Sparse-Coded Dynamic Mode Decomposition on Graph for Prediction of River Water Level Distribution

MLSP - 17

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ABSTRACT

Propose combing extended dynamic mode decomposition (**EDMD**) and graph filter banks (**GFBs**).

- **EDMD** is a data driven modeling method for nonlinear dynamic systems.
- We introduce DMD on graph to predict **multi-point river water levels**.
- **GFBs** work in combination with a sparse approximation algorithm.
- **Graph** is used to **construct GFBs** for analyzing and synthesizing water levels.
- We conduct river water level prediction for **real web-scraped data**.
- Performance evaluation shows the superiority to the normal DMD approach.

Index Terns – *Extended dynamic mode decomposition, graph signal processing, sparse coding, river disaster prevention*

I. INTRODUCTION

Problem

- **EDMD** takes no account of **graph structure**.
- **GFB** reflects no **temporal variation** for time series data.

Purpose

- To predict temporal variation of graph signal w/ **graph structure** and **dynamics**.

Figure 1: Part of Shinano River system, Japan (left), and its flooding in Ojiya on Oct. 13, 2020 (right)

II. REVIEW OF CSC-DMD AND GFB & III. SPARSE CODED DMD ON GRAPH

Convolutional-sparse-coded dynamic mode decomposition (CSC-DMD) [1]

- Variant of **EDMD**: High dimensional time-series data analysis method.
- Utilize **convolutional synthesis dictionary** for promoting **sparse representation**.

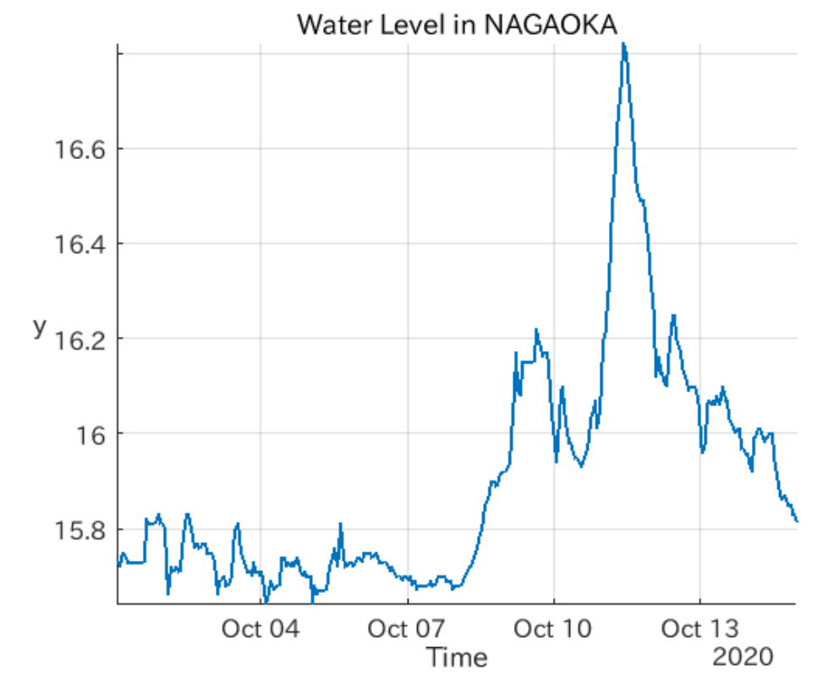


Figure 2: Example of time-series data

Graph filter bank (GFB) [2]

- Decompose and reconstruct signals on graph
  ⟶ Enable us to extract **features of graph signals**.

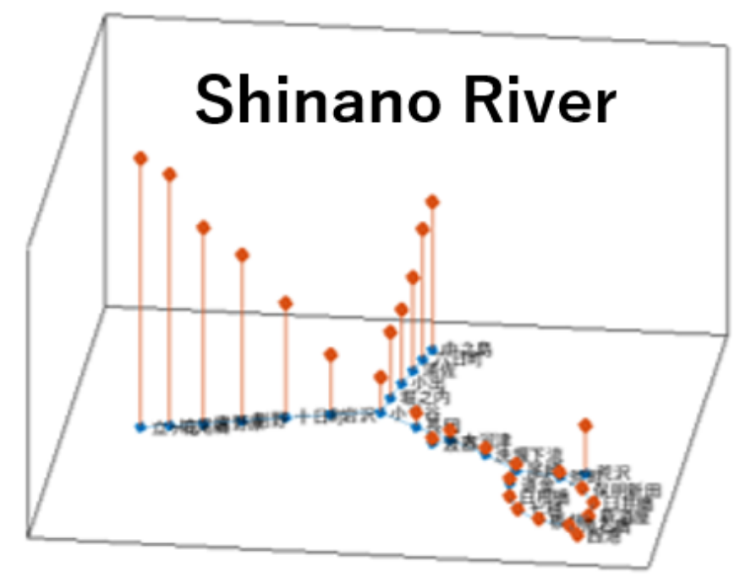


Figure 3: Example of graph signal

P-channel undecimated synthesis dictionary G

$$\mathbf{G} = \left(\mathbf{H}_0\ \mathbf{H}_1 \cdots \mathbf{H}_{P-1}\right) \qquad (1)$$

