

Introduction

Silhouettes or 2D shapes can be extracted as connected components of upper or lower sets of any image. Silhouettes are essential for human communication, being used as fonts, logos, symbols. They require a structured vectorization for scale independent transmission. To this aim, the position of corners along its outlines are an essential feature to detect and communicate.

Vector graphics consist of primitive components, such as line segments, circular arcs, and Bezier curves. Each involved primitive element is specified by a small number of 2D vectors, called control points, and the encoded shape can be scaled independently from the resolution. The conversion from a pixel image to a vector graphic is called vectorization.

Our Goal is to propose a mathematically sounded vectorization method that produces geometrically meaningful control points.

Affine Shortening Flow

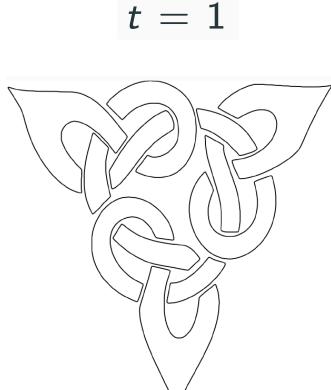
The affine shortening evolution of a curve x follows a partial differential equation (PDE):

$$\frac{\partial x}{\partial t}(s,t) = k(s,t)^{1/3} \vec{N}(s,t) , \quad x(s,0) = x_0(s)$$

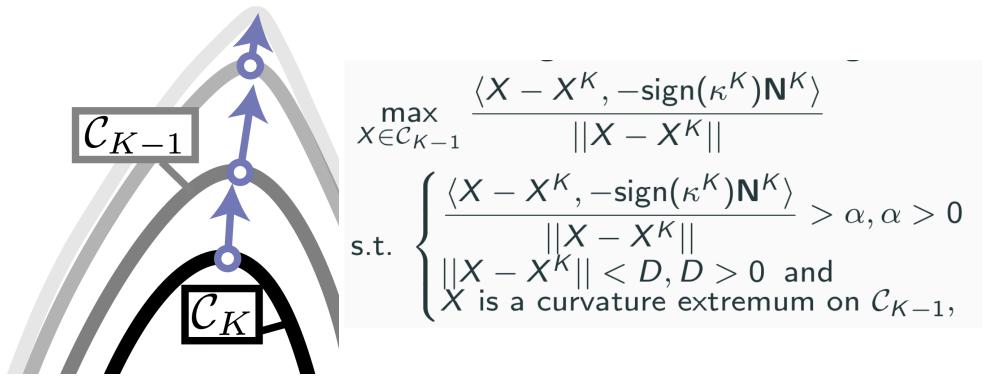
Here, k represents the curvature, and N represents the normal direction. Sapiro G., Tannenbaum A., Affine invariant scale-space. IJCV. 1993

