On the Behavior of Delay Network Reverberator Modes

Orchisama Das, Elliot K. Canfield-Dafilou, Jonathan S. Abel

Center for Computer Research in Music and Acoustics, Stanford University, Stanford, USA

IEEE WASPAA

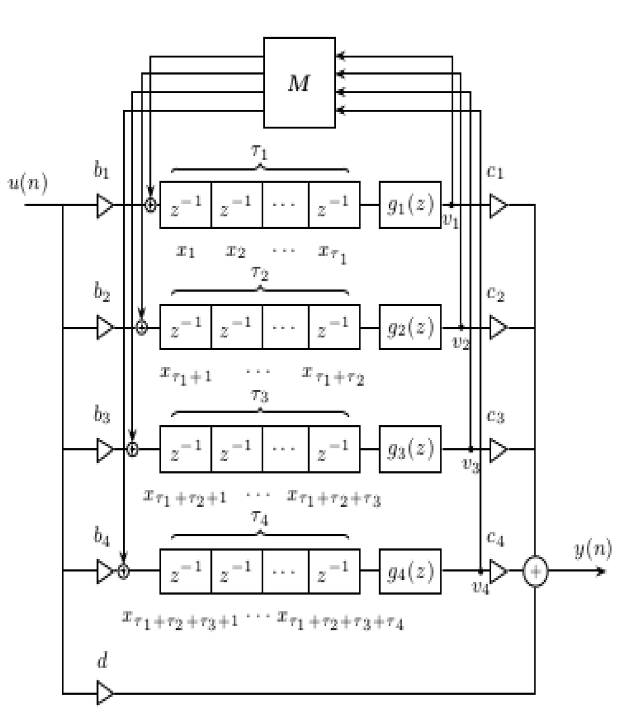
New Paltz, NY, October 20-23, 2019.

Objectives

- Find the modal decomposition of Delay Network Reverberators using a state space formulation.
- Introduce a a parameterized orthonormal mixing matrix which can be continuously varied from identity to Hadamard.
- Investigate the effect of smoothly varying mixing matrix on frequency and damping of modes of Delay Network Reverberators, including Feedback Delay Network (FDN) reverberators.
- Quantify the perceptual effect of increasing mixing by calculating the Normalized Echo Density (NED) of Feedback Delay Network (FDN) impulse responses over time.
- Derive an empirical relationship between mixing matrix, average delay line length and mixing time of the FDN reverberator.

Delay Network Reverberators

- Used to synthesize reverberation.
- Impulse response has a set of sparse early reflections with increasing density over time.
- Feedback delay networks have a unitary feedback matrix (mixing matrix), M, to mix the outputs of the delay lines [1].



State space FDN block diagram.

- FDN reverberators have delay line filters, $g_i(z)$, to yield a single desired frequency dependent T_{60} response [2].
- Delay Network Reverberator design parameters are mixing matrix, shelf filters and number and length of delay lines.

References

[1] M. Gerzon, "Unitary (energy-preserving) multichannel networks with feedback," *Electronics Letters*, vol. 12, no. 11, pp. 278–279, 1976.

[2] J.-M. Jot and A. Chaigne, "Digital delay networks for designing artificial reverberators," in *Audio Engineering Society Convention 90*. Audio Engineering Society, 1991.

[3] G. Weinreich, "Coupled piano strings," *The Journal of the Acoustical Society of America*, vol. 62, no. 6, pp. 1474–1484, 1977.

[4] J. S. Abel and P. Huang, "A simple, robust measure of reverberation echo density," in *Audio Engineering Society Convention 121*. Audio Engineering Society, 2006.

[5] S. J. Schlecht and E. A. Habets, "Feedback delay networks: Echo density and mixing time," *IEEE/ACM Transactions on Audio, Speech, and Language Processing*, vol. 25, no. 2, pp. 374–383, 2017.

