Rotation-robust beamforming based on sound field interpolation with regularly circular microphone array

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IEEE ICASSP 2021

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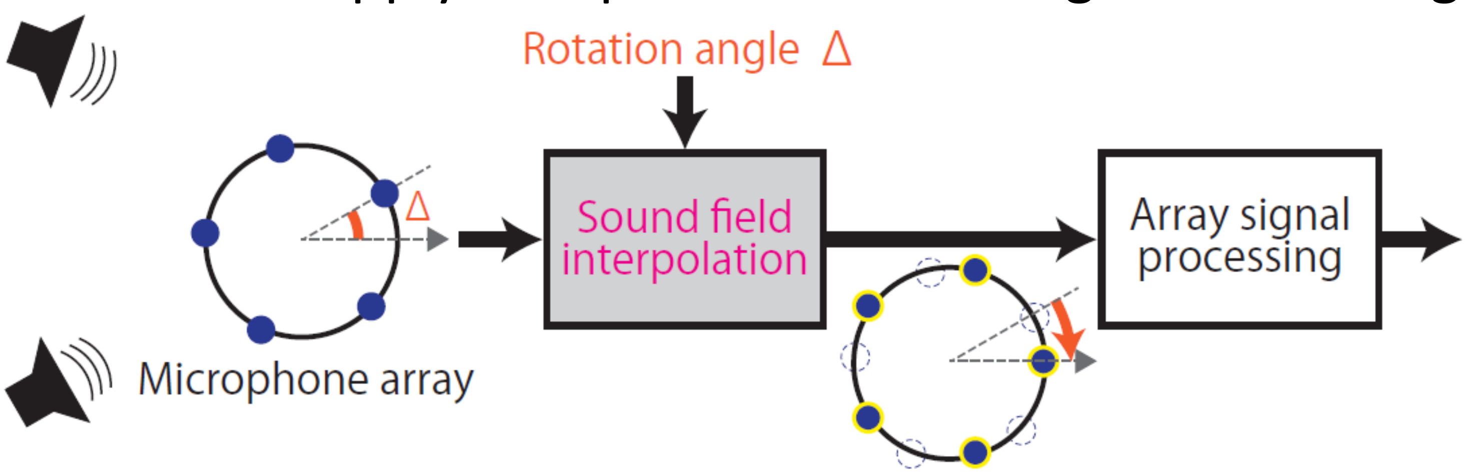
Introduction

- Background
  - Array signal processing assumes a time-invariant ATS*.
  - ATS's variation makes re-estimation of spatial information (e.g., spatial filter and DOA†) necessary.
    ⇒makes online processing difficult.

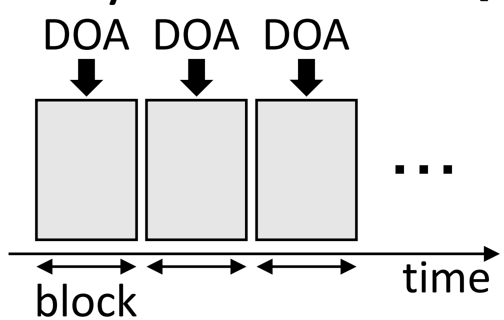
*: acoustic transfer system
†: direction of arrival
**: circular microphone array

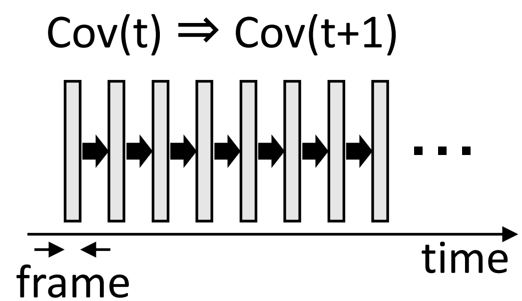
- Motivation
  - We want to follow the ATS's variation caused by CMA** rotation.
  - We want to apply interpolation to existing beamformings.



Prior work for ATS's variation

- Case of source movement
  - Blockwise spatial filter estimation using DOA information [Nikunen+, 2018], [Naqvi+, 2011]
  - Sequential covariance estimation every time-frequency bins with a Bayesian tracker [Taseska+, 2018]





