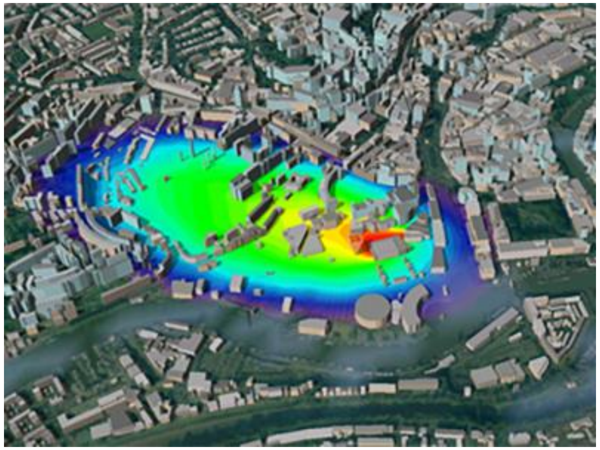
Australian Government
Department of Defence
Science and Technology

Estimation of Fields Using Binary Measurements From a Mobile Agent

Alex Leong, Mohammad Zamani
Defence Science and Technology Group, Australia

IEEE International Conference on Autonomous Systems, Aug. 2021

Introduction



- In the presence of a CBRN (chemical, biological, radiological, and nuclear) event, mapping out the contaminated area / field is an important task in allowing operations to be carried out
  - Humanitarian, disaster relief, military …
  - Want to do it as quickly and accurately as possible

Outline

- Introduction
  - Mobile Autonomous Agents
  - System Model
- Field Estimation
- Active Sensing
- Simulation Results
- Conclusion

Use of Mobile Autonomous Agents

- Significant previous research on using wireless sensor networks for environment monitoring
  - Large number of (mostly) static sensors spread out over an area
- We often won't know when and where CBRN attacks occur
  - Assumption of a large number of sensors already in place may not be realistic
  - Assume instead the use of a mobile autonomous vehicles / agents with sensors onboard, which can move around to collect measurements at different locations

Previous work

- Source localization using mobile agents
  - Non-binary measurements [Ristic, Morelande, Gunatilaka, 2010]
  - Binary measurements, single source [Selvaratnam et.al., 2019]
- Field estimation with static sensors, binary measurements
  - [Battistelli et.al., 2019]
- Field estimation using mobile agents, non-binary measurements
  - [La, Sheng, Chen, 2015], [Razak, Sukumar, Chung, 2019]

- Our work: Field estimation using mobile agents, binary measurements
  - Concentrate on single agent case

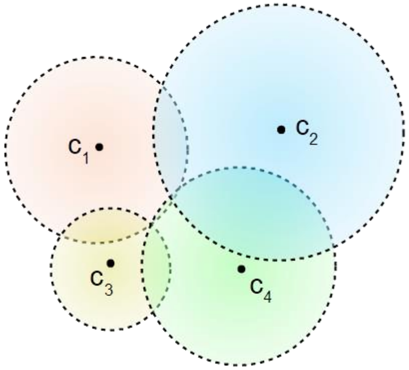
Measurement Model

- Radiological sources can be measured pretty accurately

- Chemicals used in attacks may be of very low concentration
- Current chemical sensing technologies cannot give very precise readings of such concentrations
  - Noisy and time varying
  - Sensor outputs could be one of several levels
- Assume noisy and coarsely quantized measurements
  - $z(\mathbf{x}) = q(\phi(\mathbf{x}) + v(\mathbf{x}))$, where $\mathbf{x}$ is position, ϕ is field value, v is Gaussian noise, q is quantizer
  - In this work will consider special case of noisy binary measurements - reading is above or below a known threshold τ

    $z(\mathbf{x}) = \mathbb{1}(\phi(\mathbf{x}) + v(\mathbf{x}) > \tau)$ (algorithms will still hold for general case)

Field Model



- Approximate the field as a sum of basis functions
- Field model: $$\phi(\mathbf{x}) = \sum_{j=1}^{J} \beta_j K_j(\mathbf{x})$$

  where β_j are weights, $K_j(\mathbf{x}) = \exp\left(-\frac{||\mathbf{c}_j - \mathbf{x}||^2}{\sigma_j^2}\right)$ are (radial) basis functions

  – Used in works such as [Morelande, Skvortsov, 2009], [La, Sheng, Cheng, 2015], [Razak, Sukumar, Chung, 2019]

  – Mathematical results prove that for J large enough, can approximate many fields to arbitrary accuracy

- Parameter estimation approach: Pick a (large) J, choose $\mathbf{c}_j$'s and σ_j's, and estimate the β_j's

