Overcoming Measurement Inconsistency in Deep Learning for Linear Inverse Problems

Applications in Medical Imaging


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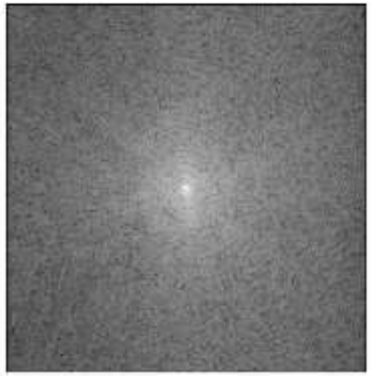
Linear Inverse Problems

MRI machine

$x^\star$

$\xrightarrow{A}$

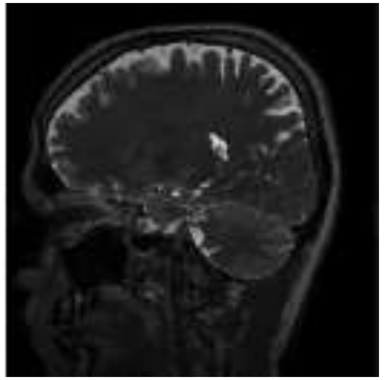
measurements



b

$\longrightarrow$

image



$\hat{x}$

Other applications: Remote sensing, super-resolution, etc.

Classic vs. DNNs

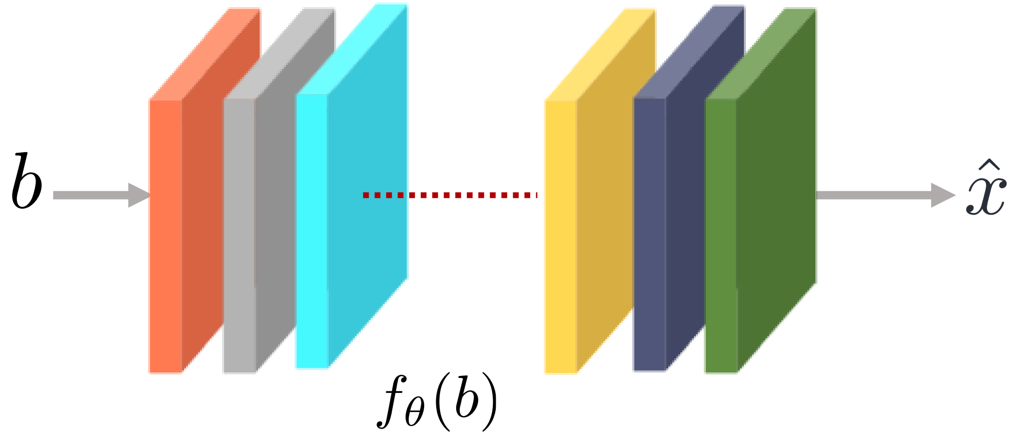
$$\hat{x} = \underset{x}{\arg\min} \quad r(x) \qquad \text{s.t.} \quad Ax = b$$

