Plug-and-Play Image Reconstruction Meets Stochastic Variance-Reduced Gradient Methods

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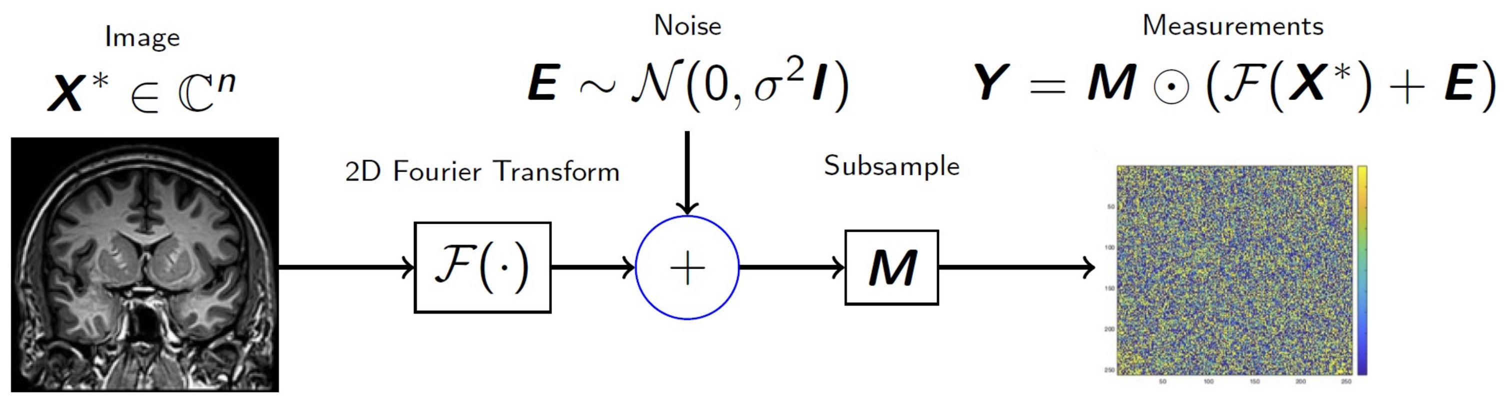
CS-MRI Problem Formulation

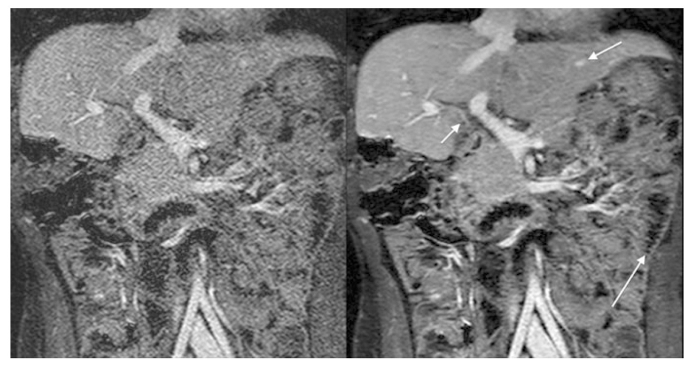
In **C**ompressed **S**ensing for **M**agnetic **R**esonance **I**maging, we observe subsampled, noisy, Fourier coefficients of an image:



The Importance of Image Priors

How can we get accurate reconstruction in an **undersampled regime?**

- Leverage prior knowledge on images!
- Compressive sensing example: exploit sparsity



Left: linear regression reconstruction.

Right: sparsity based reconstruction

Recipe for an Iterative Algorithm

Typical optimization set up to incorporate priors:

$$\min_{\boldsymbol{x}} \ \ell(\boldsymbol{x}) + \lambda r(\boldsymbol{x}).$$

Proximal **G**radient **D**escent: $\boldsymbol{x}_t = \text{prox}_{\lambda r}(\boldsymbol{x}_{t-1} - \eta \nabla \ell(\boldsymbol{x}_{t-1}))$, where,

$$\text{prox}_{\lambda g}(\boldsymbol{v}) = \arg\min_{\boldsymbol{z} \in \mathbb{R}^n} \left(\frac{\lambda}{2} \|\boldsymbol{z} - \boldsymbol{v}\|_2^2 + g(\boldsymbol{z}) \right)$$

I want an image that is close to my input...