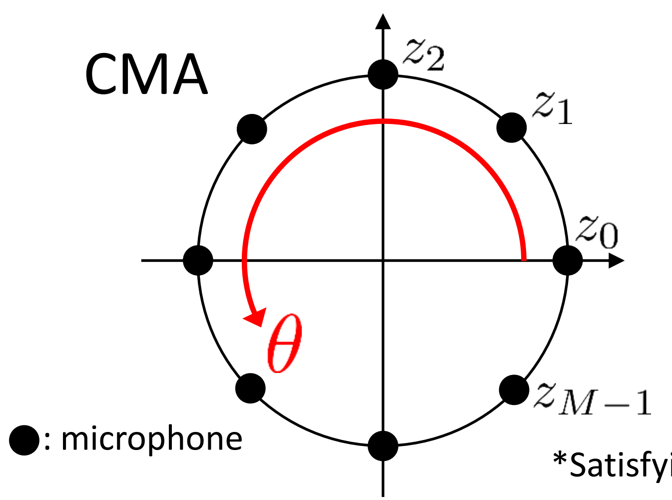
- Case of sensor movement
  - Motion compensation method [Tourbabin+, 2015]
    - Formulating circular harmonic domain rotation matrix
    - Applying to DOA estimation using a CMA

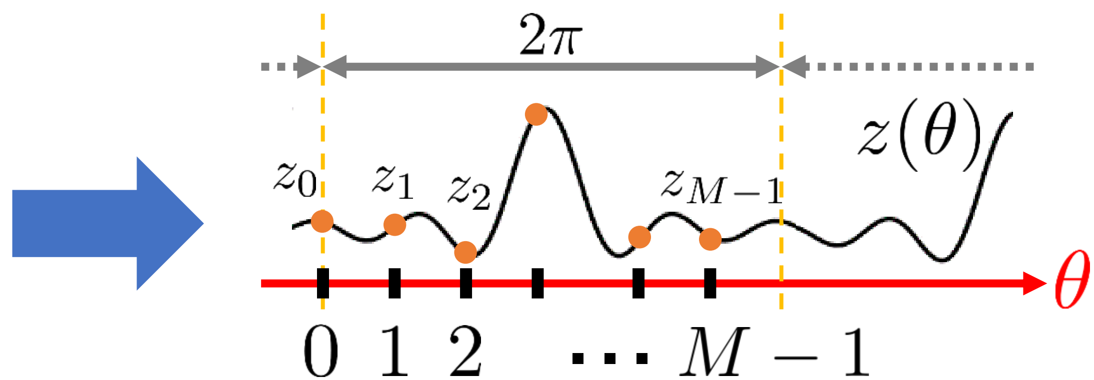
Idea of sound field interpolation

- Continuous sound field on circle's circumference, $z(\theta)$
  - $z(\theta)$ is a periodic function with 2π
  - Discretizing $z(\theta)$ with $2\pi/M$ intervals
    = Observing the sound field using an M-ch CMA

$$z_m = z\left(2\pi\frac{m}{M}\right), \quad m = 0, \ldots, M-1.$$

- $z(\theta)$ can be reconstructed by the discrete signal z_m if the sampling theorem is satisfied*, resulting in **sound field interpolation** possible.





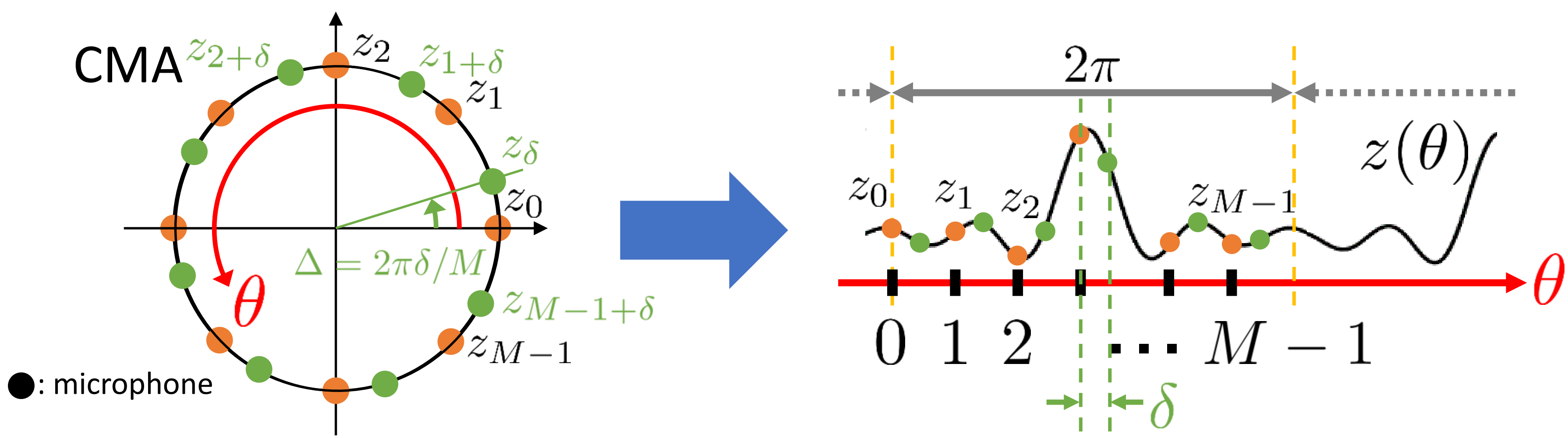
*Satisfying the sampling theorem = Band limitation of circular harmonics spectrum

Sound field interpolation using noninteger sample shift

- Relationship b/w CMA rotation and sample shift
  - Sound field observed by a CMA in the reference position, z_m
  - Sound field observed by the CMA rotated Δ (=$2\pi\delta/M$) [rad]
    = δ-sample shifted z_m in the spatial axis

$$z_{m+\delta} = z\left(2\pi\frac{m}{M} + \Delta\right)$$

- The above equation enables estimating $z_{m+\delta}$ from z_m, m=0,…, M-1.