SC-DMD-G for prediction of river water level distribution

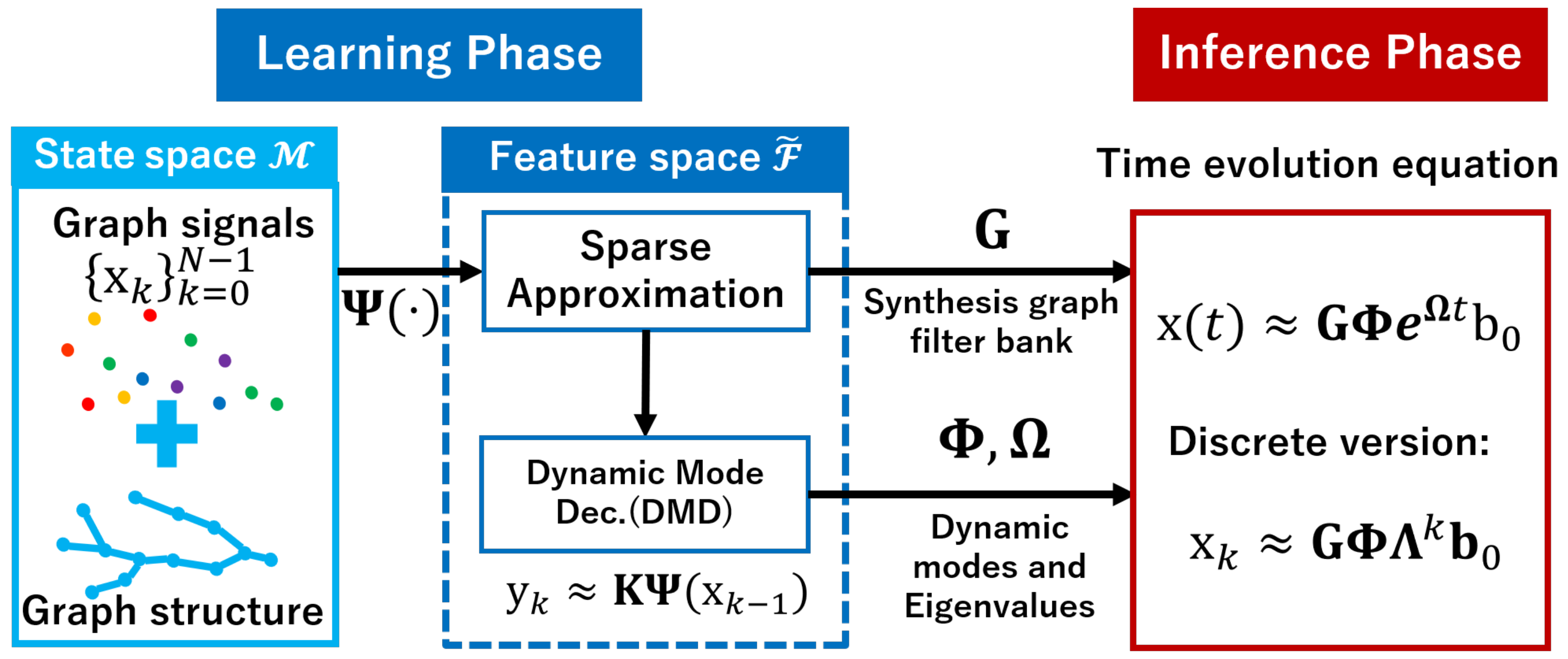


Figure 4: Framework of SC-DMD-G

Learning and inference steps:

Step 1: Prepare a training set. $\{\mathbf{x}_k\}_{k=0}^{N-1}$ of graph signals.
Step 2: Map $\{\mathbf{x}_k\}_{k=0}^{N-1}$ with $\mathbf{\Psi}(\cdot)$ to $\{\mathbf{y}_k\}_{k=0}^{N-1}$.
Step 3: Find the transfer matrix $\mathbf{K}$ in $\tilde{\mathcal{F}}$.
Step 4: Find the dynamic modes $\mathbf{\Phi}$ and eigenvalues $\mathbf{\Lambda}$ of $\mathbf{K}$.
Step 5: Predict by the time evolution equation.