Field Estimation

- Field estimation problem reduces to problem of estimating parameters $\boldsymbol{\theta} = (\beta_1, \ldots, \beta_J, \sigma_v^2)$ where σ_v^2 is measurement noise variance
- We want to compute the posterior pdf $p(\boldsymbol{\theta}|z_1, \ldots, z_k; \mathbf{x}_1, \ldots, \mathbf{x}_k)$ where z_k is the *k*-th measurement collected
  - Parameter estimates can then be derived from the posterior pdfs
- Exact computation of posterior pdfs is generally intractable
- Posterior pdfs can be computed approximately using sequential Monte Carlo / particle filtering techniques
  - Use approach of [Liu, West, 2001] suitable for estimation of constant parameters (rather than time-varying states)

Field Estimation

Algorithm 1 Sequential Monte Carlo algorithm for parameter estimation

Algorithm Parameters: $N \in \mathbb{N}$, $a \in (0,1)$, $h = \sqrt{1-a^2}$, $\eta \geq 0$, prior pdf $p_0(\boldsymbol{\theta})$
Inputs: Measurement locations $\{\mathbf{x}_k\}$
Outputs: Particles $\{\boldsymbol{\theta}_k^{(i)}\}$ and weights $\{\boldsymbol{w}_k^{(i)}\}$
Sample particles $\boldsymbol{\theta}_0^{(i)}, i = 1, \ldots, N$ from $p_0(\boldsymbol{\theta})$, and assign weights $\boldsymbol{w}_0^{(i)} = \frac{1}{N}, i = 1, \ldots, N$
for $k = 1, 2, \ldots,$ **do**
    Observe z_k at location $\mathbf{x}_k$
    for $i = 1, \ldots, N$ **do**
        Compute $\mathbf{m}_{k-1}^{(i)} = a\boldsymbol{\theta}_{k-1}^{(i)} + (1-a)\bar{\boldsymbol{\theta}}_{k-1}$, where $\bar{\boldsymbol{\theta}}_{k-1} = \sum_{i=1}^{N} \boldsymbol{w}_{k-1}^{(i)} \boldsymbol{\theta}_{k-1}^{(i)}$
        Assign $\tilde{\boldsymbol{w}}_k^{(i)} \propto p(z_k | \mathbf{m}_{k-1}^{(i)}; \mathbf{x}_k) \boldsymbol{w}_{k-1}^{(i)}$
    end for
    Normalize $\{\tilde{\boldsymbol{w}}_k^{(i)}\}$ such that $\sum_{i=1}^{N} \tilde{\boldsymbol{w}}_k^{(i)} = 1$
    Sample N times with replacement a set of indices $\{i^- : i = 1, \ldots, N\}$, from a distribution with probabilities $\mathbb{P}(i^- = j) = \tilde{\boldsymbol{w}}_k^{(j)}$
    for $i = 1, \ldots, N$ **do**
        Sample a particle $\boldsymbol{\theta}_k^{(i)} \sim \mathcal{N}(\mathbf{m}_{k-1}^{(i^-)}, h^{2-\eta}\mathbf{V}_{k-1})$, where $\mathbf{V}_{k-1} = \sum_{i=1}^{N} \boldsymbol{w}_{k-1}^{(i)} (\boldsymbol{\theta}_{k-1}^{(i)} - \bar{\boldsymbol{\theta}}_{k-1})(\boldsymbol{\theta}_{k-1}^{(i)} - \bar{\boldsymbol{\theta}}_{k-1})^T$
        Assign weights $\boldsymbol{w}_k^{(i)} \propto \frac{p(z_k | \boldsymbol{\theta}_k^{(i)}; \mathbf{x}_k)}{p(z_k | \mathbf{m}_{k-1}^{(i^-)}; \mathbf{x}_k)}$
    end for
    Normalize $\{\boldsymbol{w}_k^{(i)}\}$ such that $\sum_{i=1}^{N} \boldsymbol{w}_k^{(i)} = 1$
end for

Active Sensing

- Given a set of measurements $(z_1, \ldots, z_k)$ and their locations $(\mathbf{x}_1, \ldots, \mathbf{x}_k)$ the particle filtering approach computes posterior pdfs and hence parameter estimates
- Can we "optimize" the locations in which measurements are made, to reduce the time needed to accurately map the field?
- Active sensing – actively choose locations for the sensor measurements, based on measurements currently collected
- One approach to active sensing is based on Renyi divergence
  - [Kreucher et.al, 2007], [Ristic, Morelande, Gunatilaka, 2010]