Formulation of linear interpolation

- Linear representation of δ-sample shifted sound field using the sample shift theorem in the DFT

$$z_{m+\delta} = \mathscr{F}_{\mathrm{D}}^{-1}\left[\mathscr{F}_{\mathrm{D}}\left[z_m\right]e^{j\Delta k}\right]$$

$$= \frac{1}{M}\sum_{k=-M/2+1}^{M/2}\left(Z_k e^{j\Delta k}\right)e^{-j2\pi\frac{mk}{M}} \stackrel{\mathrm{def}}{=} \sum_{n=0}^{M-1} z_n u_{m,n,\delta}$$

$\mathscr{F}_{\mathrm{D}}$: DFT operation

- Sound field interpolation using sinc function (It is derived from the equation above*.)

*: The detailed derivation is appended on the last page.

$$u_{m,n,\delta} = \begin{cases} \frac{1+(-1)^{n-m}e^{j\delta\pi}}{M} + \frac{\mathrm{sinc}\left(L/2\right)}{\mathrm{sinc}\left(L/M\right)} \cdot \cos\left(\frac{M+2}{2M}L\pi\right), & M \text{ is even}, \\ \frac{1}{M} + \frac{M-1}{M}\frac{\mathrm{sinc}\left(L(M-1)/2M\right)}{\mathrm{sinc}\left(L/M\right)} \cdot \cos\left(\frac{M+1}{2M}L\pi\right), & M \text{ is odd}. \end{cases}$$

$$L = n - m - \delta$$

Remark of formulation

- How to handle Nyquist frequency (NyqF) component

$$z_{m+\delta} = \sum_{n=0}^{M-1} z_n u_{m,n,\delta} \quad (*)$$

$$u_{m,n,\delta} = \begin{cases} \frac{1+(-1)^{n-m}}{M} + \frac{\mathrm{sinc}\left(L/2\right)}{\mathrm{sinc}\left(L/M\right)} \cdot \cos\left(\frac{M+2}{2M}L\pi\right), & M \text{ is even}, \\ \frac{1}{M} + \frac{M-1}{M}\frac{\mathrm{sinc}\left(L(M-1)/2M\right)}{\mathrm{sinc}\left(L/M\right)} \cdot \cos\left(\frac{M+1}{2M}L\pi\right), & M \text{ is odd}. \end{cases}$$

  - When M is even, a noninteger sample shift results in a complex-valued term even if the sign of the NyqF is positive or negative, and which causes some contradiction;
    e.g., sample shift of a real-valued even point signal by $(*)$ translates to a complex-valued signal.
  - In this study, we neglect the NyqF component by setting δ=0.

Matrix representation

- Formulation of the relationship b/w the observation by a Δ-rotated CMA, **x**, and by the CMA w/o rotation, $\mathbf{x}_0$