...that also satisfies my prior knowledge

Resembles **image denoising**!

replace proximal operator with image denoiser → Plug-and-Play (PnP) [3]!

Stochastic Variance Reduced Gradients for PnP

Algorithm 1 PnP-SVRG

Input: $\boldsymbol{x}_0, \eta, T_1, T_2, B, \hat{\sigma}$.

Initialize: $\boldsymbol{x}_0$. `% e.g. back projection`

for $s = 1, 2, \cdots, T_1$ **do**

  $\tilde{\boldsymbol{x}} = \boldsymbol{x}_{s-1}$; `% set reference point`

  $\boldsymbol{w} = \nabla \ell(\tilde{\boldsymbol{x}})$; `% calculate batch gradient at` $\tilde{\boldsymbol{x}}$

  $\boldsymbol{z}_0 = \tilde{\boldsymbol{x}}$.

  for $t = 1, 2, \cdots, T_2$ **do**

    pick a set $\mathcal{I}_t \subset \{1, ..., m\}$ of cardinality B uniformly at random;

    $\boldsymbol{v}_t = \frac{1}{B} \sum_{i \in \mathcal{I}_t} (\nabla \ell_i(\boldsymbol{z}_{t-1}) - \nabla \ell_i(\tilde{\boldsymbol{x}})) + \boldsymbol{w}$; `% calculate variance-reduced gradient [1]`

    $\boldsymbol{z}_t = \text{denoise}_{\hat{\sigma}}(\boldsymbol{z}_{t-1} - \eta \boldsymbol{v}_t)$. `% denoise iterate`

  end for

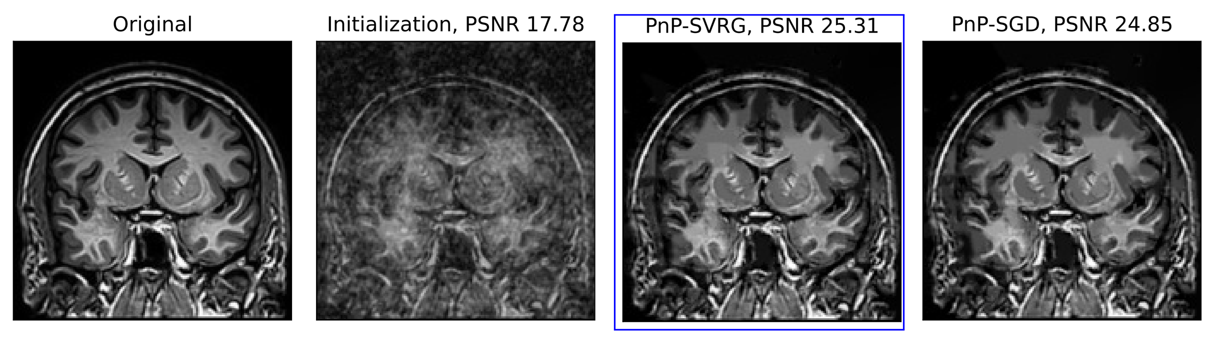
  $\boldsymbol{x}_s = \boldsymbol{z}_{T_2}$. `% choose new reference point`

end for

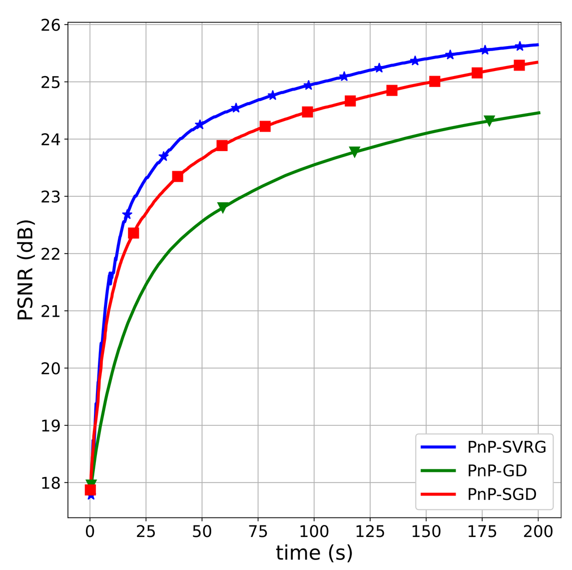
Output: $\hat{\boldsymbol{x}} = \boldsymbol{x}_{T_1}$.

