theta, \phi\}$. Does not exploit parametric channel model.

$N_\mathrm{t} N_\mathrm{r}$ unknowns

3 unknowns

$$N_\mathrm{t} N_\mathrm{r} >> 3$$

Quantization noise modeling in sigma-delta quantizer

- Usual method of noise modelling: $Q(r_i) = r_i + e_i$

Property - I:
When the dynamic range of r_i is large and Q is a multi-level quantizer with large number of levels, we can assume that e_i is uniformly distributed and uncorrelated with r_i.

- Not reasonable to assume that r_i and e_i are uncorrelated for one-bit quantizers.
- Closed form expressions relating e_i to r_i. *[*R. M. Gray, et. al. *1989]*
- We propose a new noise-model for spatial sigma-delta ADC.

R. M. Gray, W. Chou, and P. W. Wong, "Quantization noise in single-loop sigma-delta modulation with sinusoidal inputs," IEEE Trans. Commun., vol. 37, no. 9, pp. 956–968, Sept. 1989.

Quantization noise modeling in sigma-delta quantizer

- Linearize the quantizer : $y_i = \mathcal{Q}(r_i) = r_i + e_i$ → No assumptions about the pdf !!

- We can show that

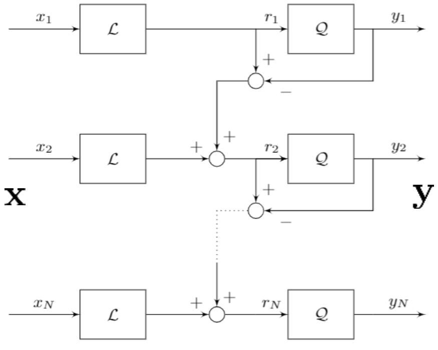
$$\Re(e_i) = b - (2b)\left\langle \frac{(i-1)}{2} + \sum_{k=1}^{i} \frac{\Re(x_k)}{2b} \right\rangle, \forall\, i = 1, 2, \ldots, N_{\mathrm{r}}$$

$$\langle x \rangle = x - \mathrm{floor}(x),\ \forall\, x \in \mathbb{R}$$

$$\Downarrow$$

$$\frac{1}{2b}\mathbf{U}\Re(\mathbf{y}) + \frac{1}{2}\mathbf{V}\mathbf{1} - \frac{1}{2}\mathbf{1} = \mathrm{floor}(\frac{1}{2b}\mathbf{U}\Re(\mathbf{x}) + \frac{1}{2}\mathbf{V}\mathbf{1})$$

$$\frac{1}{2b}\mathbf{U}\Im(\mathbf{y}) + \frac{1}{2}\mathbf{V}\mathbf{1} - \frac{1}{2}\mathbf{1} = \mathrm{floor}(\frac{1}{2b}\mathbf{U}\Im(\mathbf{x}) + \frac{1}{2}\mathbf{V}\mathbf{1})$$



$$\mathcal{Q}(x) = b\mathrm{sign}(\Re(x)) + jb\mathrm{sign}(\Im(x))$$

$$\mathbf{y} = \mathcal{Q}(\mathbf{U}\mathbf{x} - \mathbf{V}\mathbf{y})$$

Quantization noise modeling in sigma-delta quantizer

- Linearize the floor function

$$\text{floor}(\frac{1}{2b}\mathbf{U}\Re(\mathbf{x})+\frac{1}{2}\mathbf{V1})=\frac{1}{2b}\mathbf{U}\Re(\mathbf{x})+\frac{1}{2}\mathbf{V1}+\Re(\mathbf{q})$$

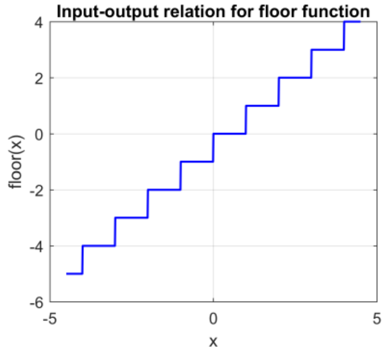
$\Re(q_i)\in(-1,0]\ \forall\ i=1,...,N_{\text{r}}$

- Dynamic range of $[\frac{1}{2b}\mathbf{U}\Re(\mathbf{x})+\frac{1}{2}\mathbf{V1}]_i$ can be large for large i

- $\text{floor}(.)$ is like a multi-level quantizer

- Using **Property- I**, we have

$\text{floor}(u)=u+q\ \forall u\in\mathbb{R}$



For large antenna indices, q_i is uniformly distributed and is uncorrelated with $[\frac{1}{2b}\mathbf{U}\mathbf{x}+\frac{1}{2}\mathbf{V1}]_i$

Quantization noise modeling in sigma-delta quantizer

- Re-arranging

$$\mathbf{y} = \mathbf{x} + (2b)\mathbf{U}^{-1}\tilde{\mathbf{q}},$$
$$\tilde{\mathbf{q}} = (\Re(\mathbf{q}) + \frac{1}{2}\mathbf{1}) + j(\Im(\mathbf{q}) + \frac{1}{2}\mathbf{1}).$$

- In practice, we can consider **Property-2** to be satisfied by all antennas in massive MIMO systems.