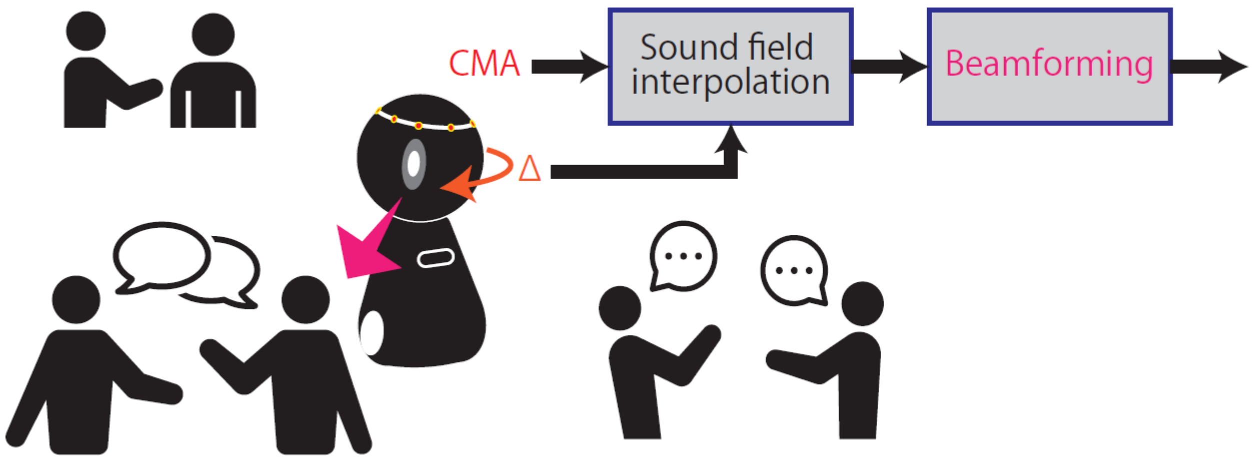
$$
\begin{aligned}
\boldsymbol{x} &= \begin{bmatrix} x_1 & \cdots & x_M \end{bmatrix}^{\mathrm{T}} \\
&= \begin{bmatrix} z\Big(2\pi(0+\delta)/M\Big) & \cdots & z\Big(2\pi(M-1+\delta)/M\Big) \end{bmatrix}^{\mathrm{T}} \\
&= \begin{bmatrix} u_{0,0,\delta} & u_{0,1,\delta} & \cdots & u_{0,M-1,\delta} \\ u_{1,0,\delta} & u_{1,1,\delta} & \cdots & u_{1,M-1,\delta} \\ \vdots & \vdots & \ddots & \vdots \\ u_{M-1,0,\delta} & u_{M-1,1,\delta} & \cdots & u_{M-1,M-1,\delta} \end{bmatrix} \begin{bmatrix} z_0 \\ z_1 \\ \vdots \\ z_{M-1} \end{bmatrix} \\
&= \boldsymbol{U}_{\Delta} \boldsymbol{x}_0 .
\end{aligned}
$$

- Note: $\mathbf{U}_{\Delta}$ is a cyclic matrix & does not depend on frequencies

[cf.] Our method has a tight relationship to the circular harmonics domain rotation matrix used by the motion compensation method [Tourbabin+, 2015].

Applying interpolation to beamforming

- Situation: A user or humanoid robot
  - wears a CMA on the head.
  - rotates the head to listen to ambient conversations attentively.



- Problem:
  - The rotation angle Δ is given.
  - We estimate the observation **before CMA rotation**.
  - We do beamforming using interpolated M-ch signals.

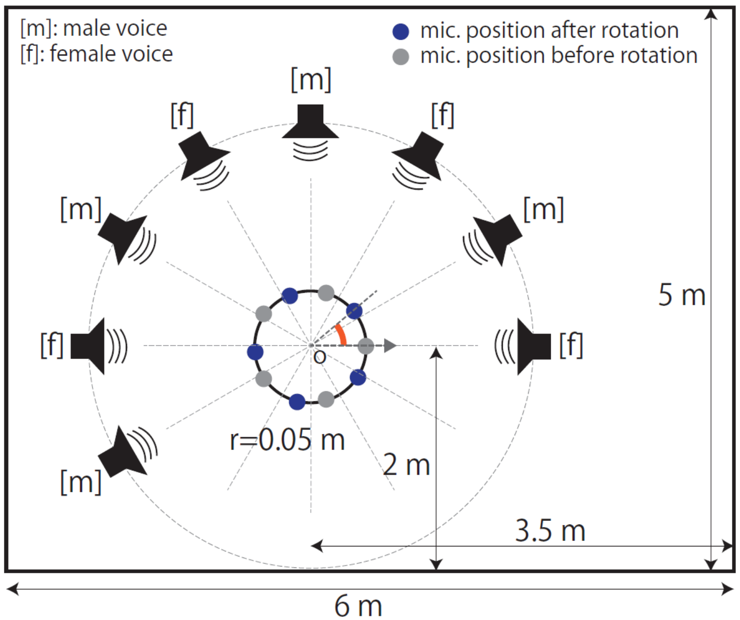
Evaluation condition

- Mixture : 2 sources from SiSEC [Araki+, 2012], fs=16 kHz, 12 envs.
- RIR : Simulator [Habets, 2008], RT60≒100 ms
- STFT : 64 ms Hamming window with 1/8 shifts

- **Exp. 1 : interpolation performance**
  - Array rot. Δ : 10, 20, and 30 deg
  - Measure : SER (signal-to-error ratio)

$$\mathrm{SER}_{m,k} = 10\log_{10}\left(\frac{\sum_t |x_{m,t,k}|^2}{\sum_t |\hat{x}_{m,t,k} - x_{m,t,k}|^2}\right)$$

$x_{m,t,k} \in \mathbb{C}$



- **Exp. 2 : signal enhancement**
  - Array rot. Δ : 10, 20, and 36 deg
  - Beamformer : MPDR* + RTF** [Doclo+, 2015]
    - 2 methods of applying beamformer.
  - Measure : SDR, SIR [Vincent+, 2006]

