Millimeter Wave MIMO Channel Estimation with 1-bit Spatial Sigma-delta Analog-to-Digital Converters

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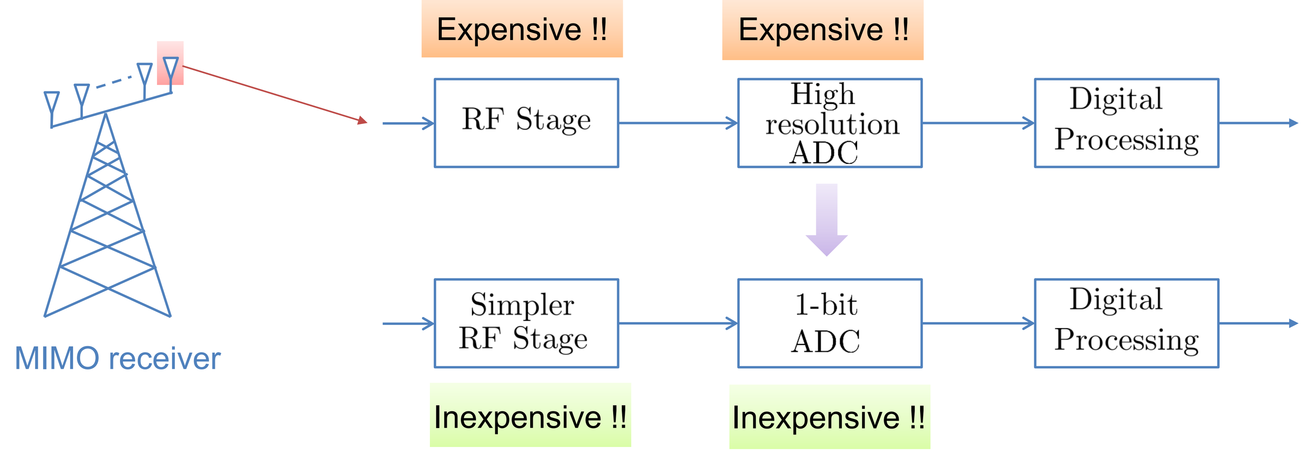
mmWave MIMO

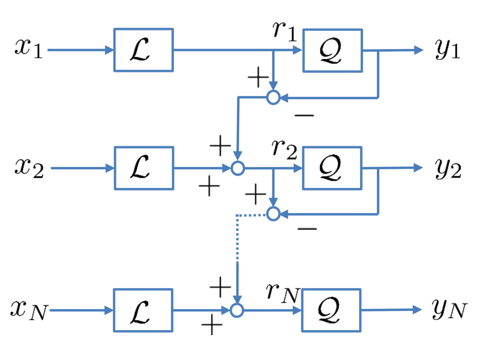
- Gbps data-rates.



- Use low-resolution quantizers.