r : prior, tractable

no training data

theoretical guarantees

consistent: $A\hat{x} = b$



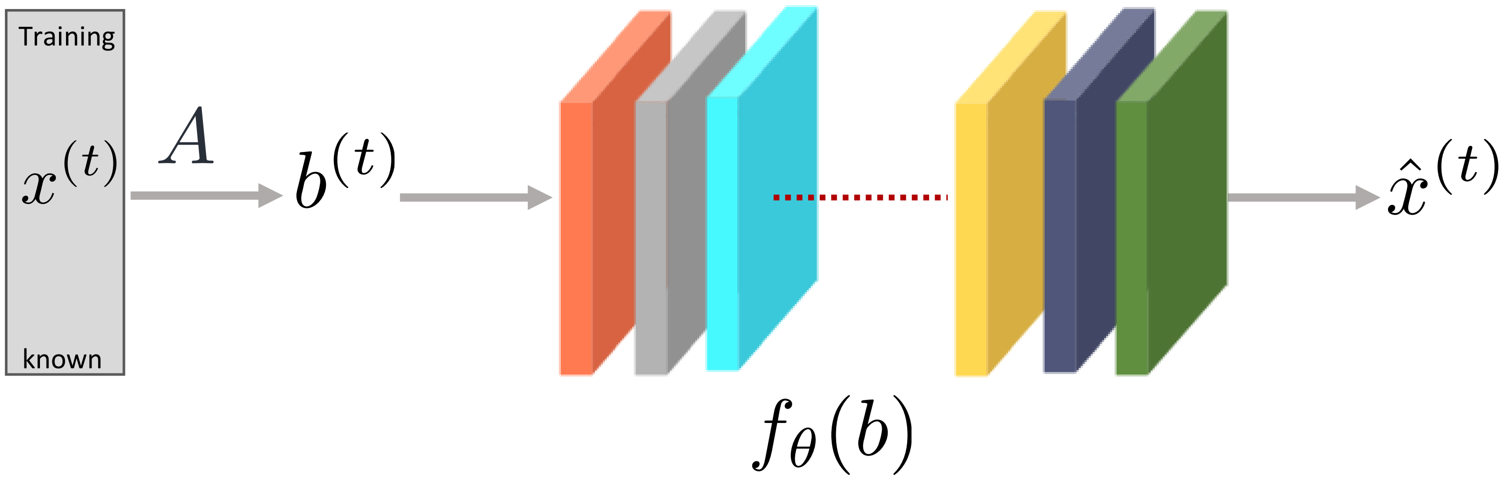
state-of-the-art performance

requires large training datasets

no theoretical guarantees

inconsistent: $A\hat{x} \neq b$ (in general)

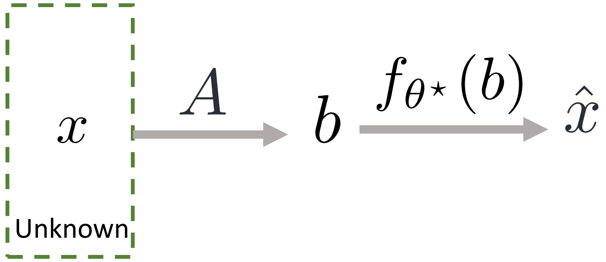
Measurement Inconsistency in DNNs



Training

$$\theta^\star \in \arg\min_\theta \frac{1}{T}\sum_{t\in\mathcal{T}} \left\| x^{(t)} - f_\theta\left(b^{(t)}\right)\right\|_2^2$$ (empirical loss)

Deployment



Numerical Evidence

Measurement consistency of SR DNNs.

Method	Image	$\|A\hat{x} - b\|_2$	
SRCNN	*Baboon*	5.29×10^{-1}	CNN
	38092	4.32×10^{-1}	
	img005	14.93×10^{-1}	
FSRCNN	*Baboon*	3.26×10^{-1}	
	38092	2.91×10^{-1}	
	img005	10.32×10^{-1}	
IRCNN	*Baboon*	1.09×10^{-1}	P-n-P
	38092	9.15×10^{-2}	
	img005	5.33×10^{-1}	

Measurement consistency of MRI DNNs.

Method	$\|A\hat{x} - b\|_2$	
MoDL	3.10×10^{-1}	P-n-P
CRNN	2.06×10^{-6}	RNN

MoDL: DC layer minimize the sum of the trained CNN denoiser and $\|A\hat{x} - b\|_2^2$

CRNN: Embeds a DC layer than minimizes $\|A\hat{x} - b\|_2^2$

DNNs "ignore" measurements during deployment!