- Thus, $\tilde{\mathbf{q}}$ is uncorrelated to $\mathbf{x}$ and $\mathbb{E}[\tilde{\mathbf{q}}\tilde{\mathbf{q}}^H] = \frac{1}{6}\mathbf{I}$

$$\Re(\tilde{q}_i), \Im(\tilde{q}_i) \sim \mathrm{Unif}(-0.5, 0.5)\ and\ i.i.d$$

$$\mathrm{floor}(\frac{1}{2b}\mathbf{U}\Re(\mathbf{x}) + \frac{1}{2}\mathbf{V}\mathbf{1}) = \frac{1}{2b}\mathbf{U}\Re(\mathbf{x}) + \frac{1}{2}\mathbf{V}\mathbf{1} + \Re(\mathbf{q})$$
$$\mathrm{floor}(\frac{1}{2b}\mathbf{U}\Im(\mathbf{x}) + \frac{1}{2}\mathbf{V}\mathbf{1}) = \frac{1}{2b}\mathbf{U}\Im(\mathbf{x}) + \frac{1}{2}\mathbf{V}\mathbf{1} + \Im(\mathbf{q})$$

$$\mathbf{y} = \mathcal{Q}(\mathbf{U}\mathbf{x} - \mathbf{V}\mathbf{y})$$

Property – 2 :

For large antenna indices, q_i is uniformly distributed and is independent with $[\frac{1}{2b}\mathbf{U}\mathbf{x} + \frac{1}{2}\mathbf{V}\mathbf{1}]_i$

Pilot selection at the MS

- We desire $\mathbf{S} \in \{-1,1\}^{N_t \times M}$ to be an orthogonal matrix with $\mathbf{S}\mathbf{S}^H = \mathbf{S}^H\mathbf{S} = 2N_t\mathbf{I}_{N_t}$

Assumed to be a power of 2

- Let $\mathbf{S} = \mathbf{G} + j\mathbf{G}$ → Hadamard matrix
- Choose $\mathbf{T}$ to obtain desired $\mathbf{S}$
- We have

All ones vector

$$\frac{1}{2}\mathbf{U}\mathbf{G} + \frac{1}{2}\mathbf{V}\mathbf{1}\mathbf{1}^T - \frac{1}{2}\mathbf{1}\mathbf{1}^T = \text{floor}(\frac{1}{2}\mathbf{U}\Re(\mathbf{T}) + \frac{1}{2}\mathbf{V}\mathbf{1}\mathbf{1}^T)$$

$$\frac{1}{2}\mathbf{U}\mathbf{G} + \frac{1}{2}\mathbf{V}\mathbf{1}\mathbf{1}^T - \frac{1}{2}\mathbf{1}\mathbf{1}^T = \text{floor}(\frac{1}{2}\mathbf{U}\Im(\mathbf{T}) + \frac{1}{2}\mathbf{V}\mathbf{1}\mathbf{1}^T)$$

- Many ways to choose $\mathbf{T}$. By noting $\text{floor}(x) = x,\ \forall x \in \mathbb{Z}$, we may choose

$$\mathbf{T} = (\mathbf{G} - \mathbf{U}^{-1}\mathbf{1}\mathbf{1}^T) + j(\mathbf{G} - \mathbf{U}^{-1}\mathbf{1}\mathbf{1}^T)$$

Channel estimation at the BS

- Received signal at the BS

$$\mathbf{Y} = \sqrt{\frac{P}{2N_\mathrm{t}}}\mathbf{HS} + \mathbf{N} \longrightarrow$$

Gaussian + quantization noise

- Noise covariance matrix

$$\mathbf{R}_n = \mathbf{I}_{N_\mathrm{r}} + \frac{2b^2}{3}\mathbf{U}^{-1}\mathbf{U}^{-H}$$

Not spatially white !