$\mathbf{x}_k$: Graph signals
$\mathbf{y}_k$: Sparse representation of $\mathbf{x}_k$
$\mathbf{\Phi}$: Dynamic modes
$\mathbf{G}$: Synthesis graph filter bank
$\mathbf{\Psi}(\cdot)$:Sparse approx.
$\mathbf{\Lambda}$: Eigenvalues, $\mathbf{\Omega} = \ln(\mathbf{\Lambda})/\mathbf{\Delta}t$

Problem setting of sparse approx.

$$\mathbf{\Psi}(\mathbf{x}) = \arg\min_{\mathbf{y}\in\mathbb{R}^L} \frac{1}{2}\|\mathbf{x} - \mathbf{G}\mathbf{y}\|_2^2 + \alpha\rho(\mathbf{y}) \qquad (2)$$

Time evolution equation

$$\mathbf{x}(t) \approx \mathbf{G\Phi} e^{\mathbf{\Omega}t}\mathbf{b}_0 \qquad (3)$$

Discrete version:

$$\mathbf{x}_k \approx \mathbf{G\Phi\Lambda}^k\mathbf{b}_0 \qquad (4)$$

IV. PERFORMANCE EVALUATION

IV- I. Analysis Result

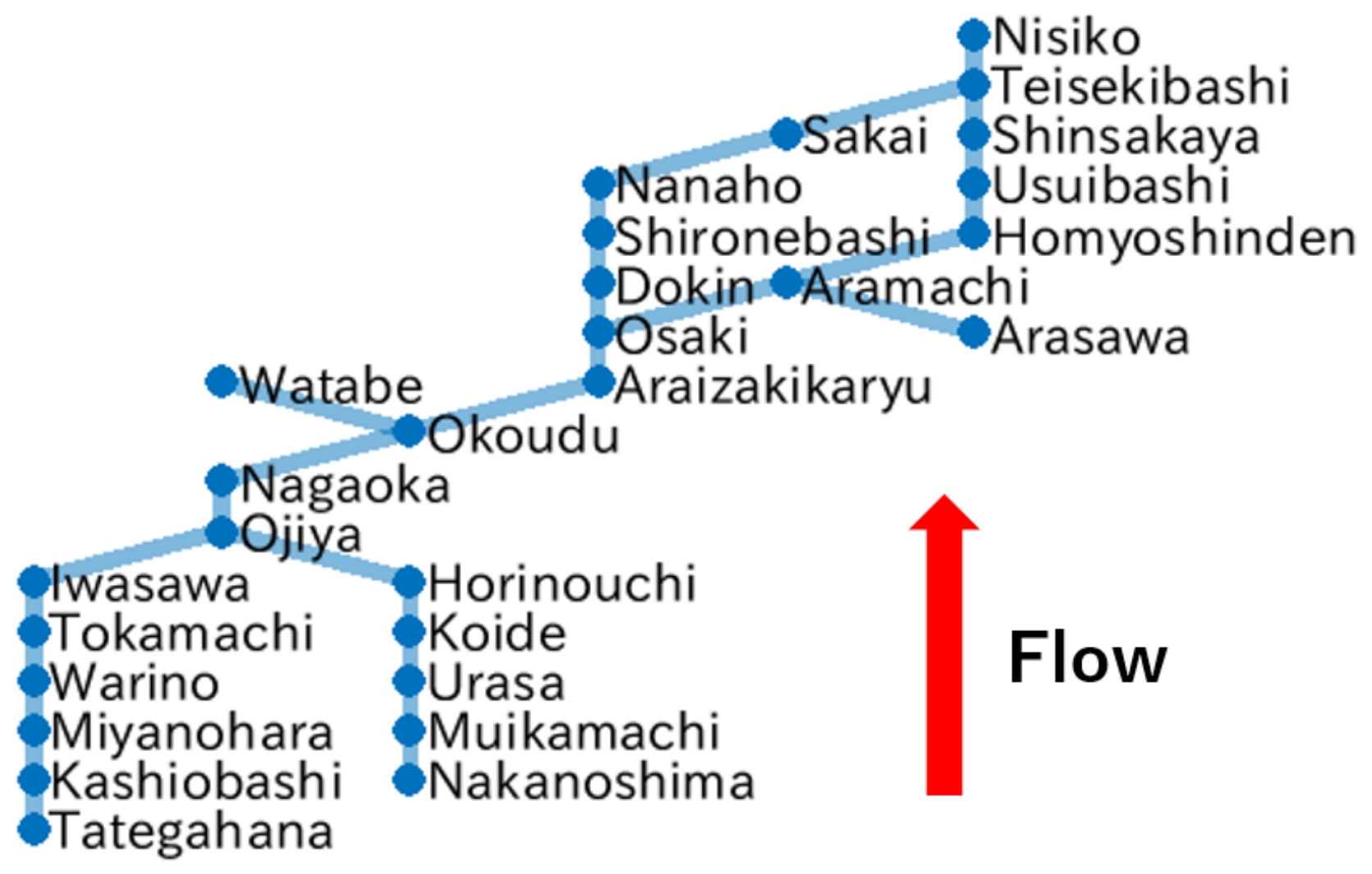
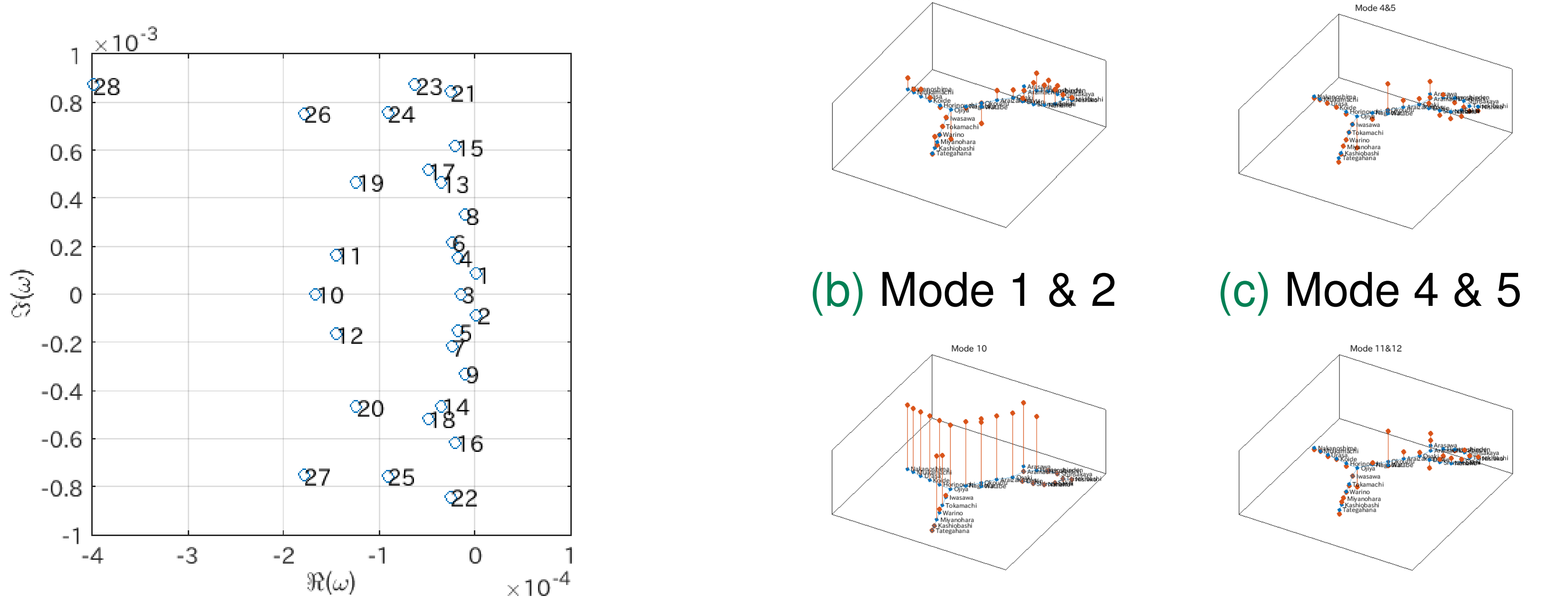


Figure 5: Graph structure of monitoring stations in Shinano River system



(a) Eigenvalues in $\mathbf{\Omega}$ (b) Mode 1 & 2 (c) Mode 4 & 5 (d) Mode 10 (e) Mode 11 & 12

Figure 6: Placement of the eigenvalues of SC-DMD-G (a), example of dynamic modes in $\mathbf{\Phi}$ (b)-(e), $\alpha = 0.2$ and step size $\gamma = 1.2$.
(Fig. 3 in the paper was wrong. The correct figure is as in (a).)