Active Sensing

- Renyi divergence between two pdfs $f_1(.), f_2(.)$ defined as

$$D_\alpha(f_1||f_0) \triangleq \frac{1}{\alpha - 1} \ln \int f_1^\alpha(\mathbf{t}) f_0^{1-\alpha}(\mathbf{t}) d\mathbf{t}$$

  – a measure of the difference between two pdfs (Kullback-Leibler divergence is a special case as $\alpha \to 1$)

- Approach to active sensing
  – look at expected Renyi divergence $\mathbb{E}[D_\alpha(p(\boldsymbol{\theta}|z_{1:k}; \mathbf{x}_{1:k})||p(\boldsymbol{\theta}|z_{1:k+1}; \mathbf{x}_{1:k+1}))]$ between posterior pdf at current location $\mathbf{x}_k$ and posterior pdf at a set of candidate future locations $\mathbf{x}_{k+1}$
  – pick the future location which maximizes this
  – Intuition: Larger divergence means more "information" can potentially be obtained at the new location

Active Sensing

Algorithm 2 Active sensing algorithm: $\mathbf{x}_{k+1} = \texttt{ActiveSensing}(\mathbf{x}_k, \{\boldsymbol{\theta}_k^{(i)}\})$

1: **Algorithm Parameters**: $\varepsilon \geq 0$, $\alpha \in [0, \infty)\backslash\{1\}$, $\rho_0 \geq 0$, $N_\rho \in \mathbb{N}$, $N_d \in \mathbb{N}$, search region $\mathcal{S}$

2: **Inputs**: $\mathbf{x}_k$, $\{\boldsymbol{\theta}_k^{(i)}\}$

3: **Output**: Next measurement location $\mathbf{x}_{k+1}$

4: With probability ε set $\mathbf{x}_{k+1}$ to a random location in $\mathcal{S}$, otherwise set

$$\mathbf{x}_{k+1} = \arg\max_{\mathbf{x}' \in \mathcal{X}_k} \frac{1}{\alpha - 1} \sum_{z_{k+1}=0}^{1} \gamma_1(z_{k+1}|\mathbf{x}') \ln \frac{\gamma_\alpha(z_{k+1}|\mathbf{x}')}{(\gamma_1(z_{k+1}|\mathbf{x}'))^\alpha}$$

where

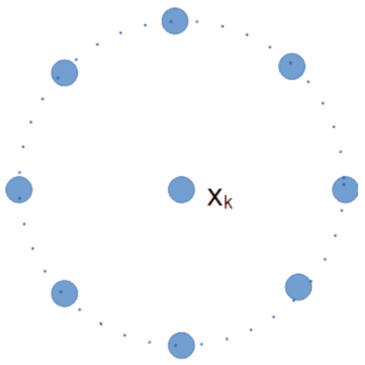
$$\mathcal{X}_k = \left\{ \mathbf{x}_k + \left(n\rho_0 \cos\left(\frac{2\pi\ell}{N_d}\right), n\rho_0 \sin\left(\frac{2\pi\ell}{N_d}\right)\right), \right.$$
$$\left. n = 0, \ldots, N_\rho, \ell = 0, 1, \ldots, N_d - 1 \right\} \cap \mathcal{S}$$

$$\gamma_\alpha(z_{k+1}|\mathbf{x}') = \frac{1}{N} \sum_{i=1}^{N} p(z_{k+1}|\boldsymbol{\theta}_k^{(i)}; \mathbf{x}')^\alpha$$

