

a Sparse Set of Measurements Using a Modal Architecture







CEBO

Motivation

- Dynamic reverberation with changing source and listener position is a requirement for realistic room acoustics simulation.
 - VR

Information about geometry, materials is available.

AR

Missing geometries, inaccurate materials. Sparse representation of the room can be obtained.

Objectives

- Generate a continuous RIR space utilizing a sparse dataset of RIRs from a room.
- Smooth, real-time interpolation, extrapolation and morphing between measured positions.
- Accurate spatial interpolation of perceptually relevant low frequency modes in rooms with simple geometries having non-rigid walls.

Related Work

- RIR interpolation with Dynamic Time Warping.¹
 - Not a function of source/listener position.
- Compressed Sensing²
 - Reconstructs early part of RIR only.
 - Requires accurate calibration of microphone array.
- Common acoustical-pole and residue model³ (derived from room modes).
 - Parameterized by source and listener locations.
 - Physics is well known for rectangular rooms.

Masterson et al. "Acoustic impulse response interpolation for multichannel systems using dynamic time warping.", AES 2009
 Mignot et al. "Room reverberation reconstruction: Interpolation of the early part using compressed sensing." IEEE TALSP 2013.
 Haneda et al. "Common-acoustical-pole and residue model and its application to spatial interpolation and extrapolation of a room transfer function." IEEE TASLP, 99

Room Modes I

- Standing waves are solutions to the 3D wave equation
- RIR can be characterized as a sum of M room modes⁴

$$h(x, y, z, t) = \sum_{\substack{m=1 \\ \text{Complex mode}}} \gamma_m(x, y, z) e^{(j\omega_m - \alpha_m)t}$$
Mode frequencies and dampings determine

amplitudes depend on dampings determines spatial location of the time response source/listener



• RIRs can be efficiently synthesized with M parallel biquads

$$H(z) = \sum_{m=1}^{M} \frac{Re(\gamma_m) - e^{-\alpha_m} Re(\gamma_m e^{-j\omega_m}) z^{-1}}{1 - 2e^{-\alpha_m} \cos(\omega_m) z^{-1} + e^{-2\alpha_m} z^{-2}}$$

^[4] Abel et al., "A modal architecture for reverberation with application to room acoustics modeling", AES 2014.

Room Modes II

• Complex mode amplitudes are the solution to Helmholtz equation

Wave number

$$\nabla^2 p + k^2 p = 0; k = \frac{\omega_m}{c}$$

• For rigid walls

$$p(x, y, z) = C \cos(k_x x) \cos(k_y y) \cos(k_z z)$$

From boundary conditions
$$k_\mu = \frac{n_\mu \pi}{L_\mu}$$
$$k = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

• For non-rigid walls with finite impedance

$$p(\mu) = C_{\mu} \exp\left(ik_{\mu}\mu\right) + D_{\mu} \exp\left(-ik_{\mu}\mu\right)$$
$$k_{\mu} = \frac{n_{\mu}\pi}{L_{\mu}} + j\delta_{u} \quad \text{Wall absorption}$$
$$p(x, y, z) = p(x)p(y)p(z)$$

Pressure distributions



Kuttruff, Heinrich. Room acoustics, 4th Edition, Spon Press (2000).

Room Modes III

- Analytic solution for pressure distribution as a function of spatial location in a room (with non-rigid walls) requires
 - knowledge of room dimensions.
 - wall absorption, which is frequency dependent.
- Assuming we don't have this information, can we fit parameters to the measurements?



Modal Estimation

For a grid of RIR measurements

- Average RIRs over all locations and estimate common mode frequencies ω_m and decay rates α_m using subband ESPRIT⁵
- Calculate unique set of M mode amplitudes for L locations in grid using linear least squares.

$$\begin{pmatrix} h_1(0) & h_2(0) & \cdots & h_L(0) \\ h_1(1) & h_2(1) & \cdots & h_L(1) \\ \vdots & \vdots & \vdots & \vdots \\ h_1(T) & h_2(T) & \cdots & h_L(T) \end{pmatrix} = \begin{pmatrix} 1 & \cdots & 1 \\ e^{-(j\omega_1 - \alpha_1)1} & \cdots & e^{-(j\omega_M - \alpha_M)1} \\ \vdots & \ddots & \vdots \\ e^{-(j\omega_1 - \alpha_1)T} & \cdots & e^{-(j\omega_M - \alpha_M)T} \end{pmatrix} \begin{pmatrix} \gamma_{11} & \cdots & \gamma_{L1} \\ \gamma_{12} & \cdots & \gamma_{L2} \\ \vdots & \vdots & \vdots \\ \gamma_{M1} & \cdots & \gamma_{ML} \end{pmatrix}$$

[5] Kereliuk et al."Modal analysis of room impulse responses using subband ESPRIT." In *Proceedings of the International Conference on Digital Audio Effects*. 2018.