Spatial ΣΔ quantizer

- Classical ΣΔ applied over space

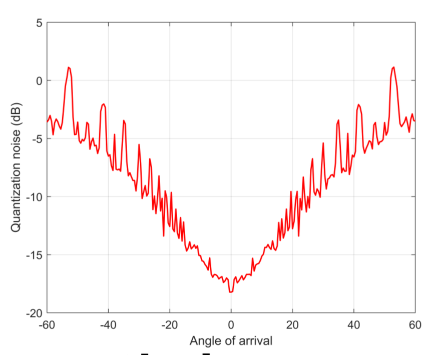


$-b \leq \Re(x_i), \Im(x_i) \leq b, i = 1, 2, \ldots, N$

$\mathcal{Q}(x) = b\text{sign}(\Re(x)) + jb\text{sign}(\Im(x))$

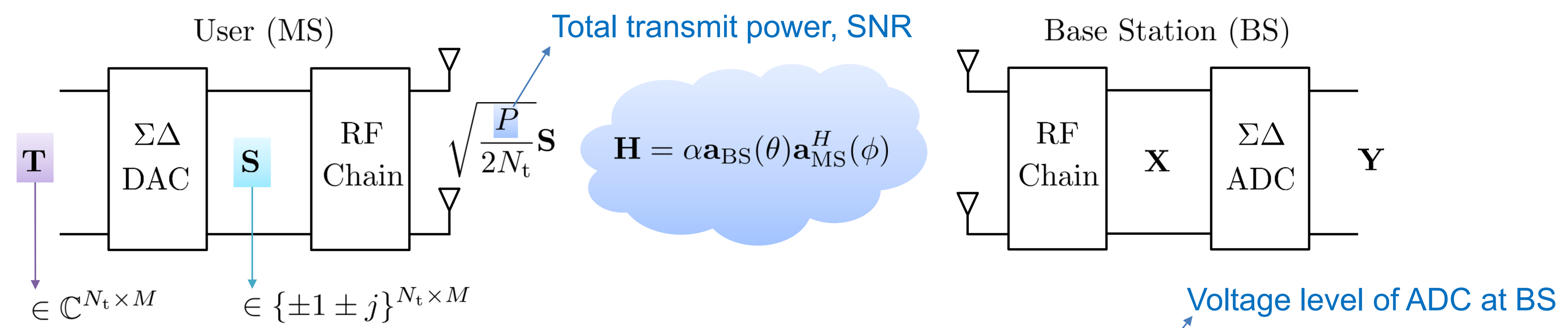
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix} \quad \mathbf{U} = \begin{bmatrix} 1 & 0 & \cdots & \cdots & 0 \\ 1 & 1 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & & 0 \\ 1 & 1 & \cdots & \cdots & 1 \end{bmatrix}$$

$\mathbf{y} = \mathcal{Q}(\mathbf{U}\mathbf{x} - \mathbf{V}\mathbf{y}) \qquad \mathbf{V} = \mathbf{U} - \mathbf{I}_N$



- Shapes noise towards higher spatial frequencies

System model



$\mathbf{X} = \sqrt{\frac{P}{2N_t}}\mathbf{H}\mathbf{S} + \mathbf{W}$, $\in \mathbb{C}^{N_r \times M}$, $\in \mathbb{C}^{N_r \times M}$

$\mathbf{Y} = \mathcal{Q}(\mathbf{U}\mathbf{X} - \mathbf{V}\mathbf{Y}) \in \{\pm b \pm jb\}^{N_r \times N_t}$, $w_{ij} \sim \mathcal{CN}(0, 1)$

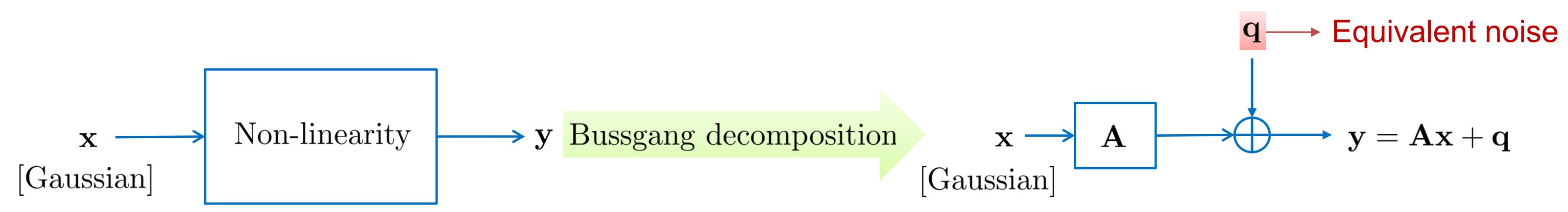
$\mathbf{H} = \alpha \mathbf{a}_{\text{BS}}(\theta)\mathbf{a}_{\text{MS}}^H(\phi)$, $\mathbf{T} \in \mathbb{C}^{N_t \times M}$, $\mathbf{S} \in \{\pm 1 \pm j\}^{N_t \times M}$

$\mathbf{a}_{\text{MS}}(\phi) = [1, \; e^{j2\pi d \sin(\phi)/\lambda}, \; \cdots, \; e^{j2\pi d(N_t-1)\sin(\phi)/\lambda}]^T \qquad \mathbf{a}_{\text{BS}}(\theta) = [1, \; e^{j2\pi d \sin(\theta)/\lambda}, \; \cdots, \; e^{j2\pi d(N_r-1)\sin(\theta)/\lambda}]^T$

- We need to estimate $\{\alpha, \theta, \phi\}$

Prior art

- Use Bussgang decomposition.



$\mathbf{A} = \mathbf{C}_{yx}\mathbf{C}_x^{-1}$

$\mathbf{C}_{yx} = \mathbb{E}[\mathbf{y}\mathbf{x}^H] \quad \mathbf{C}_x = \mathbb{E}[\mathbf{x}\mathbf{x}^H]$

- Channel correlation needs to be known.
- Amounts to knowing angles in mmWave MIMO systems.

Noise modeling

- Usual modeling: $\mathcal{Q}(r_i) = r_i + e_i$

Property - I:
When the dynamic range of r_i is large and $\mathcal{Q}$ is a multi-level quantizer with large number of levels, we can assume that e_i is uniformly distributed and uncorrelated with r_i.

