Optimal zero forcing precoder and decoder design for multi-user MIMO FBMC under strong channel selectivity

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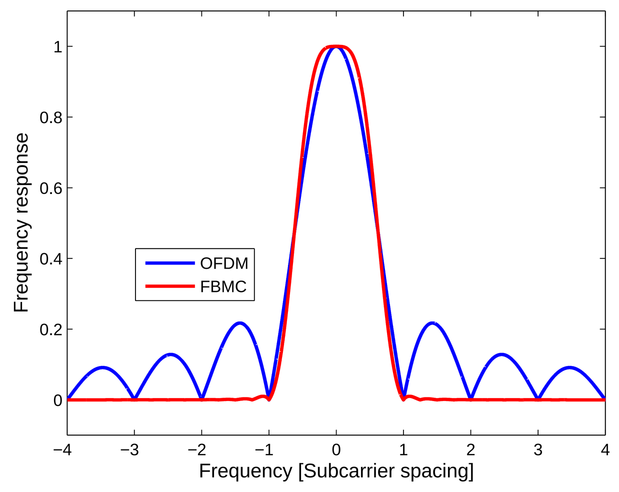
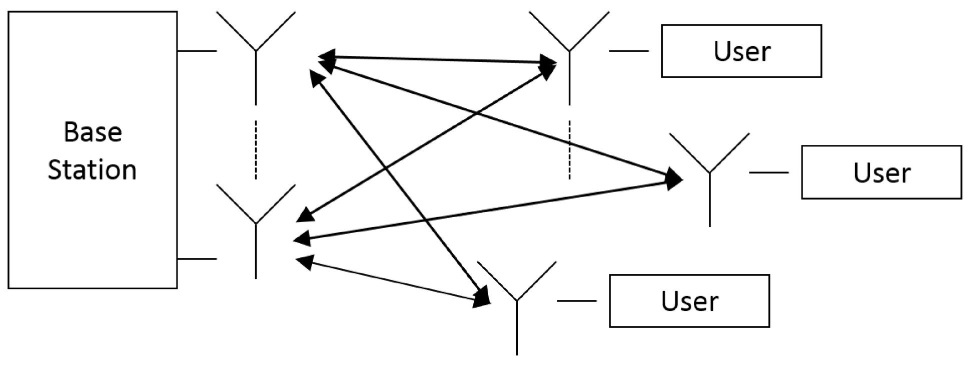
Context

FBMC-OQAM has been shown to be an interesting alternative to CP-OFDM for the physical layer of 5G, mainly thanks to its spectral efficiency and the good time-frequency localization of its prototype filter.



