Overcoming Measurement Inconsistency in Deep Learning for Linear Inverse Problems: Applications in Medical Imaging

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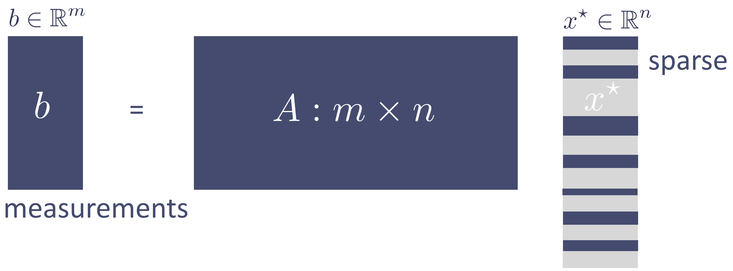
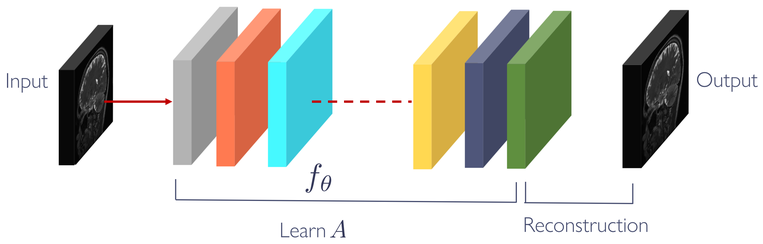
Paper and code link

Linear Inverse Problems



Reconstruct $x^\star$ from b containing m measurements

Deep Learning Methods



Cannot Guarantee $A\hat{x} = b$
Computationally efficient testing
State-of-the-art performance

$c := \ell_{\text{exp}}(f_{\theta^\star}) - \ell_{\text{emp}}(f_{\theta^\star}; \mathcal{T})$ $\quad$ $\mathcal{T}$- training set

$\epsilon := \ell_{\text{emp}}(f_{\theta^\star}; \mathcal{T}) > 0$ $\quad$ $\sigma := C/2$

$$\mathbb{P}\Big(\|AX - Af_{\theta^\star}(AX)\|_2^2 \geq \delta\Big) \geq 1 - \exp\Big(-2\frac{(c+\epsilon-\delta)^2}{C^2}\Big)$$

Probability of inconsistency

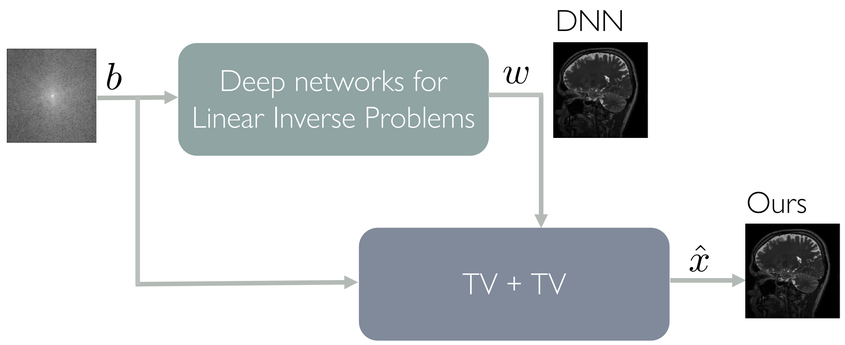
Optimization-based Methods

$$\hat{x} = \arg\min_x \|\Phi x\|_1 \quad \text{s.t.} \quad Ax = b$$

Assume Φx is sparse (DCT, TV, etc.)

Guarantees $A\hat{x} = b$
Computationally expensive
Outperformed by deep learning methods

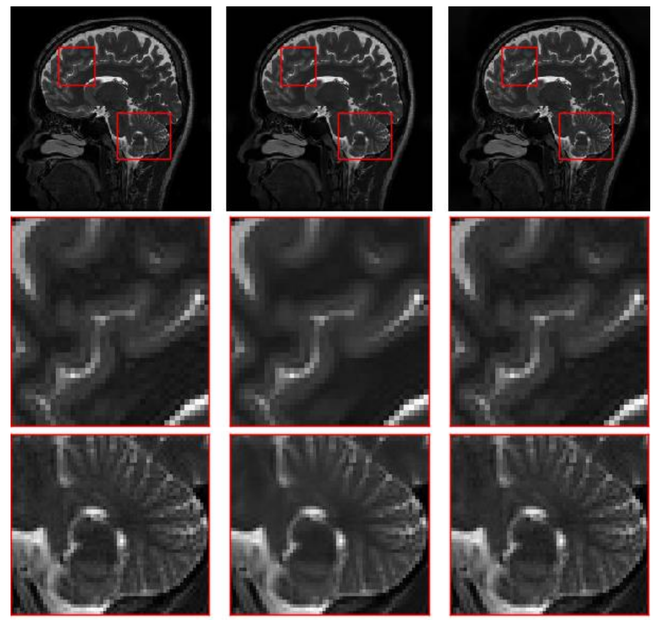
Our Approach



$$\min_x \|x\|_{\text{TV}} + \beta\|x - w\|_{\text{TV}} \quad \text{s.t.} \quad Ax = b$$

Theory indicates that $\beta = 1$

Experimental Results



Acceleration factor of 6

Measurement consistency of MoDL, CRNN and Ours

Method	$\|Aw - b\|_2$	$\|A\hat{x} - b\|_2$
MoDL (Aggarwal, 2019)	3.10×10^{-1}	9.88×10^{-5}
CRNN (Qin, 2019)	2.06×10^{-6}	7.71×10^{-15}

PSNR (SSIM) in the format average ± std

MoDL (Aggarwal, 2019)	*Ours*
39.06 ± 1.58 (0.97 ± 0.02)	45.96 ± 3.94 (0.98 ± 0.02)
CRNN (Qin, 2019)	*Ours*
24.08 ± 0.59 (0.71 ± 0.03)	25.45 ± 0.71 (0.76 ±0.02)