Modal Decomposition

- State space equations:

$$\begin{aligned} \boldsymbol{x}(n) &= \boldsymbol{A}\boldsymbol{x}(n-1) + \boldsymbol{b}u(n) \\ y(n) &= \boldsymbol{c}^T\boldsymbol{x}(n) + du(n) \end{aligned}$$

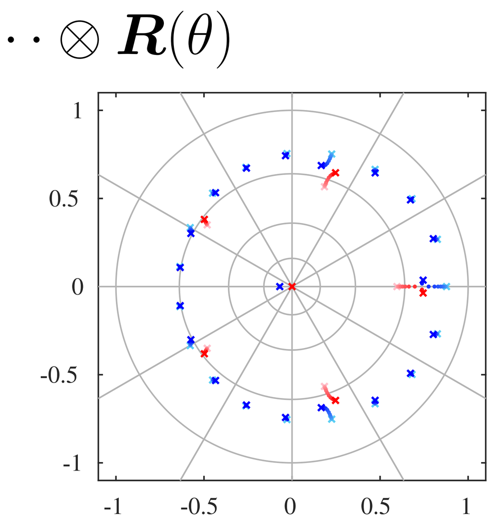
- Modes of the DNR are the eigenvalues of the state transition matrix $\boldsymbol{A}$.
- $\boldsymbol{A}$ is a sparse block matrix whose order is the sum of delay line lengths. The sub-matrix sizes are given by the delay line lengths plus the associated filter order.

Mixing Matrix Effect

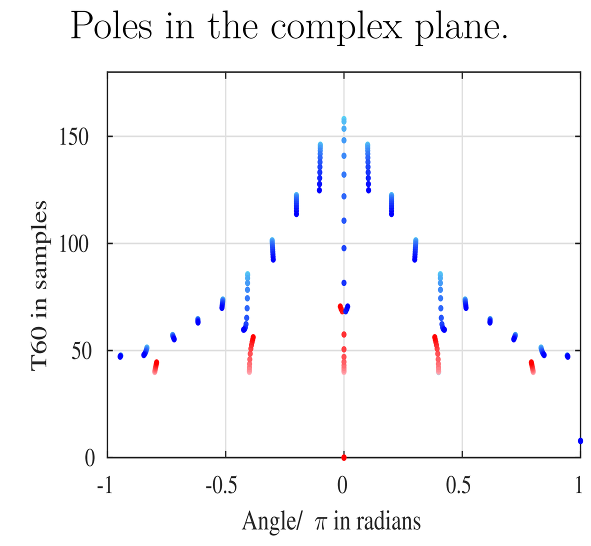
$$\boldsymbol{R}(\theta) = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

$$\boldsymbol{M}(\theta) = \boldsymbol{R}(\theta) \otimes \boldsymbol{R}(\theta) \otimes \cdots \otimes \boldsymbol{R}(\theta)$$

- $\boldsymbol{M}(0) = \boldsymbol{I}$, identity matrix (parallel delay lines).
- $\boldsymbol{M}(\frac{\pi}{4}) = \boldsymbol{H}$, Hadamard matrix (maximum mixing among delay lines).
- Vary θ (mixing angle) in small increments from 0 to $\frac{\pi}{4}$.
- With more mixing, modes that are close in frequency approach each other in damping rapidly and then deflect in frequency.



Poles in the complex plane.



Frequency dependent T_{60}.

Two delay line example. Red indicates scalar gain, blue indicates shelf filter in delay line.