- We have *[R. M. Gray, et. al. 1989]*

$\Re(e_i) = b - (2b)\left\langle \frac{(i-1)}{2} + \sum_{k=1}^{i} \frac{\Re(x_k)}{2b} \right\rangle, \forall\, i = 1, 2, \ldots, N_r$

$\langle x \rangle = x - \text{floor}(x), \forall\, x \in \mathbb{R}$

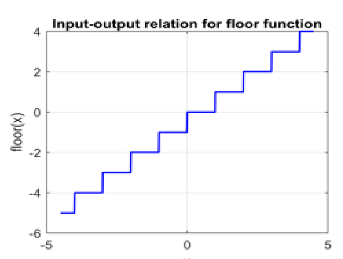
$\frac{1}{2b}\mathbf{U}\Re(\mathbf{y}) + \frac{1}{2}\mathbf{V}\mathbf{1} - \frac{1}{2}\mathbf{1} = \text{floor}(\frac{1}{2b}\mathbf{U}\Re(\mathbf{x}) + \frac{1}{2}\mathbf{V}\mathbf{1})$

$\frac{1}{2b}\mathbf{U}\Im(\mathbf{y}) + \frac{1}{2}\mathbf{V}\mathbf{1} - \frac{1}{2}\mathbf{1} = \text{floor}(\frac{1}{2b}\mathbf{U}\Im(\mathbf{x}) + \frac{1}{2}\mathbf{V}\mathbf{1})$

- Linearize the floor function

$\text{floor}(\frac{1}{2b}\mathbf{U}\Re(\mathbf{x}) + \frac{1}{2}\mathbf{V}\mathbf{1}) = \frac{1}{2b}\mathbf{U}\Re(\mathbf{x}) + \frac{1}{2}\mathbf{V}\mathbf{1} + \Re(\mathbf{q})$

$\Re(q_i) \in (-1, 0] \; \forall\, i = 1, \ldots, N_r$



For large antenna indices, q_i is uniformly distributed and is uncorrelated with $[\frac{1}{2b}\mathbf{U}\mathbf{x} + \frac{1}{2}\mathbf{V}\mathbf{1}]_i$

$\mathbf{y} = \mathbf{x} + (2b)\mathbf{U}^{-1}\tilde{\mathbf{q}}$

$\tilde{\mathbf{q}} = (\Re(\mathbf{q}) + \frac{1}{2}\mathbf{1}) + j(\Im(\mathbf{q}) + \frac{1}{2}\mathbf{1})$

$\mathbf{C}_{\tilde{\mathbf{q}}} = \mathbb{E}[\tilde{\mathbf{q}}\tilde{\mathbf{q}}^H] \approx \frac{1}{6}\mathbf{I}$

$\mathbf{Y} = \sqrt{\frac{P}{2N_t}}\mathbf{H}\mathbf{S} + \mathbf{N}$ — Gaussian + quantization noise

$\mathbf{R}_n = \mathbf{I}_{N_r} + \frac{2b^2}{3}\mathbf{U}^{-1}\mathbf{U}^{-H}$ — Not spatially white !

Channel Estimation

- Pilot selection at the MS:

$\mathbf{S}\mathbf{S}^H = \mathbf{S}^H\mathbf{S} = 2N_t\mathbf{I}_{N_t}$

$\mathbf{S} = \mathbf{G} + j\mathbf{G}$ — Hadamard matrix

$\mathbf{T} = (\mathbf{G} - \mathbf{U}^{-1}\mathbf{1}\mathbf{1}^T) + j(\mathbf{G} - \mathbf{U}^{-1}\mathbf{1}\mathbf{1}^T)$

- Voltage selection at the BS:

Voltage level b needs to be selected to satisfy

- $\mathcal{L}(x_i) \approx x_i$, i.e., error due to clipping is minimum – Choose large level
- $[\frac{1}{2b}\mathbf{U}\mathbf{x} + \frac{1}{2}\mathbf{V}\mathbf{1}]_i$, and q_i are uncorrelated – Choose small level

Conflicting requirements !!

