

Coupled Rank- (L_m, L_n, \cdot) Block Term Decomposition

Xiao-Feng Gong, Qiu-Hua Lin, Otto Debals, Nico Vervliet, Lieven De Lathauwer

E-mail: xfgong@dlut.edu.cn

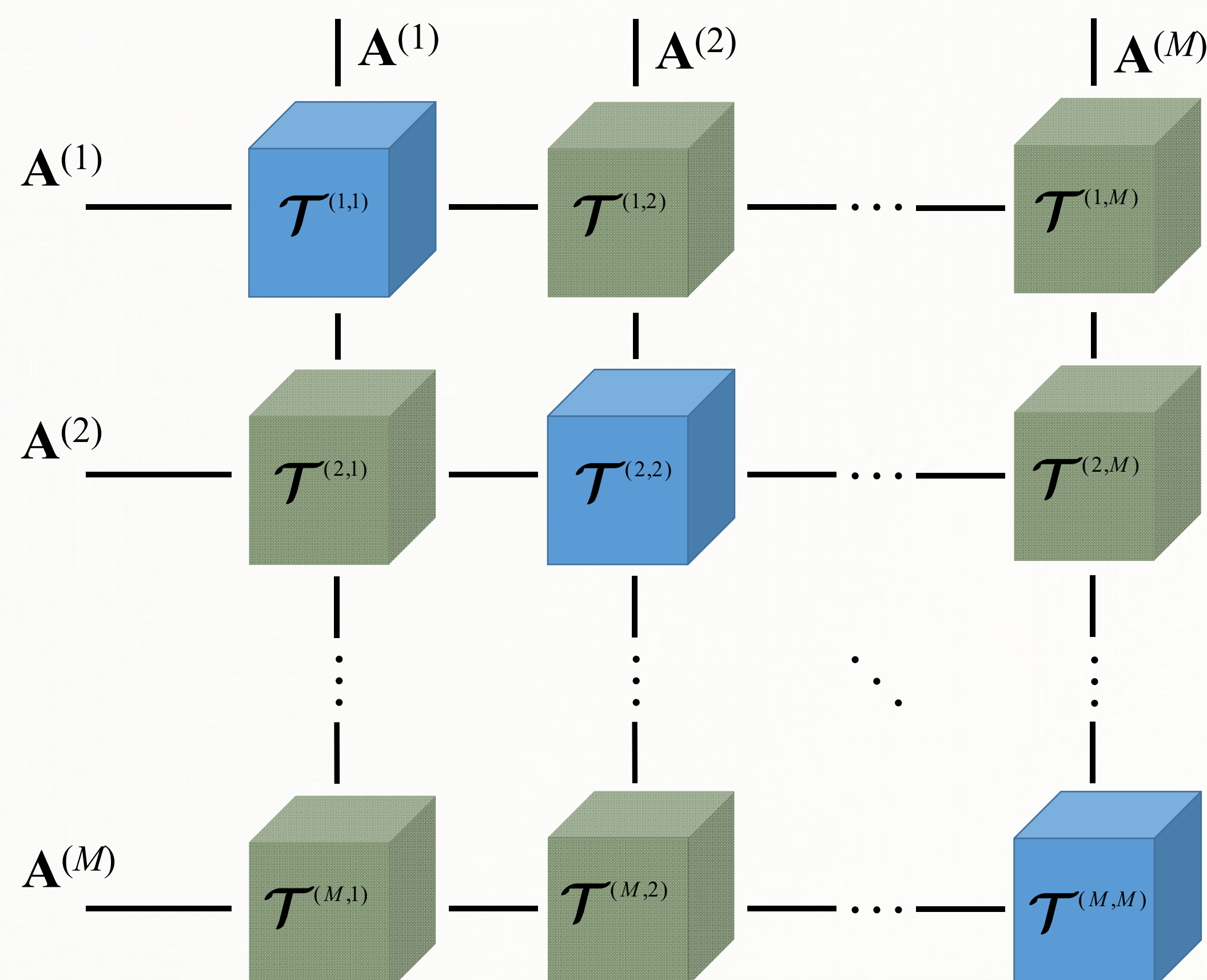


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WHAT?