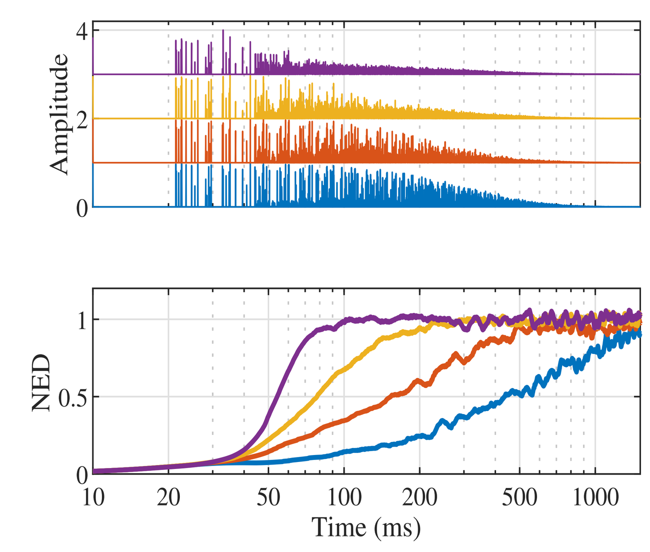
- Damping of low frequency modes affected more than damping of high frequency modes.
- Similar behavior is observed in piano strings [3].

Decay Filter Design

- In FDN reverberators, decay filters are designed such that all delay lines independently produce the same T_{60} frequency response.
- Physical rooms produce multiple, concurrent T_{60} responses.
- One set of filters can model air absorption, another set of filters can model absorption by materials in the room.
- Mixing matrix can emulate occupancy or clutter of a room, or coupled rooms.

Echo Density

- Normalized Echo Density (NED) profile [4] is a perceptual measure of echo density over time, and is used to indicate mixing time.
- NED is calculated for a 16 delay line FDN with random delay lengths uniformly distributed between 10–20 ms at a sampling frequency of 48 kHz with $T_{60_{DC}} = 4$ s and $T_{60_{Nyquist}} = 2$ s.

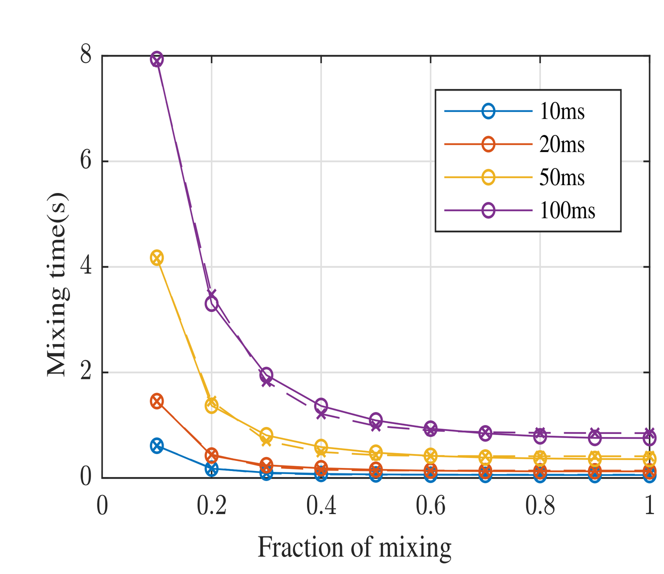


Impulse Responses and NED profiles for mixing matrices with 10%, 20%, 30%, and 100% mixing (bottom to top).

- Reverberator impulse response becomes Gaussian more quickly with increase in mixing.

Mixing Time

- Defined as the time when NED reaches the threshold of 0.9.
- Schlecht [5] has a mechanism to determine the mixing time given a mean delay line length for a random orthogonal mixing matrix.



Mixing time as a function of mixing angle θ (divided by $\frac{\pi}{4}$).

- Here we have a closed form expression for mixing time vs. mixing angle given a mean delay line length.
- Monte Carlo simulations performed for 4 mean delay line lengths, $\bar{\tau} = 10, 20, 50, 100$ ms.

$$t_{mix} = 0.25\bar{\tau}\exp\left[-(20.25 - 0.09\bar{\tau})\theta\right]$$

Conclusion

- Explicitly derived the modal decomposition of delay network reverberators.
- Described how mixing affects modal behavior, NED profile and mixing time.
- Derived closed-form expression for mixing time vs. mixing angle.