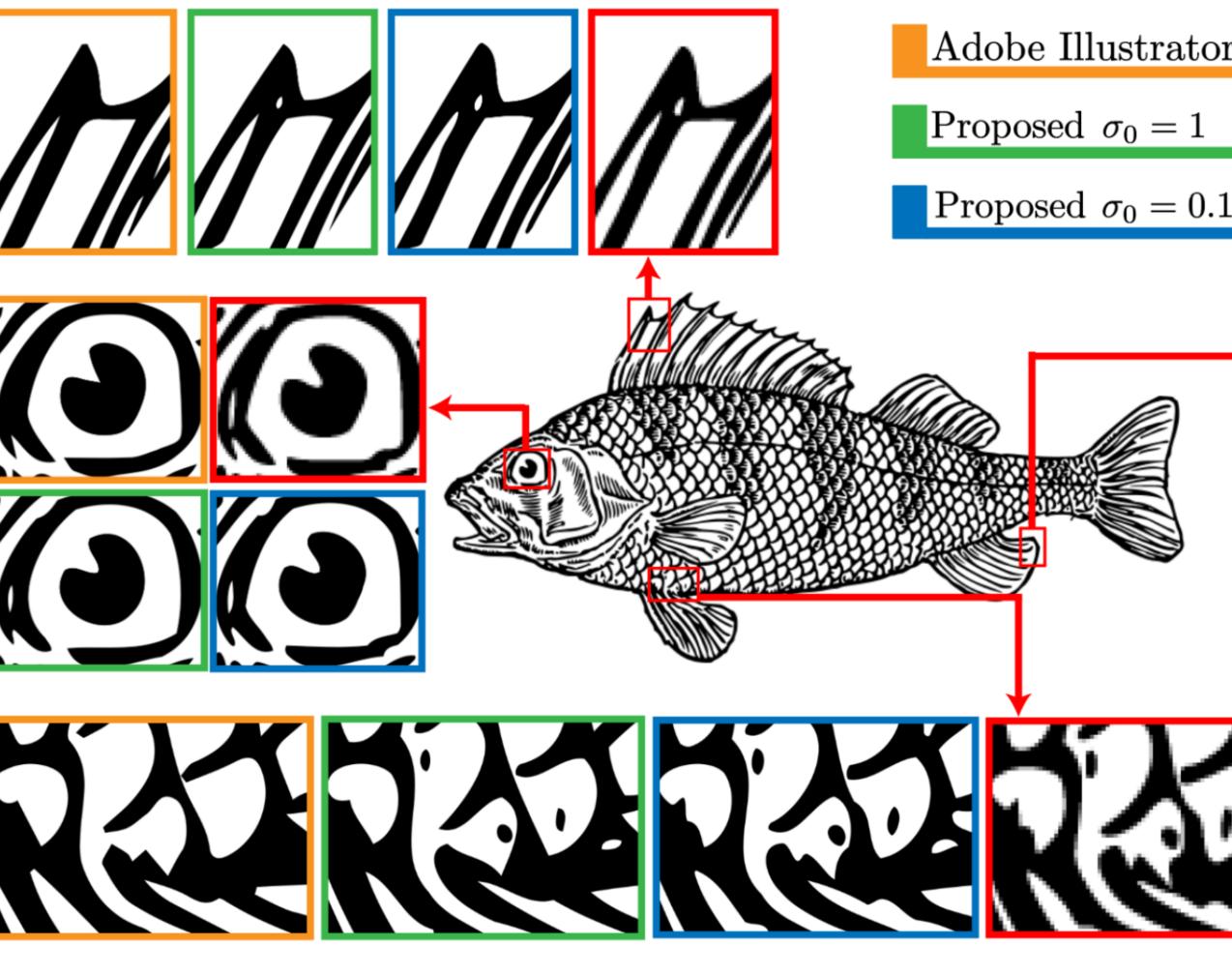
Alvarez L., Guichard F., Lions P.L., Morel J.M., Axioms and fundamental equations of image processing. Archive for rational mechanics and analysis. 1993