Our Work

Formalize that generalization errors imply inconsistency under ERM

New framework: guaranteeing consistency of DNN outputs

Application to MRI

Measurement Inconsistency in DNNs

Suppose we train a DNN $f_\theta : b \mapsto x$ for minimizing inconsistency:

$$\theta^\star \in \arg\min_\theta \ \ell_{\text{emp}}(f_\theta\,;\mathcal{T}) := \frac{1}{|\mathcal{T}|}\sum_{t\in\mathcal{T}} \Big\| \underbrace{Ax^{(t)}}_{b^{(t)}} - A\underbrace{f_\theta\big(Ax^{(t)}\big)}_{\widehat{x}^{(t)}} \Big\|_2^2$$

empirical risk

$$\ell_{\text{exp}}(f_\theta) := \mathbb{E}\Big[\big\|AX - Af_\theta(AX)\big\|_2^2\Big]$$

risk

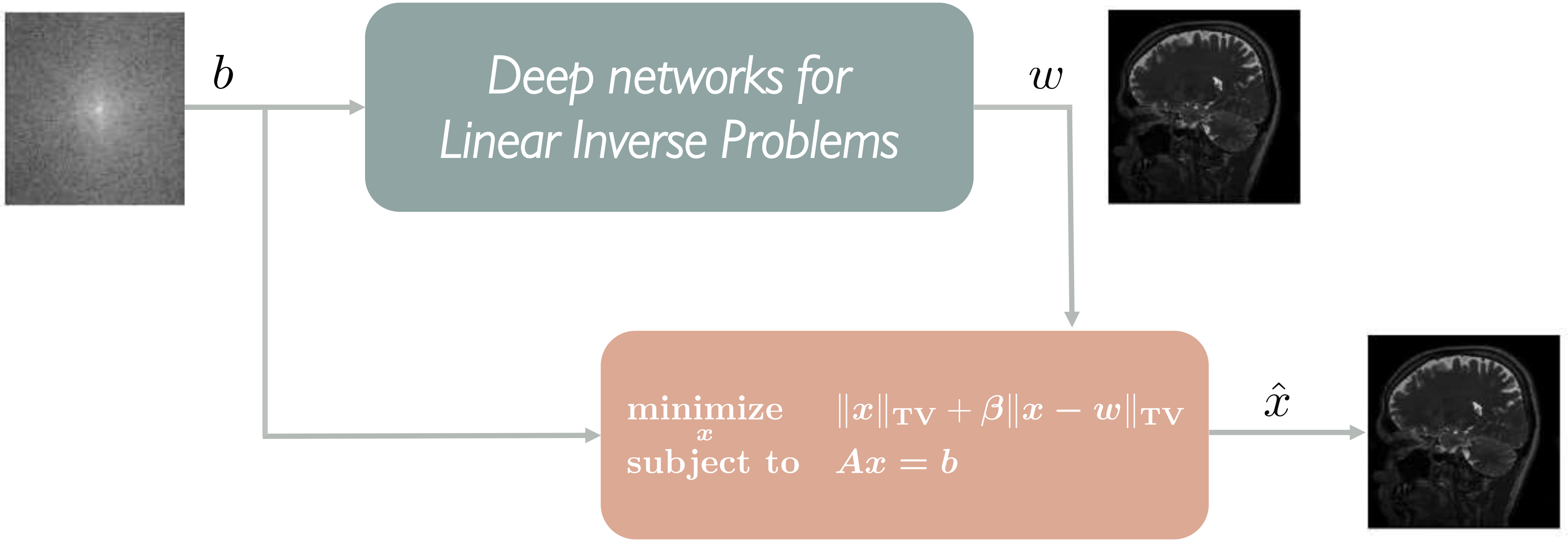
Proposition

Assume generalization error $c := \ell_{\text{exp}}(f_{\theta^\star}) - \ell_{\text{emp}}(f_{\theta^\star}\,;\mathcal{T}) > 0$ and $\epsilon := \ell_{\text{emp}}(f_{\theta^\star}\,;\mathcal{T}) > 0$