Optimization - Model

- Goal Given a sparse set of impulse responses on a 2D plane, find the complex mode amplitudes and parameterize them as functions of (x,y) location
- General solution to 2D Helmholtz equation for non-rigid walls is $\widehat{\gamma_m}(x, y) = C_{1m} e^{-j(k_{xm}x+k_{ym}y)} + D_{1m} e^{j(k_{xm}x+k_{ym}y)} + C_{2m} e^{-j(k_{xm}x-k_{ym}y)} + D_{2m} e^{j(k_{xm}x-k_{ym}y)}$
- For each location in grid $(x_1, y_1), \dots, (x_L, y_L)$

 $\left(\widehat{\gamma_{m}}(x_{1},y_{1})\right) = \left(e^{-j(k_{x_{m}}x_{1}+k_{y_{m}}y_{1})} e^{j(k_{x_{m}}x_{1}+k_{y_{m}}y_{1})} e^{-j(k_{x_{m}}x_{1}-k_{y_{m}}y_{1})} e^{-j(k_{x_{m}}x_{1}-k_{y_{m}}y_{1})}$

 $\widehat{\gamma_m} = U_m(k_{x_m}, k_{y_m})C_m$ Linear in constants

Non-linear in wave numbers

Optimization –Sequential update

- Find wave numbers and constants for each mode from the measured set of mode amplitudes.
- For modes, m = 1, 2,..., M_c,
 - Initialize : $k_{\chi_m}(0), k_{\chi_m}(0) = \frac{\omega_m + j\alpha_m}{\sqrt{2}c}$
 - Repeat, till convergence
 - Update constants : $c_m(i) = U_m(i-1)^+ \gamma_m$
 - Update wave numbers : $\operatorname{argmin}_{k_{x_m},k_{y_m}} ||20 \log_{10}(\gamma_m) 20 \log_{10}(U_m(i)c_m(i))||^2$
 - Bounds: $0 \le k_{x_m}, k_{y_m} \le \frac{\omega_m + j\alpha_m}{c}$
- Calculate mode amplitudes at arbitrary 2D location in room
 - $\widehat{\gamma_m}(x,y) = U_m\left(k_{x_m}^*,k_{y_m}^*\right)c_m^*$

Validation

- Accurate simulation of small room acoustics with FDTD simulator.⁶
- Omni source placed at (0.3,0.2,0.1). Receivers placed in XY plane at resolution of d = 20cm at a
 nominal standing height of 1.7m.
- Maximum mode frequency corresponding to M_c is $\frac{c}{2d} = 866$ Hz.



Left – Shoebox room of dimensions 3x2x3 m³

Right – Room with no parallel walls, 3m on longest edge

Front and back wall admittance - 0.9 Left and right wall admittance - 0.8 Floor and ceiling admittance - 0.99



[6] Saarelma, J., Califa, J. and Mehra, R.."Challenges of distributed real-time finite-difference time-domain room acoustic simulation for auralization." In 2018 AES International Conference on Spatial Reproduction-Aesthetics and Science. Audio Engineering Society.

Modal fit for shoebox room – 442 mics

















Modal fit for room with tilted walls – 594 mics



















Experiment with Sparse Microphones

- To replicate sparsity, vary number of microphones in each simulated room from **5** to **50**.
- Run N_{tr} = 100 trials for each set, placing the microphones in different configurations.
- Calculate and plot
 - Mean Structural Similarity Index (MSSIM) metric between [0,1] to measure similarity between actual and fit mode shapes.
 - Absolute Mean Spectral Difference Error (AMSDE) -

$$\frac{1}{N_{tr}} \sum_{n=1}^{N_{tr}} |20 \log_{10} \frac{\sum_{l=1}^{L} H_{l,n}(\omega)}{\sum_{l=1}^{L} \hat{H}_{l,n}(\omega)}|$$







Tilted Walls, Mode at 62 Hz

Results





Conclusion

- Proposed method is capable of accurate RIR interpolation in the lower frequencies with a very small number of randomly distributed microphones.
- Only a handful of parameters need to be stored for characterizing the lowfrequency spatial response of a room - mode frequencies, dampings, two wave numbers and four constants for each mode.
- RIR can be interpolated very efficiently in real-time once these parameters have been estimated offline.

Future Work

- Comparison with State-of-the-art DL method.⁷
- Perceptual evaluation.
- Extend model beyond low frequencies.