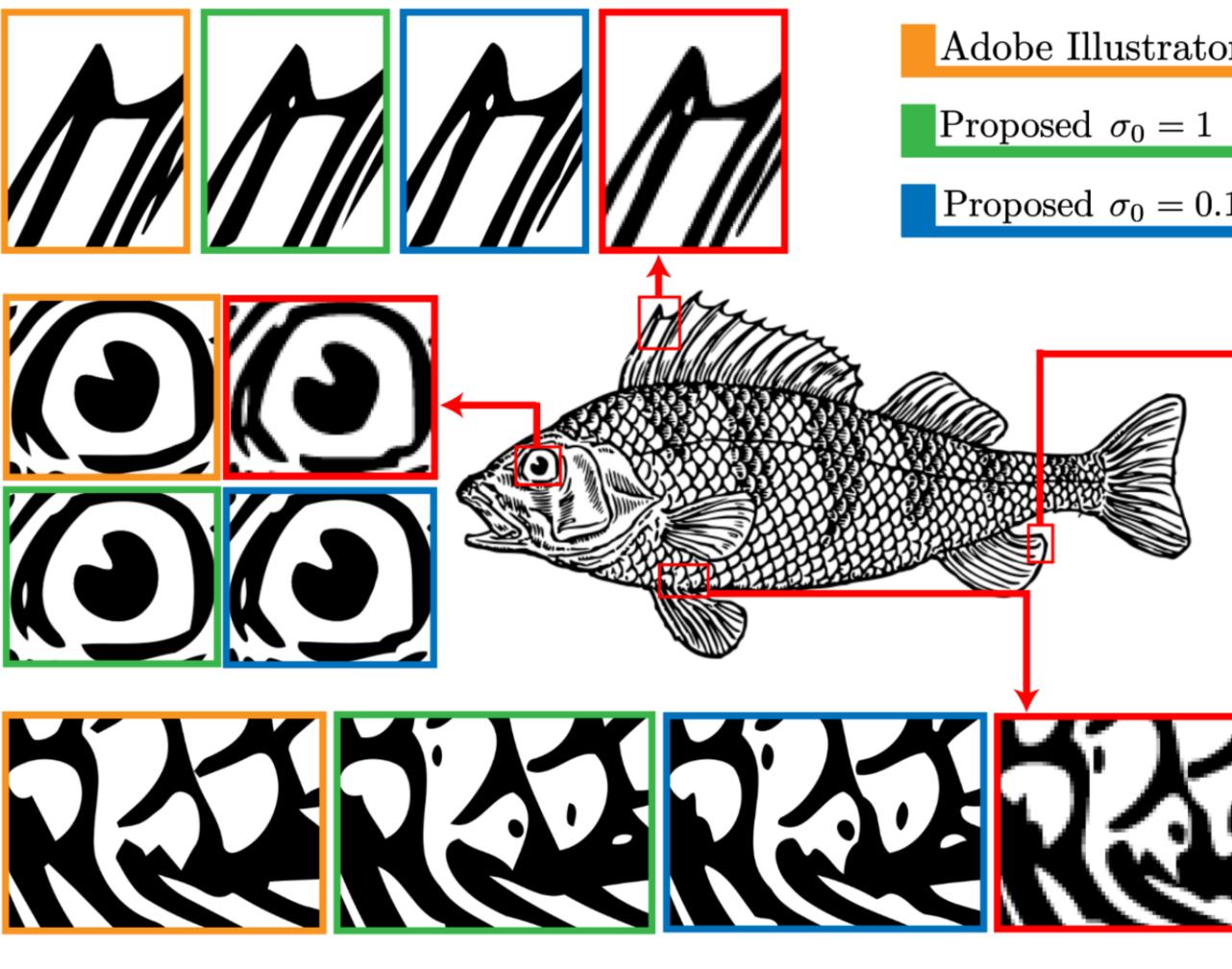
t = 8



Backtracing along the affine shortening flow



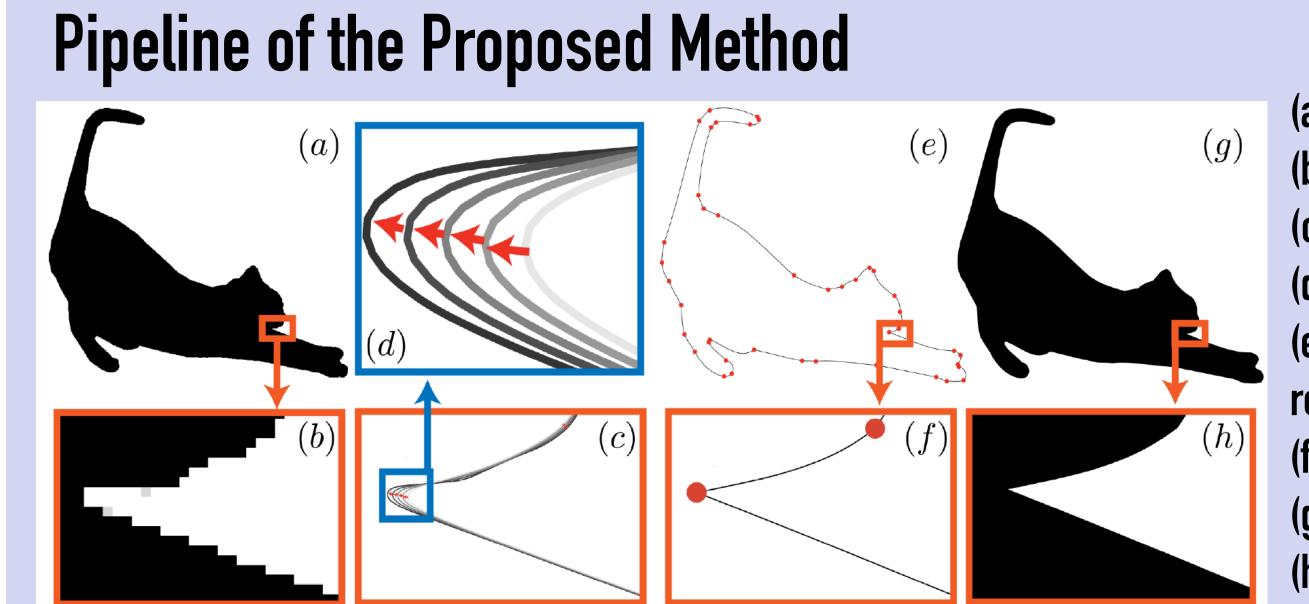








t = 10



Accurate Silhouette Vectorization by Affine Scale-space

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Qualitative comparison with Adobe Illustrator 2020