Then, for any $0 < \delta < c + \epsilon$,

$$\mathbb{P}\Big(\underbrace{\|AX - Af_{\theta^\star}(AX)\|_2^2}_{\text{inconsistency}} \geq \delta\Big) \geq 1 - \exp\Big(-2\frac{(c+\epsilon-\delta)^2}{C^2}\Big)$$

New Framework: TV-TV Minimization



Improves the quality of w + Guarantees $Ax = b$ + Easily adapted to different A operators

TV-TV Minimization

x has a small total-variation

Output from DNN

$$\min_x \quad \|x\|_{\mathrm{TV}} + \beta\|x - \boxed{w}\|_{\mathrm{TV}}$$

x is close to w

$$\text{s.t.} \quad \boxed{A}x = \boxed{b}$$

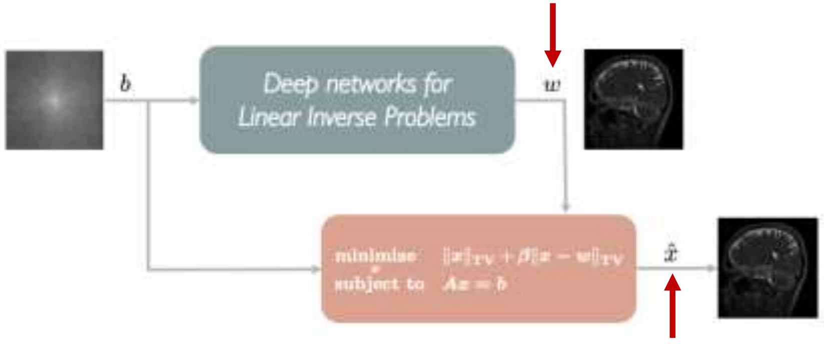
Enforce measurement consistency

Sensing Matrix

Measurement

- *Solved via ADMM*
- All sub-problems have a closed form solution
- Algorithms details are in the paper

Experiments on MRI



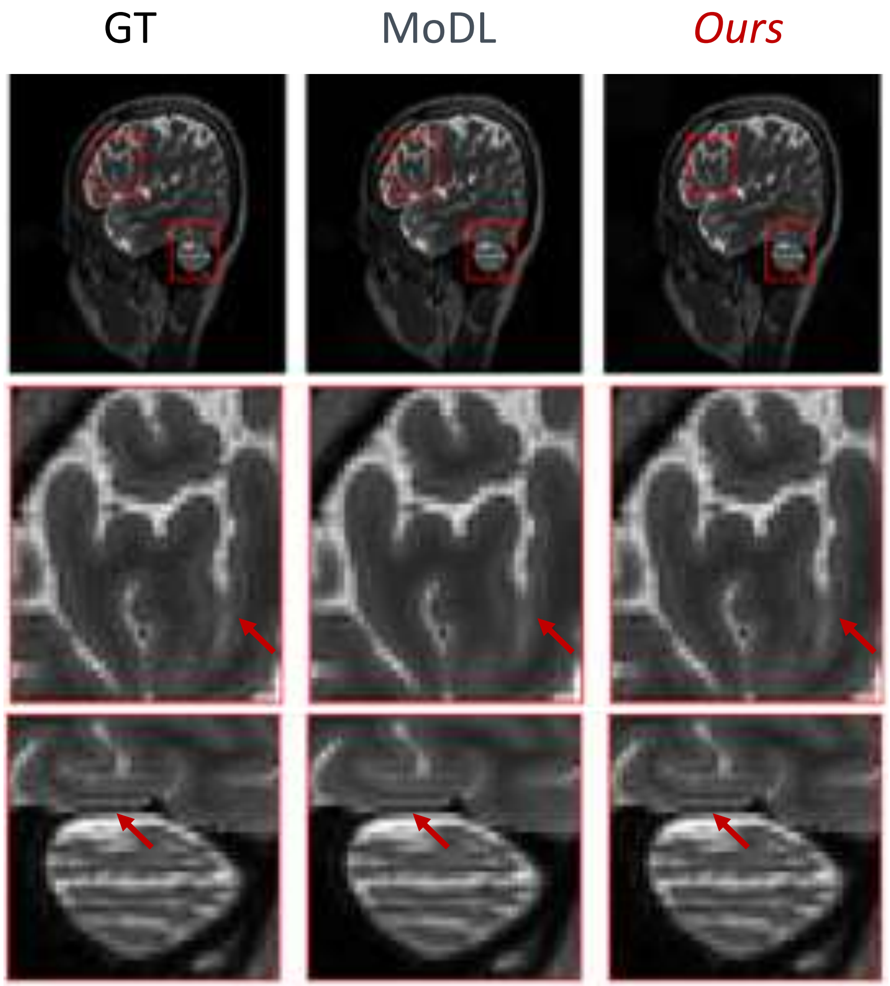
PSNR and SSIM in the format average ± std.