IV- II. Prediction Result

- **Learning for 33 hours:** from Aug. 25, 2020 at 0 a.m. to Aug. 26, 2020 at 9 a.m.
- **Inference for 87 hours:** from Aug. 26, 2020 at 10 a.m. to Aug. 29, 2020 at 11 p.m.
- **28 monitoring stations** in Shinano River system
- SC-DMD-G performs better than normal DMD.

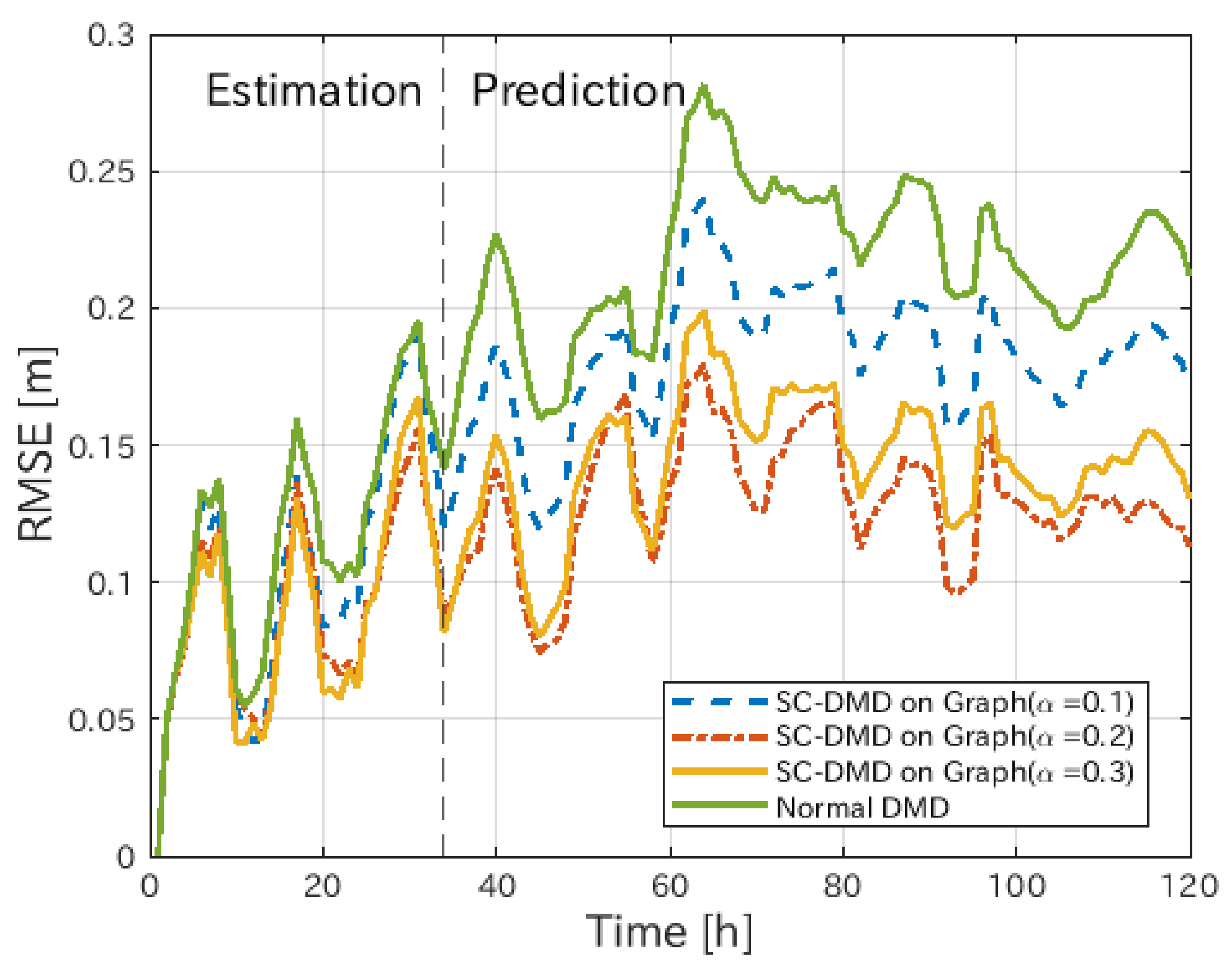


Figure 7: Result of river water level prediction at all evaluation points (RMSE), step size $\gamma = 1.2$.

V. CONCLUSION

- Proposed SC-DMD-G w/ DMD & GFBs.
- Demonstrated the proposed method performs better than the normal DMD.
- Future works: utilization of linkage with rainfall information, dictionary learning, graph neural networks and control river water level

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