CS-MRI Example Image Reconstruction

Experimental Setup: noise level of $\sigma = 5$, observe 50% of the Fourier coefficients, limit run-time to 200 seconds



Comparison of PSNR Over Time



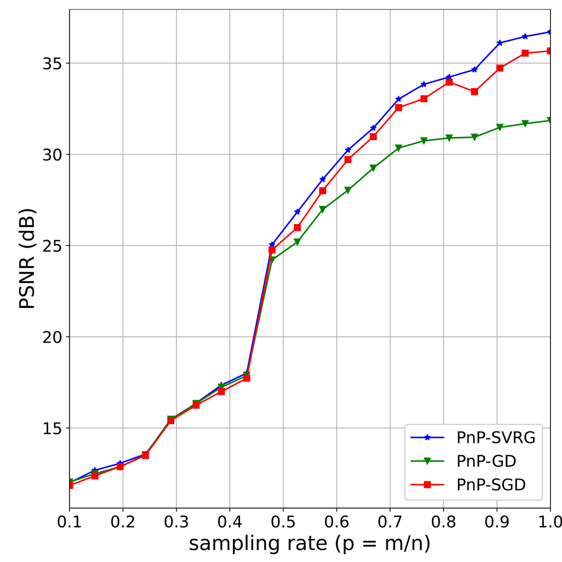
Experimental Set Up:

- noise level of $\sigma = 5$
- sampling rate = 2
- limit run-time to 200 seconds

Stochastic methods are quicker in the given time frame

- given more time, all approach a similar output

Impact of Sampling Rate on Accuracy

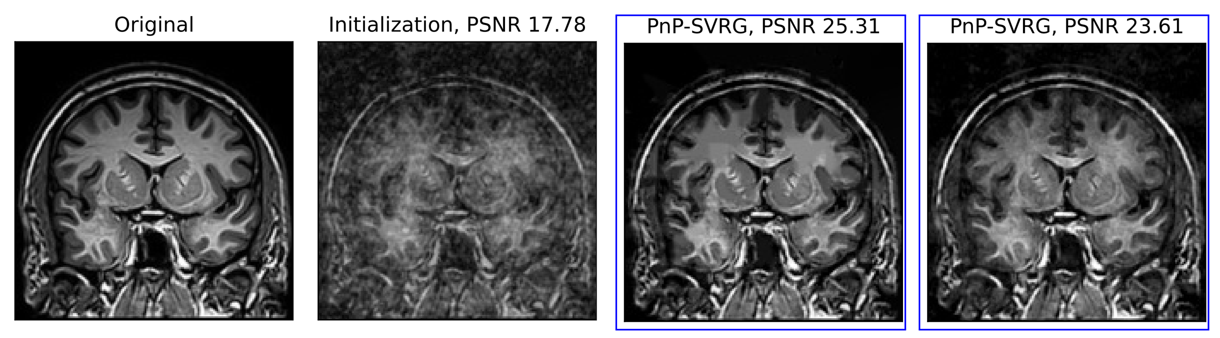


Experimental Set Up:

- noise level of $\sigma = 5$
- limit run-time to 200 seconds
- vary sampling rate

Stochastic methods obtain higher image reconstruction accuracy at varied sampling rates.

Comparing PnP-SVRG with Different Denoisers



	Non-Local Means	**RealSN-DnCNN** [2]
Run-time:	200s	**20s**

References

[1] Rie Johnson and Tong Zhang. Accelerating stochastic gradient descent using predictive variance reduction. *Advances in Neural Information Processing Systems*, 1(3):1–9, 2013.

[2] Ernest Ryu, Jialin Liu, Sicheng Wang, Xiaohan Chen, Zhangyang Wang, and Wotao Yin. Plug-and-play methods provably converge with properly trained denoisers. In *International Conference on Machine Learning*, pages 5546–5557. PMLR, 2019.

[3] Singanallur V Venkatakrishnan, Charles A Bouman, and Brendt Wohlberg. Plug-and-play priors for model based reconstruction. In *2013 IEEE Global Conference on Signal and Information Processing*, pages 945–948. IEEE, 2013.

Acknowledgements

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