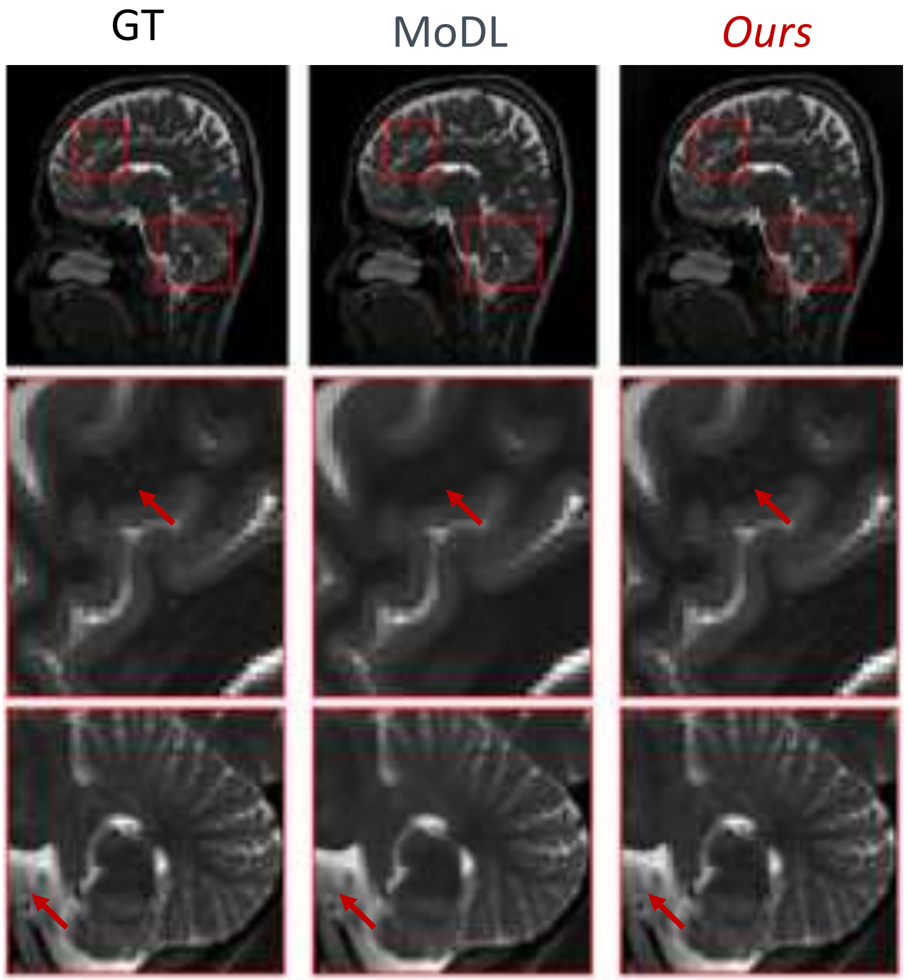
Method	PSNR	SSIM
MoDL	39.06 ± 1.58	0.97 ± 0.02
Ours	$\mathbf{45.96 \pm 3.94}$	$\mathbf{0.98 \pm 0.02}$
CRNN	24.08 ± 0.59	0.71 ± 0.03
Ours	$\mathbf{25.45 \pm 0.71}$	$\mathbf{0.76 \pm 0.02}$