*minimum power distortionless response
**relative transfer function

How to apply to beamforming

- **Method 1: Constantly use of pre-estimated spatial filter**
  - Pre-estimation of spatial filter when the CMA does not rotate

$$\boldsymbol{w} = \mathrm{MPDR}(\boldsymbol{V}, \boldsymbol{a}), \quad \boldsymbol{V} = \mathbb{E}\left[\boldsymbol{x}_0 \boldsymbol{x}_0^{\mathrm{H}}\right]$$

  - Interpolation & beamforming

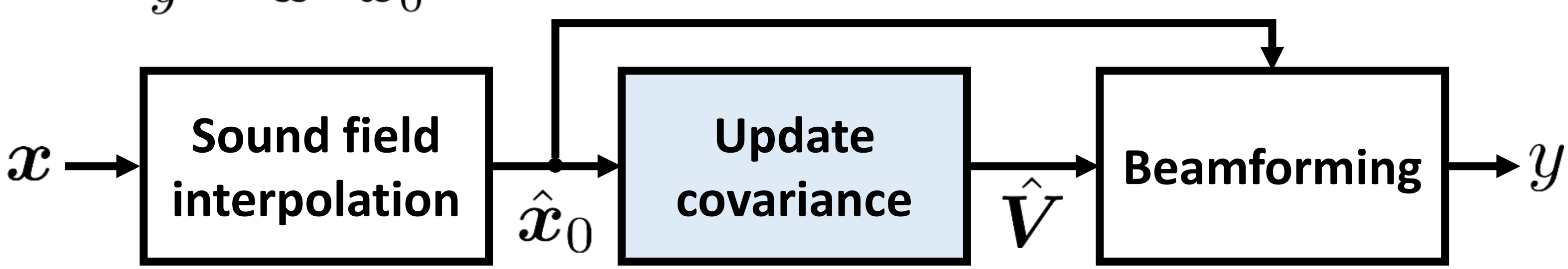
$$y = \boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{x}}_0 = \boldsymbol{w}^{\mathrm{H}} \left(\boldsymbol{U}_{-\Delta} \boldsymbol{x}\right)$$

Δ : rotation angle
$\boldsymbol{a}$: transfer function
$\boldsymbol{x}_0$: observation in the reference position

- **Method 2: Re-estimation of spatial filter**

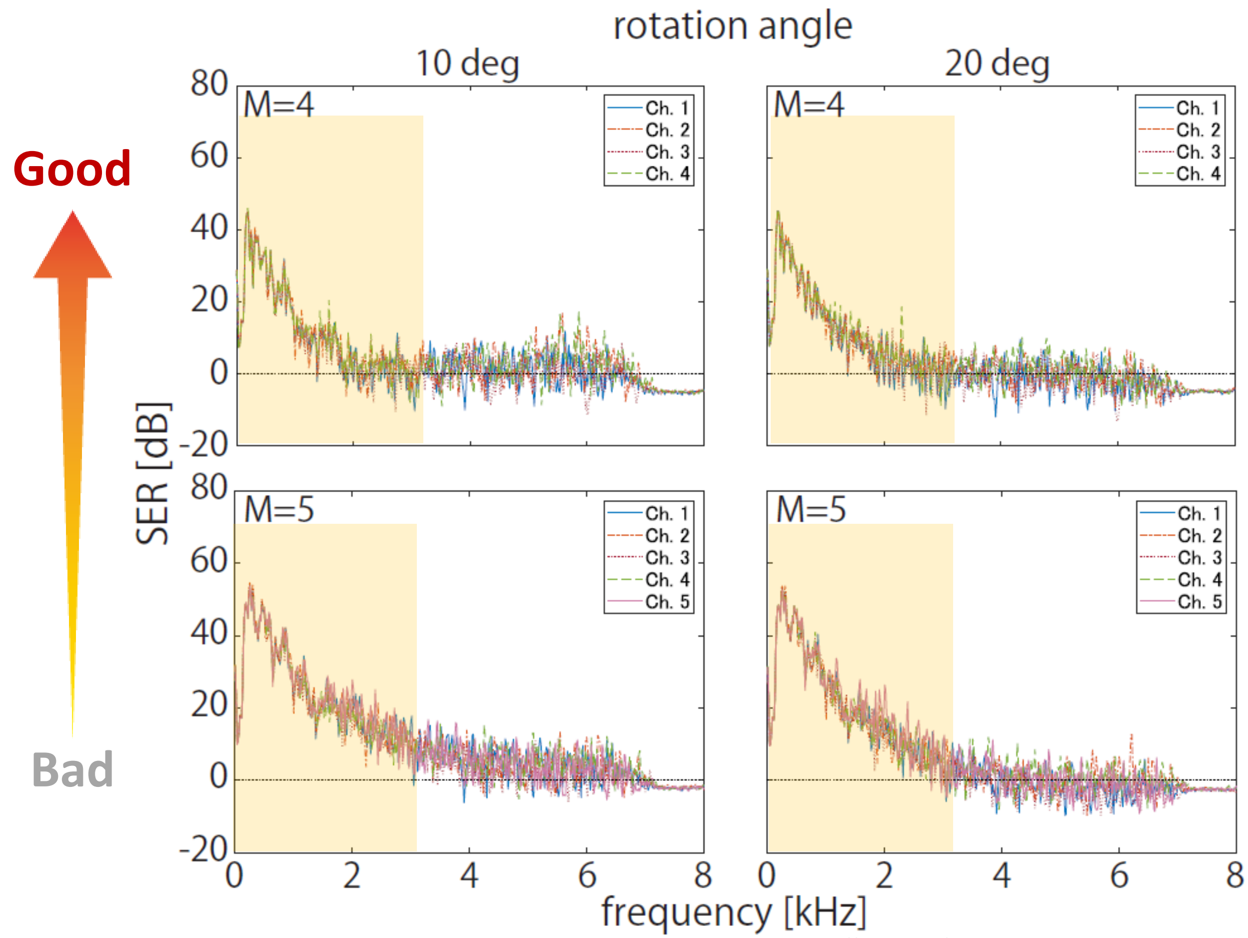
$$\tilde{\boldsymbol{w}} = \mathrm{MPDR}(\hat{\boldsymbol{V}}, \boldsymbol{a}), \quad \hat{\boldsymbol{V}} = \mathbb{E}\left[\hat{\boldsymbol{x}}_0 \hat{\boldsymbol{x}}_0^{\mathrm{H}}\right], \quad \hat{\boldsymbol{x}}_0 = \boldsymbol{U}_{-\Delta} \boldsymbol{x}$$