- To estimate angles, we can use subspace based methods, e.g., MUSIC.

$$\hat{\mathbf{H}} = \mathbf{R}_n^{-1/2}\mathbf{Y}\frac{1}{\sqrt{2PN_t}}\mathbf{S}^H = \mathbf{R}_n^{-1/2}\mathbf{H} + \mathbf{R}_n^{-1/2}\mathbf{N}\mathbf{S}^H/\sqrt{2PN_t}$$

Pre-whitening matrix

Whitened noise term $\hat{\mathbf{N}}$

- We have

$$\hat{\mathbf{H}} = \mathbf{R}_n^{-1/2}\alpha\mathbf{a}_{\mathrm{BS}}(\theta)\mathbf{a}_{\mathrm{UE}}^H(\phi) + \hat{\mathbf{N}}$$

Signal part

Noise part (white and uncorrelated with signal part)

Channel estimation at the BS

$$\hat{\mathbf{H}} = \underbrace{\mathbf{R}_n^{-1/2}\alpha\mathbf{a}_{\mathrm{BS}}(\theta)\mathbf{a}_{\mathrm{UE}}^H(\phi)}_{\text{Signal part}} + \underbrace{\hat{\mathbf{N}}}_{\text{Noise part (white and uncorrelated with signal part)}}$$

- The SVD of $\hat{\mathbf{H}}$ is given by $\hat{\mathbf{H}} = \mathbf{P}\boldsymbol{\Sigma}\mathbf{Q}^H$
- $\mathbf{P} = [\underbrace{\mathbf{p}_s}_{\in \mathbb{C}^{N_\mathrm{r}\times 1}} \quad \underbrace{\mathbf{P}_n}_{\in \mathbb{C}^{N_\mathrm{r}\times (N_\mathrm{r}-1)}}]$, $\mathbf{Q} = [\underbrace{\mathbf{q}_s}_{\in \mathbb{C}^{N_\mathrm{t}\times 1}} \quad \underbrace{\mathbf{Q}_n}_{\in \mathbb{C}^{N_\mathrm{t}\times (N_\mathrm{t}-1)}}]$
- $\mathcal{R}(\mathbf{p}_s) = \mathcal{R}(\hat{\mathbf{H}}) = \mathcal{R}(\mathbf{R}_n^{-1/2}\mathbf{a}_{\mathrm{BS}}(\theta))$, $\mathcal{R}(\mathbf{q}_s) = \mathcal{R}(\hat{\mathbf{H}}^H) = \mathcal{R}(\mathbf{a}_{\mathrm{MS}}(\phi))$
- AoA and AoD can be estimated by finding peaks of MUSIC pseudo-spectra

$$\rho_{\mathrm{BS}}(\tilde{\theta}) = \frac{1}{\|\mathbf{P}_n^H\mathbf{R}_n^{-1/2}\mathbf{a}_{\mathrm{BS}}(\tilde{\theta})\|_2^2}; \quad \rho^{\mathrm{MS}}(\tilde{\phi}) = \frac{1}{\|\mathbf{Q}_n^H\mathbf{a}_{\mathrm{MS}}(\tilde{\phi})\|_2^2}$$

- Path gain α can be estimated using least squares.

$\hat{\alpha} = \mathbf{d}^H\hat{\mathbf{h}}/\|\mathbf{d}\|_2^2$, where $\mathbf{d} = \mathrm{vec}(\mathbf{R}_n^{-1/2}\mathbf{a}_{\mathrm{BS}}(\hat{\theta})\mathbf{a}_{\mathrm{MS}}^H(\hat{\phi}))$, and $\hat{\mathbf{h}} = \mathrm{vec}(\alpha\mathbf{R}_n^{-1/2}\mathbf{a}_{\mathrm{BS}}(\hat{\theta})\mathbf{a}_{\mathrm{MS}}^H(\hat{\phi}) + \hat{\mathbf{N}})$

Voltage level selection at the BS

- Voltage level selection is crucial to ensure good performance.
- Voltage level b needs to be selected to satisfy
  - $\mathcal{L}(x_i) \approx x_i$, i.e., error due to clipping is minimum – Choose large level
  - $[\frac{1}{2b}\mathbf{U}\mathbf{x} + \frac{1}{2}\mathbf{V}\mathbf{1}]_i$, and q_i are uncorrelated – Choose small level

Conflicting requirements !!