$x_i \sim \mathcal{CN}(\mu, 1)$

$\mu = \sqrt{\frac{P}{2N_t}}\alpha e^{j2\pi d(i-1)\sin(\theta)/\lambda}\mathbf{a}_{MS}^H(\phi)\mathbf{s}$

$|\mu| \leq \sqrt{2PN_t} \quad |\alpha| = 1$

$b = \sqrt{2PN_t} + 3\sqrt{0.5}$ — Variance of $\Re(x_j), \Im(x_j)$

- Angle estimation: Using MUSIC

$\hat{\mathbf{H}} = \mathbf{R}_n^{-1/2}\alpha\mathbf{a}_{\text{BS}}(\theta)\mathbf{a}_{\text{UE}}^H(\phi) + \tilde{\mathbf{N}}$

Signal part; Noise part (white and uncorrelated with signal part)

$\hat{\mathbf{H}} = \mathbf{P}\boldsymbol{\Sigma}\mathbf{Q}^H \quad \mathbf{P} = [\mathbf{p}_s \;\; \mathbf{P}_n] \quad \mathbf{Q} = [\mathbf{q}_s \;\; \mathbf{Q}_n]$

$\mathcal{R}(\mathbf{p}_s) = \mathcal{R}(\hat{\mathbf{H}}) = \mathcal{R}(\mathbf{R}_n^{-1/2}\mathbf{a}_{\text{BS}}(\theta))$

$\mathcal{R}(\mathbf{q}_s) = \mathcal{R}(\hat{\mathbf{H}}^H) = \mathcal{R}(\mathbf{a}_{\text{MS}}(\phi))$

$\rho_{\text{BS}}(\tilde{\theta}) = \frac{1}{\|\mathbf{P}_n^H\mathbf{R}_n^{-1/2}\mathbf{a}_{\text{BS}}(\tilde{\theta})\|_2^2}; \quad \rho^{\text{MS}}(\tilde{\phi}) = \frac{1}{\|\mathbf{Q}_n^H\mathbf{a}_{\text{MS}}(\tilde{\phi})\|_2^2}$

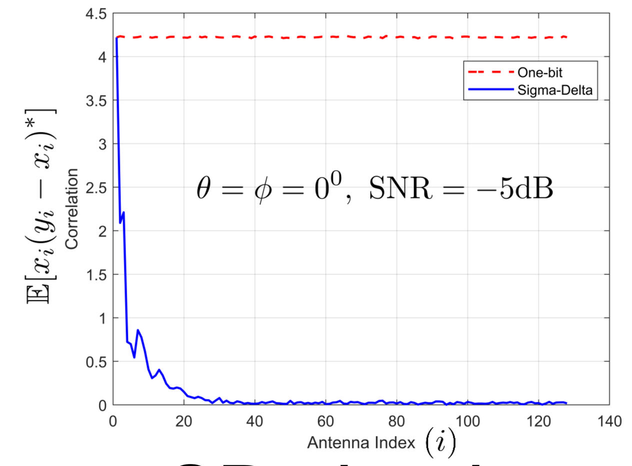
- Path gain estimation: Using LS

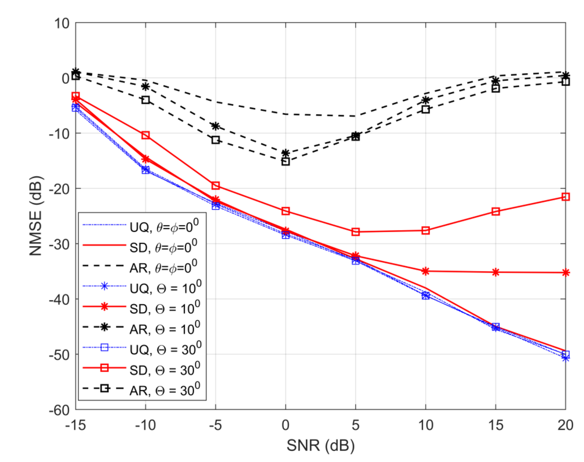
$\hat{\alpha} = \mathbf{d}^H\hat{\mathbf{h}}/\|\mathbf{d}\|_2^2$, where $\mathbf{d} = \text{vec}(\mathbf{R}_n^{-1/2}\mathbf{a}_{\text{BS}}(\hat{\theta})\mathbf{a}_{\text{MS}}^H(\hat{\phi}))$, and $\hat{\mathbf{h}} = \text{vec}(\alpha\mathbf{R}_n^{-1/2}\mathbf{a}_{\text{BS}}(\theta)\mathbf{a}_{\text{MS}}^H(\phi) + \tilde{\mathbf{N}})$

Simulations

$N_t = 8, \; N_r = 128, \; d = \frac{\lambda}{8}, \; \theta, \phi \in [-\Theta^0, \Theta^0], |\alpha| = 1$

UQ – unquantized (infinite resolution quantization)
AR – One-bit MIMO channel estimation. *[C. Qian, et al. 2019]*
SD – Proposed method with spatial ΣΔ





- SD clearly outperforms AR and is comparable to that of UQ.

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