$$y = \tilde{\boldsymbol{w}}^{\mathrm{H}} \hat{\boldsymbol{x}}_0$$



Exp. 1: Interpolation performance 1/2

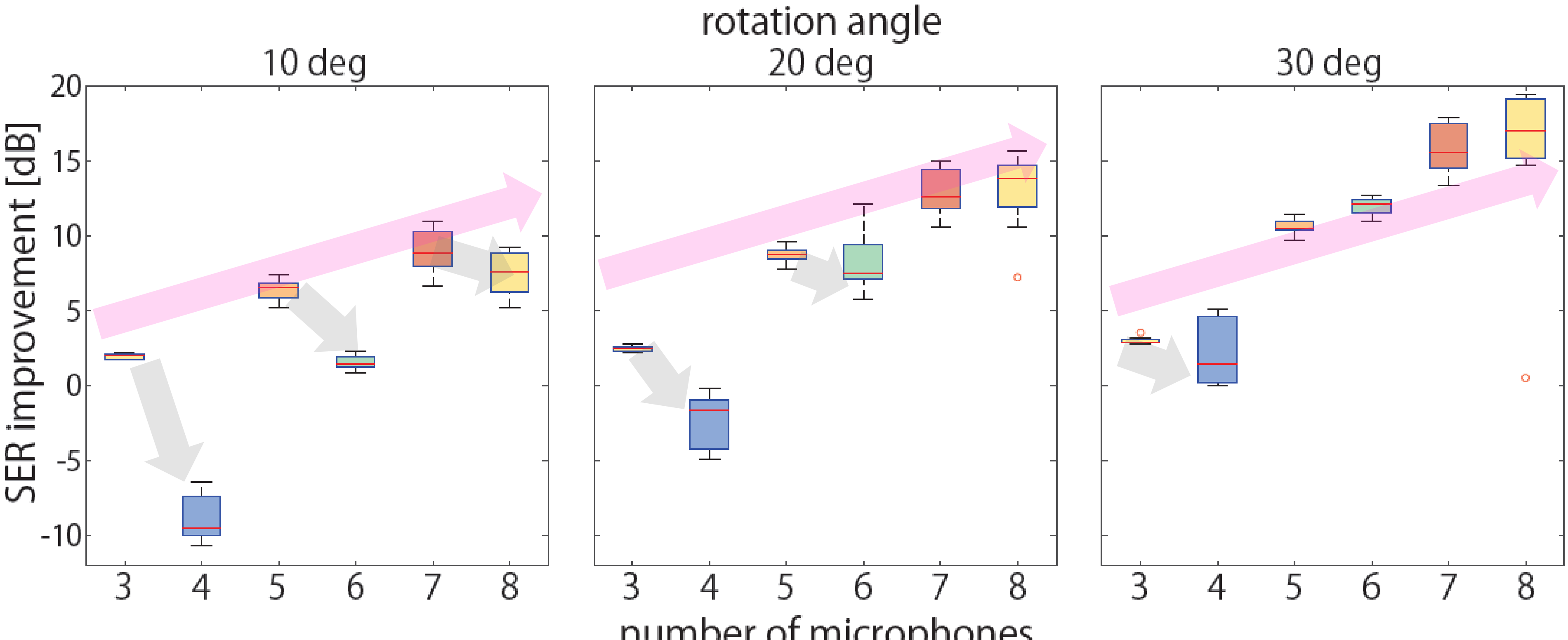
- Examples of SER when 2 sources are active
  - Lower band interpolation worked well, but not higher one.