- Each tensor $\mathcal{T}^{(m,n)}$ has a rank- (L_m, L_n, \cdot) BTD formulation

$$\text{Tensor} = \text{Matrix}_1 \times \text{Matrix}_2 \times \text{Matrix}_3 + \dots + \text{Matrix}_1 \times \text{Matrix}_2 \times \text{Matrix}_3$$

$$\mathcal{T}^{(m,n)} = \sum_r \mathcal{C}_r^{(m,n)} \times_1 \mathbf{A}_r^{(m)} \times_2 \mathbf{A}_r^{(n)}$$

Factor matrix: $\mathbf{A}^{(m)} = [\mathbf{A}_1^{(m)}, \dots, \mathbf{A}_R^{(m)}]$

- **Double coupling structure:**

- Tensors $\mathcal{T}^{(m,1)}, \mathcal{T}^{(m,2)}, \dots, \mathcal{T}^{(m,M)}$ have common factor matrix $\mathbf{A}^{(m)}$ in the first mode;
- Tensors $\mathcal{T}^{(1,n)}, \mathcal{T}^{(2,n)}, \dots, \mathcal{T}^{(M,n)}$ have common factor matrix $\mathbf{A}^{(n)}$ in the second mode.

WHY?

$$\mathbf{X}^{(1)} = \mathbf{A}_1^{(1)} \mathbf{S}_1^{(1)T} + \dots + \mathbf{A}_R^{(1)} \mathbf{S}_R^{(1)T}$$

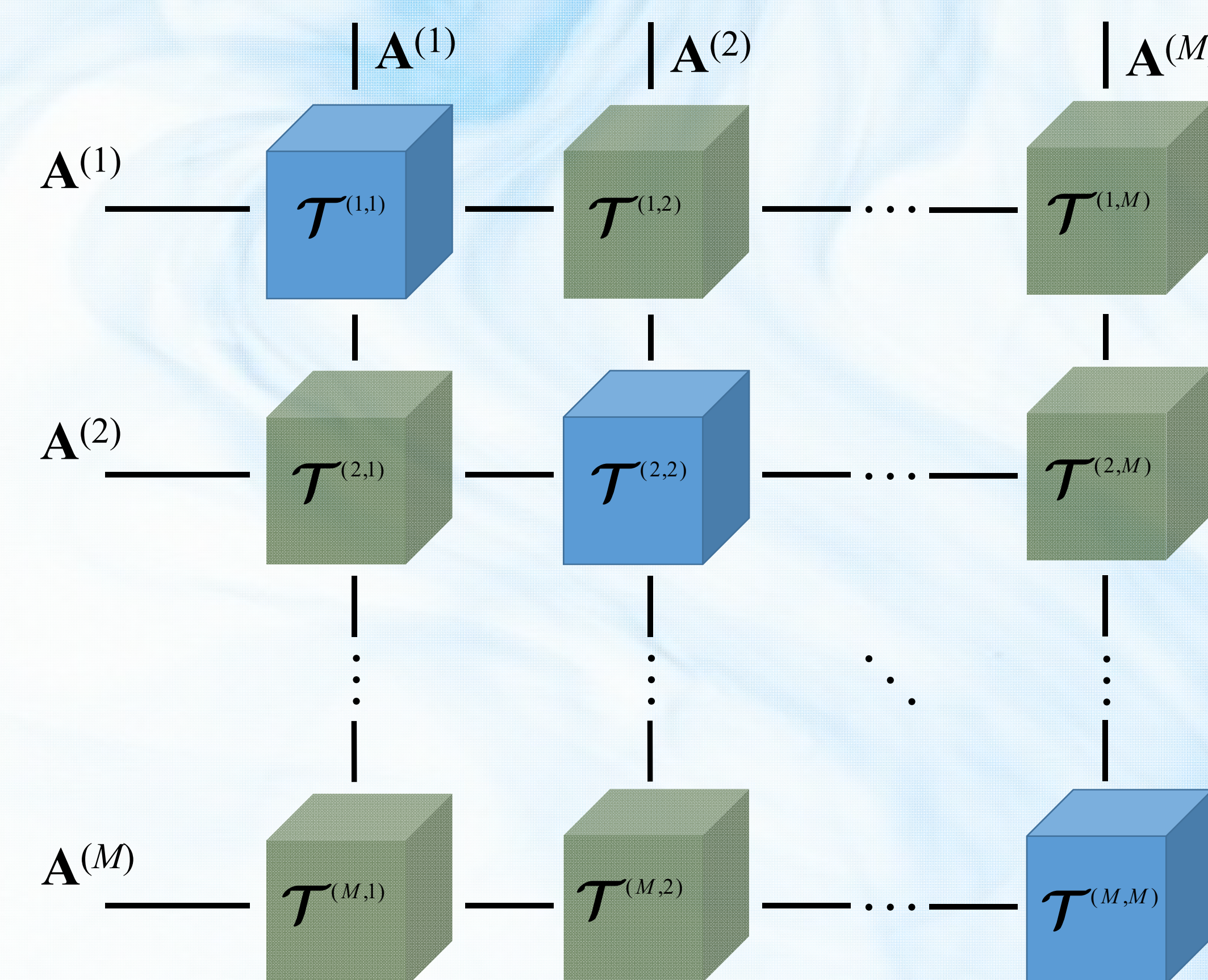
$$\vdots$$

$$\mathbf{X}^{(M)} = \mathbf{A}_1^{(M)} \mathbf{S}_1^{(M)T} + \dots + \mathbf{A}_R^{(M)} \mathbf{S}_R^{(M)T}$$