Control points as feature detectors (d)

We compare our control points (red dots) with some well-es tablished feature point extractors (green crosses).

(a) Harris Detector Harris C., Stephens M. A combined corner and edge detector. In AVC 1988.

(b) FAST Rosten E., Drummond T. Fusing points and lines for high performance tracking. In ICCV 2005.

(c) SURF Bay H., Tuytelaars T,. Van Gool L. Surf: Speeded up robust features. In ECCV 2006.

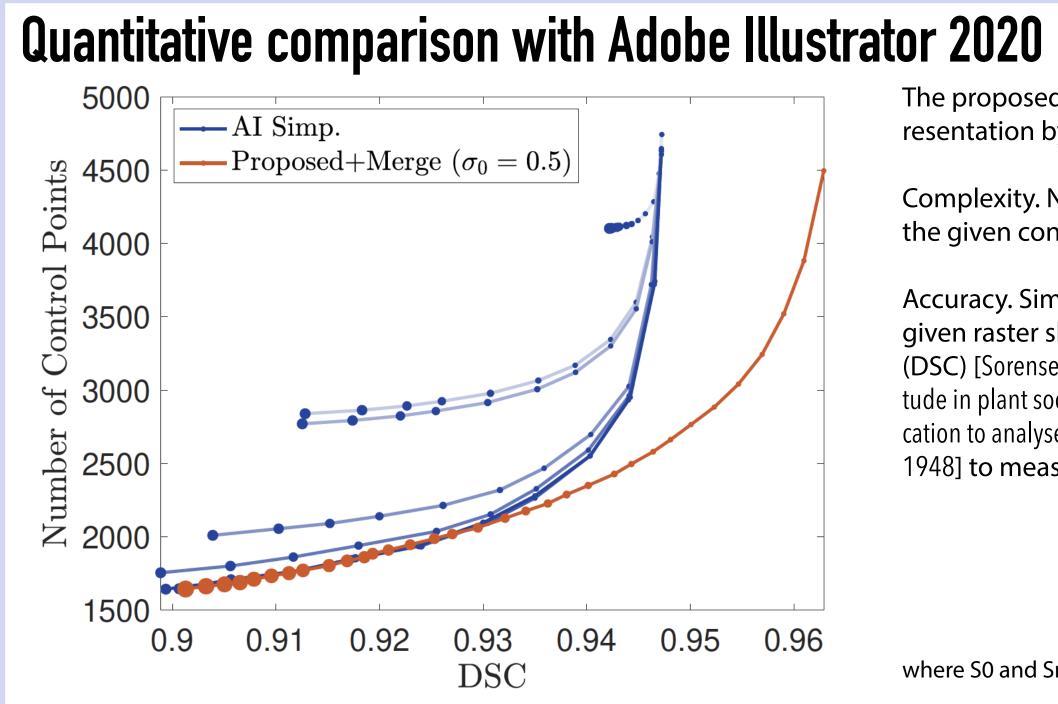
(d) SIFT Lowe D.G. Object recognition from local scale-invariant features. In ICCV 1999.