Exp. 1: Interpolation performance 2/2

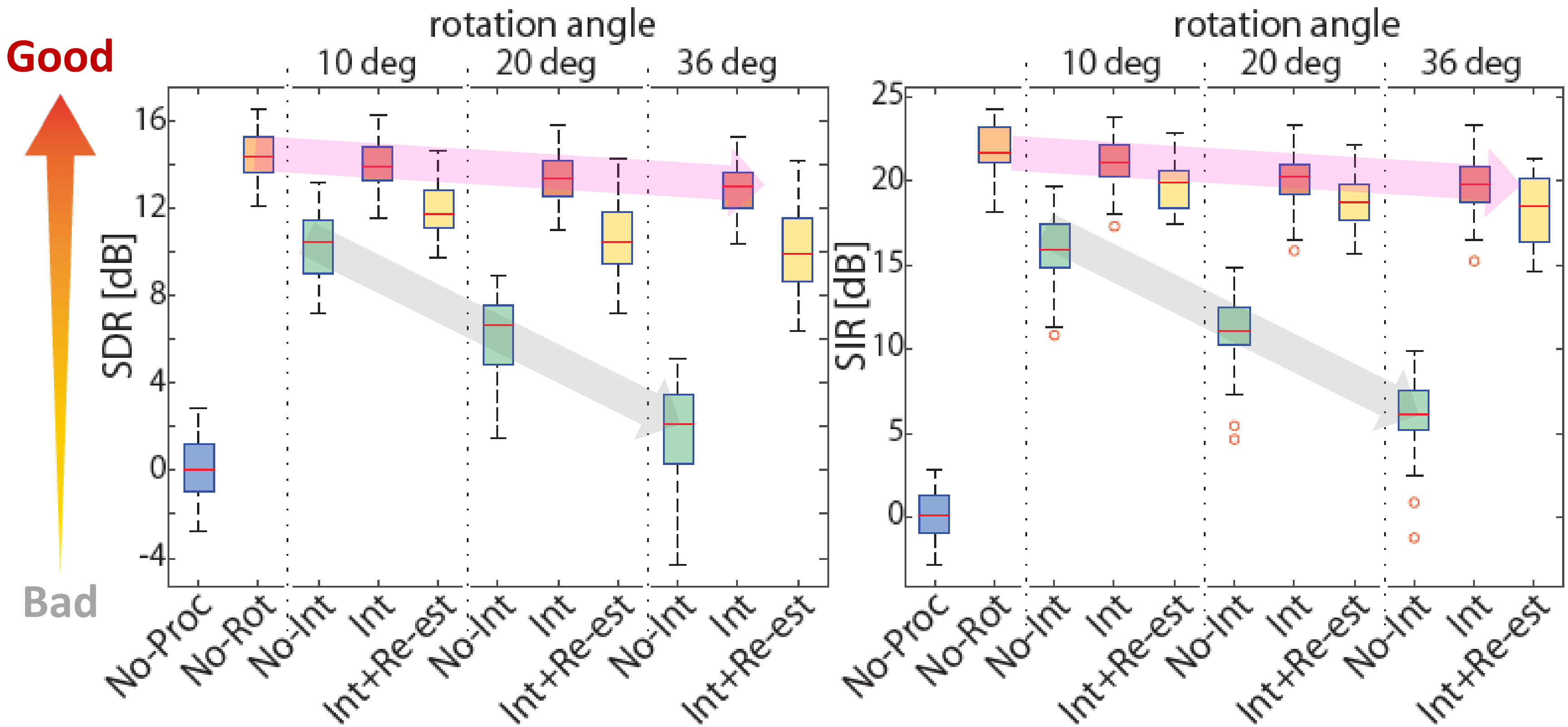
- Averaged SER improvement (up to 3 kHz)
  - More the number of microphones improves performance.↑
  - NyqF components decrease performance when M=4, 6↓

    e.g., when M=4, only 3 components contribute to interpolation.



Exp. 2: Signal enhancement

- SDR & SIR, M=5
  - In **No-Int** case, bigger rotation decreases SDR & SIR.
  - **Int** & **Int+Re-est** come close to the performance of **No-Rot**.