Multi-set convolutive mixture

Pairwise covariance

$$\mathcal{T}_{(:, :, t)}^{(m, n)} = \text{cov}\{\mathbf{X}_{(:, t)}^{(m)}, \mathbf{X}_{(:, t)}^{(n)T}\}$$



Coupled rank- (L_m, L_n, \cdot) BTD

Via pairwise cross covariance between each pair of datasets we convert a joint independent subspace analysis (J-ISA) problem to a coupled rank- (L_m, L_n, \cdot) BTD problem.

HOW?

• Coupled Block Simultaneous Generalized Schur Decomposition (CB-SGSD):

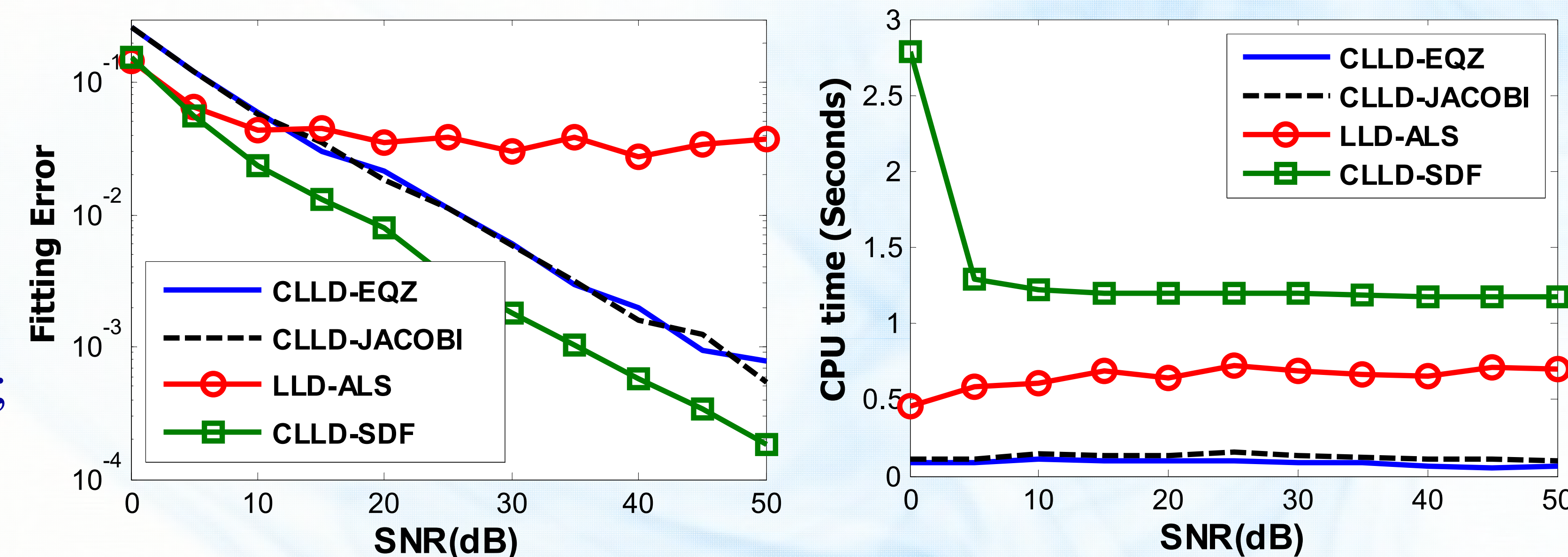
- Extension of existing works on SGSD [A.-J. van der Veen, etc. 1996, L. De Lathauwer, etc. 2004, A. Stegeman, 2009] to the block and coupled case.
- We proposed two algorithms for its computation: (1) extended QZ iteration; (2) Jacobi.

• Structured data fusion (SDF):

- The coupled rank- (L_m, L_n, \cdot) BTD can be implemented via SDF (www.tensorlab.net).

• Proposed vs. SDF:

- SDF is more accurate but sensitive to initialization. Proposed algorithms are faster, and thus can be used to provide low-cost initialization for SDF.



Proposed (CLLD-EQZ, CLLD-JACOBI) vs. SDF implementation (CLLD-SDF) and single tensor based uncoupled ALS algorithm (LLD-ALS).