* H. K. Aggarwal, M. P. Mani, and M. Jacob,
MoDL: Model based deep learning architecture for inverse problems
IEEE Transaction on Medical Imaging, Vol 38, No 2, 2019

* C. Qin, J. Schlemper, J. Caballero, A. N. Price, J. V. Hajnal, D. Rueckert
Convolutional recurrent neural networks for dynamic MR image reconstruction
IEEE Transaction on Medical Imaging, Vol 38, No 1, 2019

Experiments on MRI





Experiments on MRI

Measurement consistency of MoDL, CRNN and ours.

Method	$\|\|Aw - b\|\|_2$	$\|\|A\hat{x} - b\|\|_2$
MoDL*	3.10×10^{-1}	9.88×10^{-5}
CRNN*	2.06×10^{-6}	7.71×10^{-15}

Ours

TV-TV Minimization achieves better consistency

Translates to more reliable results

* C. Qin, J. Schlemper, J. Caballero, A. N. Price, J. V. Hajnal, D. Rueckert
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Conclusions

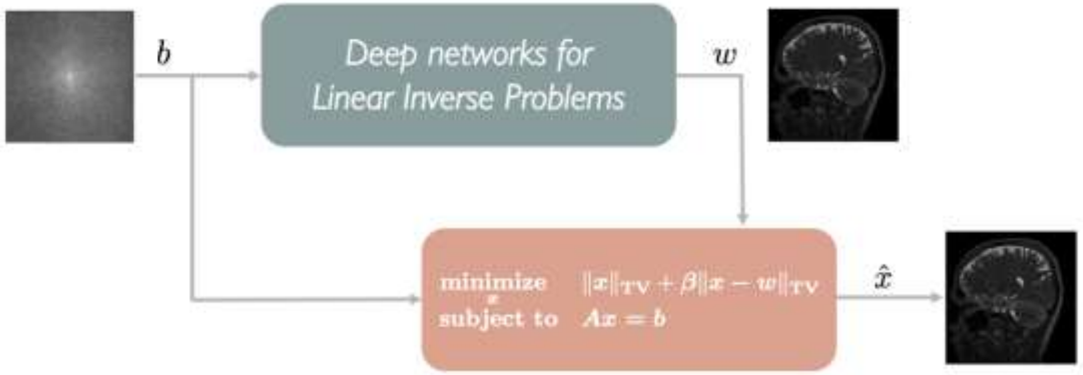
- Optimization-based methods have high measurement consistency guarantee

- DNNs are ***measurement inconsistent*** but provide good quality outputs

- Generalization error implies inconsistency

$$\mathbb{P}\Big(\big\|AX - Af_{\theta^\star}(AX)\big\|_2^2 \geq \delta\Big) \geq 1 - \exp\Big(-2\frac{(c+\epsilon-\delta)^2}{C^2}\Big)$$

- ***TV-TV minimization:*** better quality outputs while ensuring measurement consistency



- Step forward towards applying DNNs to critical areas

Relevant Papers

1. M. Vella, J. F. C. Mota, ***Single Image Super-Resolution via CNN Architectures and TV-TV Minimization***, BMVC 2019

2. M. Vella, J. F. C. Mota, ***Robust Single-Image Super-Resolution via CNNs and TV-TV Minimization***, submitted

3. M. Vella, J. F. C. Mota, ***Overcoming Measurement Inconsistency in Deep Learning for Linear Inverse Problems: Applications in Medical Imaging***, ICASSP 2021

Link to papers

Link to codes

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