No-Proc : unprocessed, **No-Rot : no rotation**, **No-Int : no interpolation**, **Int : interpolation**, **Int＋Re-est : interpolation + filter re-estimation**

Conclusion

- Summary
  - Sound field interpolation method for rotation-robust beamforming using CMA.
  - Lower band interpolation accuracy was high when M is odd.
  - Higher band interpolation accuracy was low, but applying to beamforming worked well.
- Future work
  - Applying to different array processing, e.g., source separation
  - Clarifying relation to circular harmonics domain beamforming

Derivation (1/2)

When M is even

$$W = e^{-j2\pi/M}$$
$$0 < \delta < 1$$

$$z_{m+\delta} = z\left(2\pi\frac{m}{M} + \Delta\right) = \mathscr{F}_{\mathrm{D}}^{-1}\left[\mathscr{F}_{\mathrm{D}}\left[z_m\right]e^{j\Delta k}\right]$$

$$= \frac{1}{M}\sum_{k=-M/2+1}^{M/2} Z_k e^{j\Delta k} W^{-mk}$$

DC, Positive freq., NyqF component, Negative freq.

$$= \frac{1}{M}\left\{Z_0 + \sum_{k=1}^{M/2-1} Z_k e^{j\Delta k} W^{-mk} + Z_{\frac{M}{2}} e^{j\Delta\frac{M}{2}} W^{-m\frac{M}{2}} + \sum_{k=-M/2+1}^{-1} Z_k e^{j\Delta k} W^{-mk}\right\}$$

$$= \frac{1}{M}\left\{\left(\sum_{n=0}^{M-1} z_n\right) + \sum_{k=1}^{M/2-1}\left(\sum_{n=0}^{M-1} z_n W^{nk} e^{j\Delta k}\right)W^{-mk} + \left(\sum_{n=0}^{M-1} z_n W^{n\frac{M}{2}} e^{j\Delta\frac{M}{2}}\right)W^{-m\frac{M}{2}}\right.$$
$$\left. + \sum_{k=1}^{M/2-1}\overline{\left(\sum_{n=0}^{M-1} z_n W^{nk} e^{j\Delta k}\right)}W^{mk}\right\}$$

This term is a complex value because δ is a noninteger.

$$= \frac{1}{M}\sum_{n=0}^{M-1} z_n \underbrace{\left\{1 + \left(\sum_{k=1}^{M/2-1} W^{(n-m-\delta)k}\right) + W^{(n-m-\delta)\frac{M}{2}} + \left(\sum_{k=1}^{M/2-1} W^{-(n-m-\delta)k}\right)\right\}}_{u_{m,n,\delta}}$$

Derivation (2/2)

Set $L = n - m - \delta$

$$
\begin{aligned}
u_{m,n,\delta} &= \frac{1}{M}\left\{1 + \underline{W^{L\frac{M}{2}}} + \sum_{k=1}^{M/2-1} W^{Lk} + \sum_{k=1}^{M/2-1} W^{-Lk}\right\} \\
&= \frac{1}{M}\left\{1 + W^{L\frac{M}{2}} + W^{L}\frac{1-W^{L\frac{M}{2}}}{1-W^{L}} + W^{-L}\frac{1-W^{-L\frac{M}{2}}}{1-W^{-L}}\right\} \\
&= \frac{1}{M}\left\{1 + W^{L\frac{M}{2}} + W^{\frac{L}{4}(M+2)}\frac{W^{-L\frac{M}{4}}-W^{L\frac{M}{4}}}{W^{-\frac{L}{2}}-W^{\frac{L}{2}}} + W^{-\frac{L}{4}(M+2)}\frac{W^{L\frac{M}{4}}-W^{-L\frac{M}{4}}}{W^{\frac{L}{2}}-W^{-\frac{L}{2}}}\right\} \\
&= \frac{1}{M}\left(1 + W^{L\frac{M}{2}} + W^{\frac{L}{4}(M+2)}\frac{\sin(\pi L/2)}{\sin(\pi L/M)} + W^{-\frac{L}{4}(M+2)}\frac{\sin(\pi L/2)}{\sin(\pi L/M)}\right) \\
&= \frac{1}{M}\left(1 + W^{L\frac{M}{2}}\right) + \frac{1}{2}\cdot\frac{\operatorname{sinc}(L/2)}{\operatorname{sinc}(L/M)}\left(W^{\frac{L}{4}(M+2)} + W^{-\frac{L}{4}(M+2)}\right) \\
&= \frac{1}{M}\left(1 + e^{-jL\pi}\right) + \frac{\operatorname{sinc}\Big(L/2\Big)}{\operatorname{sinc}\Big(L/M\Big)}\cdot\cos\left(\frac{M+2}{2M}L\pi\right).
\end{aligned}
$$