Results

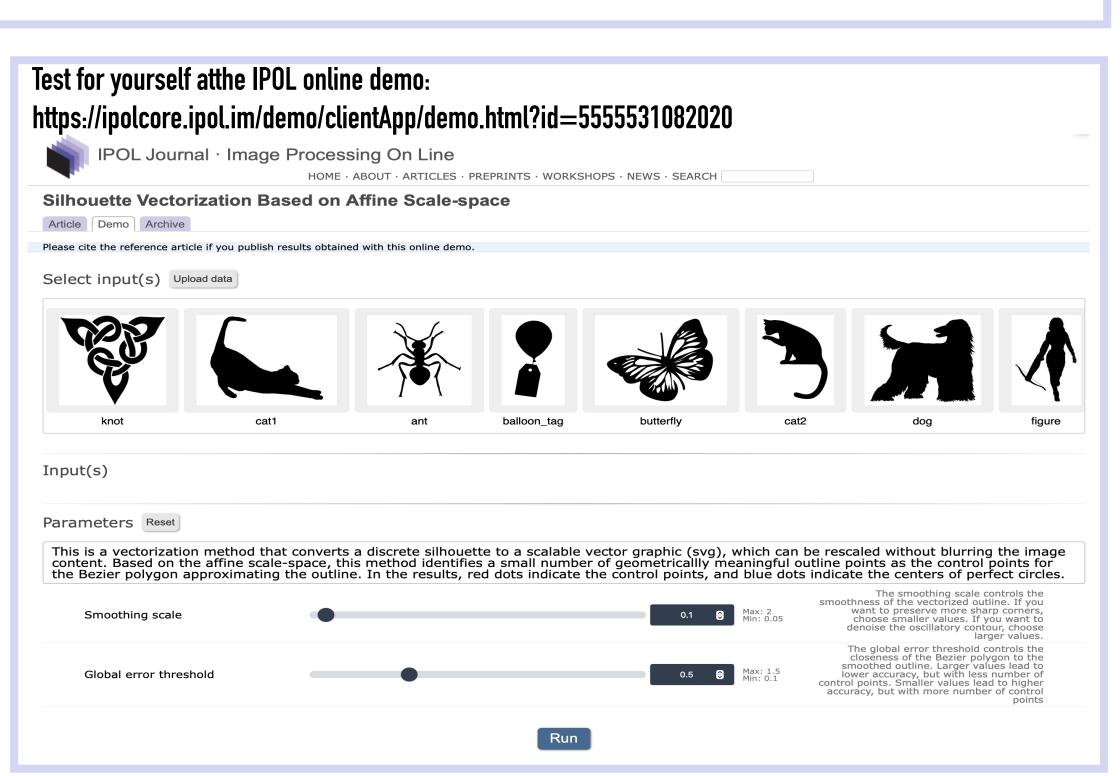
r 2020)	#C	=487	74
$ au_e =$: 1	#C	= 281	0
$1 \tau_e =$	0.5	#C	=416	5
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	Number of Control Points $(\#C)$					
Test Image	Original	VM	IS	AI	Proposed	
	405	248/256/245	330	$280~(193^{\dagger})$	168	
	611	359/343/325	383	$340~(293^{\dagger})$	222	
J. J.	682	296/294/263	272	$211~(128^{\dagger})$	120	
	1434	915/828/715	932	$698~(462^{\dagger})$	379	
	4434	2789/2582/2370	3292	$2120~(1431^{\dagger})$	1407	
	6664	5470/5218/4955	6493	$4870~(3441^{\dagger})$	2810	

Comparison with image vectorization software in terms of the number of control points. We compared with Vector Magic (VM), Inkspace (IS), and Adobe Illustrator 2020 (AI). For VM, we report the number of control points using three settings: High/Medium/Low. For AI, the values with dagger indicate the numbers of control points produced by the automatic simplification. MRR stands for mean relative reduction of the number of control points for the results above. These figures are given for realizations with approximately equal accuracy.



- (a) The input of a raster silhouette.
- (b) Zoom-in of (a).
- (c) Extracted bilinear outline of (a).
- (d) Inverse tracing of outline's corner in affine scale space.
- (e) The vectorized outline of (a) with control points marked as red dots.
- (f) Zoom-in of (e).
- (g) Vectorized result of (a) by the proposed method.
- (h) Zoom-in of (g).



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The proposed method shows superior efficiency in shape representation by the following criteria.

Complexity. Number of Bezier curves used for approximating the given contour.

Accuracy. Similarity between the vectorized shape and the given raster shape. We employ the dice similarity coefficient (DSC) [Sorensen T.A., A method of establishing groups of equal amplitude in plant sociology based on similarity of species content and its application to analyses of the vegetation on Danish commons. Biol. Skar. 1948] to measure

 $\mathsf{DSC} = \frac{2|S_0 \cap S_r|}{|S_0| + |S_r|}$

where S0 and Sr are two binary images.