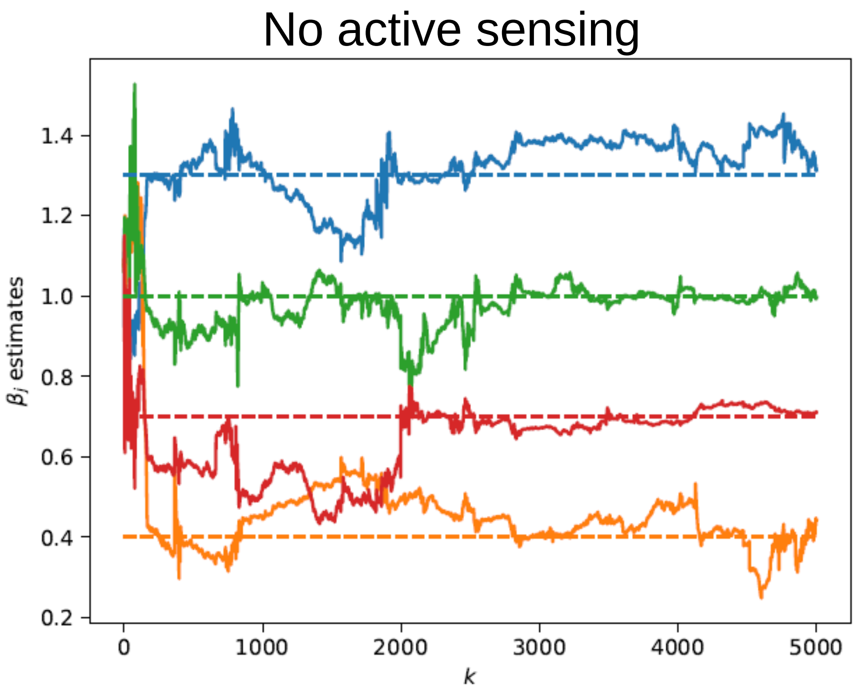
Simulation Studies – Example 1

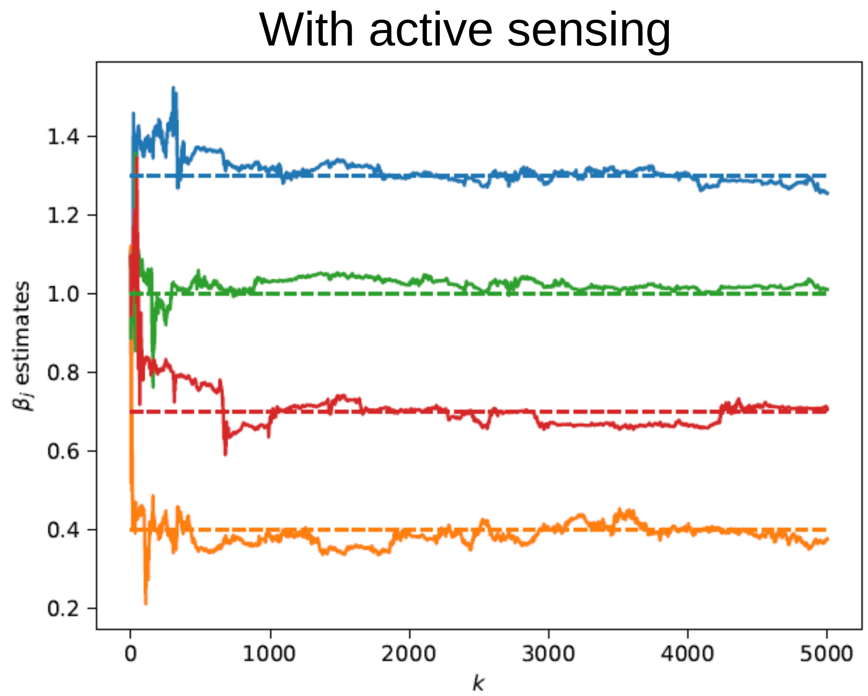
- $J = 4$ basis functions
- True values of $\mathbf{c}_j$'s and σ_j's known

- Candidate future locations to optimize over in active sensing algorithm
  - Current location plus eight directions