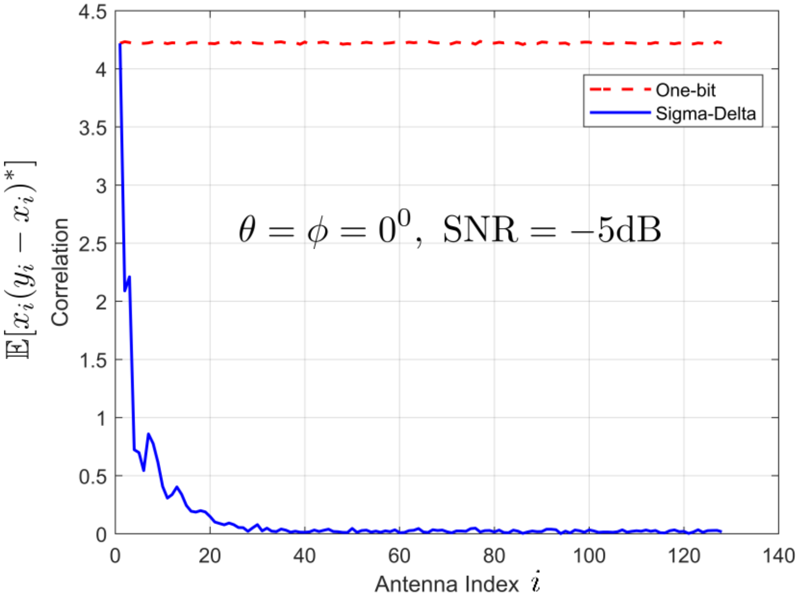
- Optimal selection is difficult.
- We have $x_i \sim \mathcal{CN}(\mu, 1)$, $\mu = \sqrt{\frac{P}{2N_\mathrm{t}}}\alpha e^{j2\pi d(i-1)\sin(\theta)/\lambda}\mathbf{a}_{MS}^H(\phi)\mathbf{s}$

  Column of $\mathbf{S}$

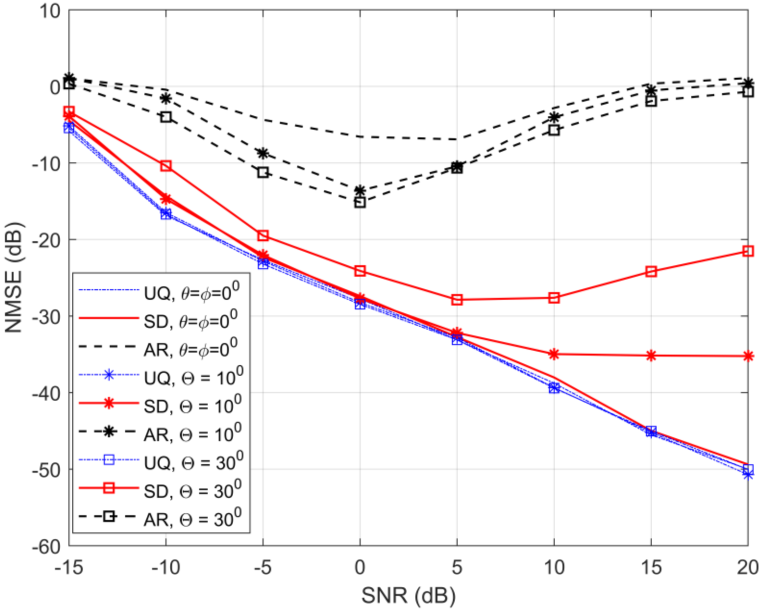
- $|\mu| \leq \sqrt{2PN_\mathrm{t}}$, if $|\alpha| = 1$
- Choose $b = \sqrt{2PN_\mathrm{t}} + 3\sqrt{0.5} \xrightarrow{} \text{Variance of } \Re(x_j), \Im(x_j)$
- Choice works well for LoS channels.

Simulations

- $N_\mathrm{t} = 8,\ N_\mathrm{r} = 128,\ d = \dfrac{\lambda}{8}, \theta, \phi \in [-\Theta^0, \Theta^0], |\alpha| = 1$



Correlation between input and quantization noise



Channel estimation error

C. Qian, X. Fu, and N. D. Sidiropoulos, “Amplitude retrieval for channel estimation of MIMO systems with one-bit ADCs,” IEEE Signal Process. Lett., vol. 26, no. 11, pp. 1698–1702, Nov. 2019.

Conclusions

- We have proposed angular channel estimation algorithm for LoS mmWave SU-MIMO systems with one-bit spatial $\Sigma\Delta$ quantizers.
- Usual i.i.d. noise assumption not suitable for one-bit quantizers.
- Presented a new noise modeling for spatial $\Sigma\Delta$quantizers by linearizing the $\mathrm{floor}(.)$ function.
- Input and quantization noise are uncorrelated for antennas away from phase reference.
- Proposed algorithm does not require prior knowledge of channel correlation.
- Significantly outperforms conventional one-bit MIMO systems, comparable to unquantized systems.

Thank you!

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