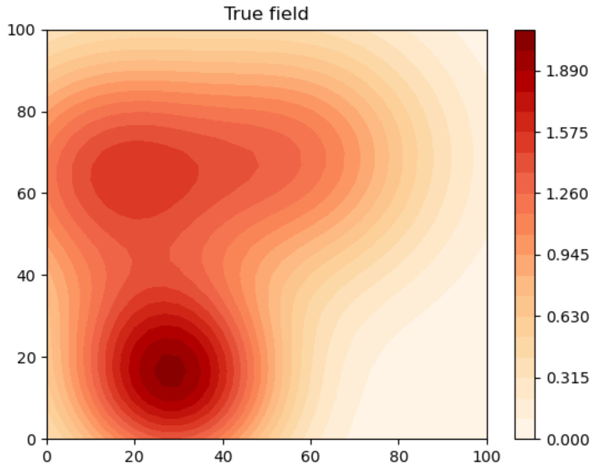
Simulation Studies – Example 1





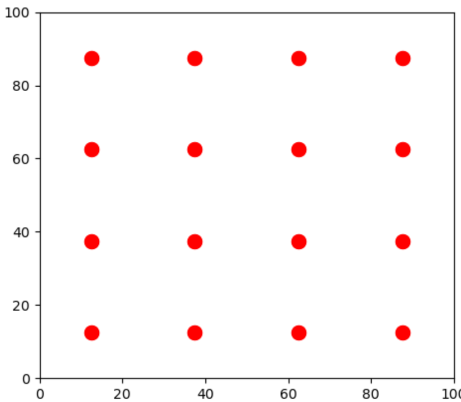
Simulation Studies – Example 2

- True field as shown

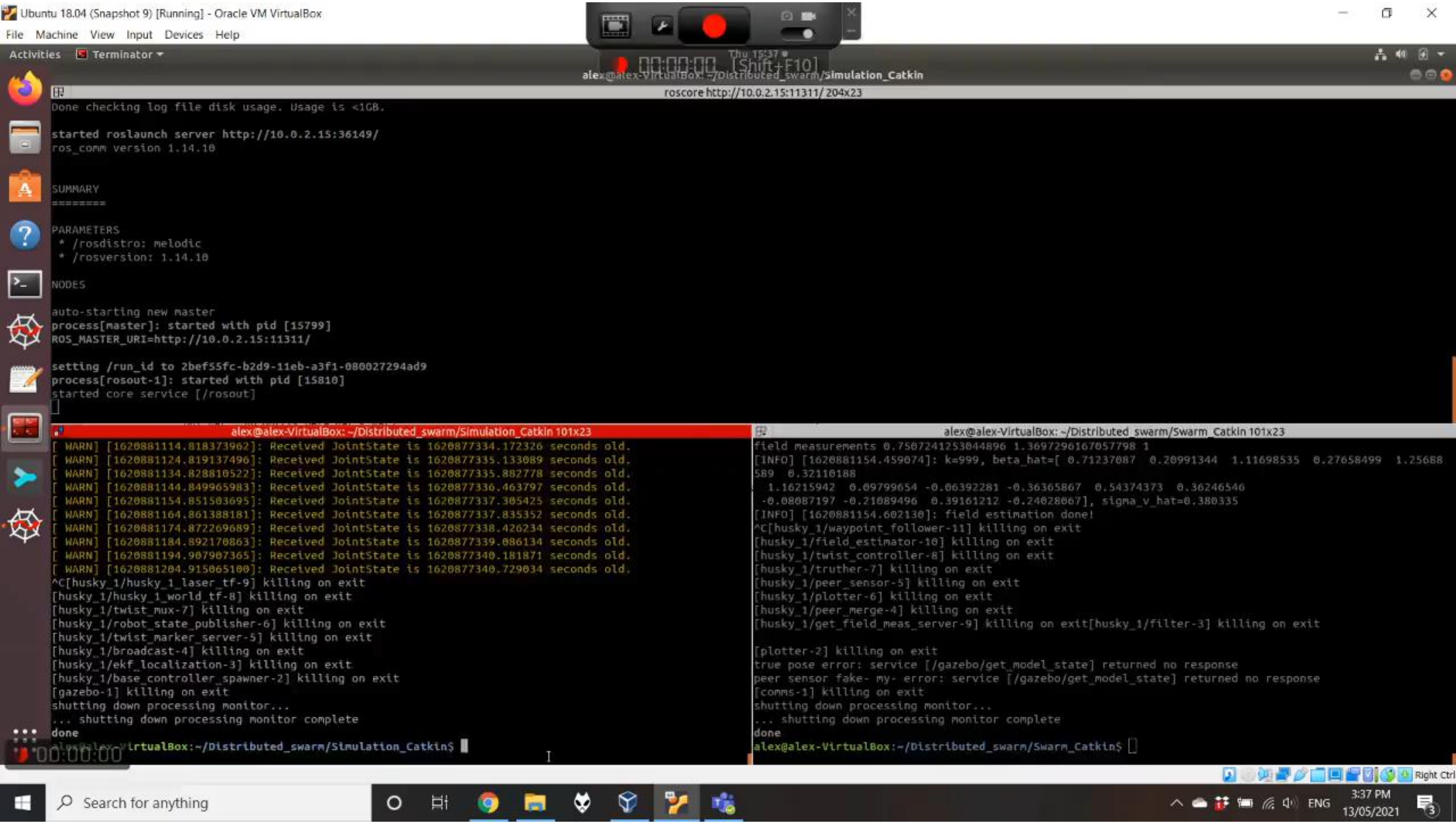


- True values of $\mathbf{c}_j$'s and σ_j's not known

- For field estimation, use $J = 16$ basis functions, $\mathbf{c}_j$'s located on a "grid", $\sigma_j = 25, \forall j$



Simulation Results – Example 2



Conclusion

- Field estimation can be done even with coarsely quantized / binary measurements
- Active sensing mechanism can be incorporated into estimation algorithm

- Extensions
  - Multiple agents: Reduce the amount of time needed to estimate field, both centralized and decentralized schemes
  - Time-varying fields: Adapt approach of [Nemeth, Fearnhead, Mihaylova, 2014]