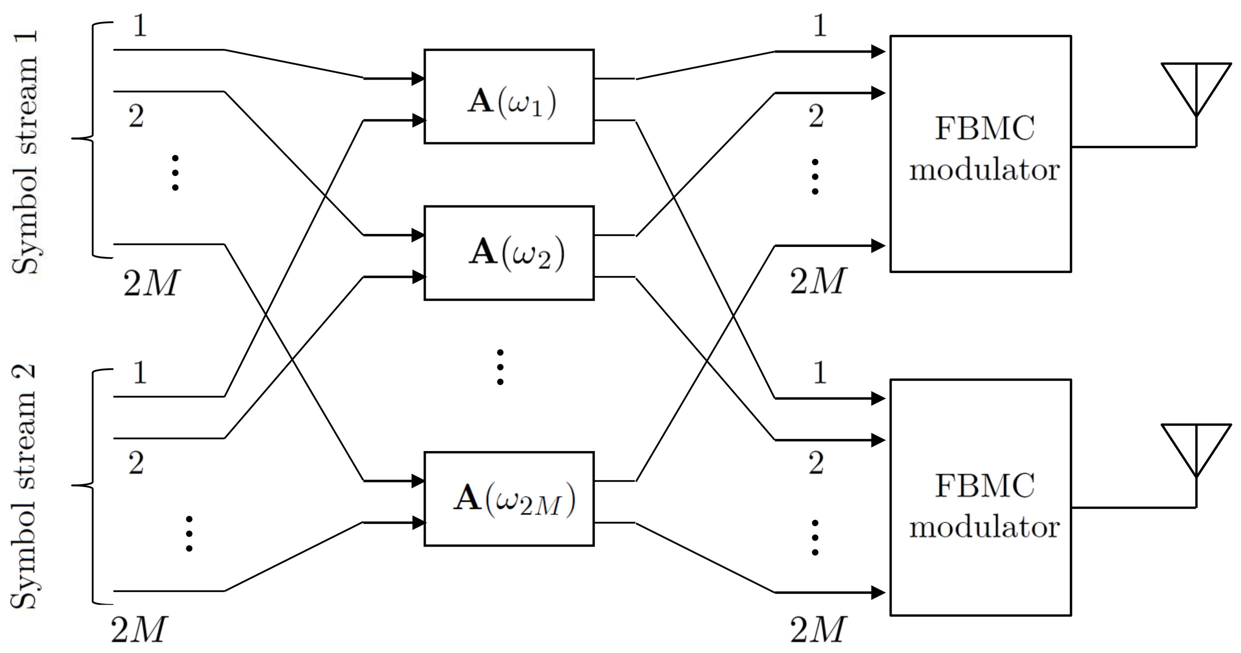


Problem: FBMC-OQAM application to MIMO systems is not as straightforward as in CP-OFDM, especially in the case of strong channel selectivity.

Proposed solution

State of the art: most of the approaches to mitigate high channel frequency selectivity are based on the design of multi-tap precoding and decoding matrices, increasing the system complexity.
Proposed approach: very low complexity design based on single tap per-subcarrier precoding and decoding matrices.
Result: as long as the number of base station (BS) antennas is larger than the number of users, the optimized precoder/decoder can compensate for the channel frequency selectivity and restore the system orthogonality.

System model



- Multi-user MIMO scenario with one BS equipped with N antennas and N_U users, each one equipped with a single antenna. Users cannot collaborate.
- If the number of subcarriers $2M$ is large w.r.t. the channel delay spread, one may assume that the channel is approximately frequency flat inside each sub-band.
- In downlink (DL), the BS applies a single tap per-subcarrier precoding matrix $\boldsymbol{A}(\omega_k) \in \mathbb{C}^{N\times N_U}$ with $\omega_k = \frac{2\pi(k-1)}{2M}$ to pre-equalize the channel. In uplink (UL), the BS applies a single tap per-subcarrier decoding matrix $\boldsymbol{B}(\omega_k) \in \mathbb{C}^{N_U\times N}$ to equalize the channel.
- $\boldsymbol{H}(\omega)$ denotes the channel frequency response matrix. More specifically, $\boldsymbol{H}_{\mathrm{DL}}(\omega) \in \mathbb{C}^{N_U\times N}$ in DL and $\boldsymbol{H}_{\mathrm{UL}}(\omega) \in \mathbb{C}^{N\times N_U}$ in UL.
- Zero forcing (ZF) designs, i.e. $\boldsymbol{B}(\omega)\boldsymbol{H}(\omega)\boldsymbol{A}(\omega) = \boldsymbol{I}_{N_U}$.

MSE formulation

When the variation of the channel becomes non-negligible, distortion appears and the orthogonality is progressively destroyed.
Proposition: When the number of subcarriers grows large and for identical transmit and receive pulses, the total MSE at subcarrier k can be written as

$$\begin{aligned}\mathrm{MSE}(k) &= \alpha\mathrm{tr}\left[\left(\boldsymbol{B}\boldsymbol{H}'\boldsymbol{A}\right)\left(\boldsymbol{B}\boldsymbol{H}'\boldsymbol{A}\right)^H\right] \\ &\quad - (2\alpha+2\beta)\mathrm{tr}\left[\Im(\boldsymbol{B}\boldsymbol{H}\boldsymbol{A}')\Im(\boldsymbol{B}'\boldsymbol{H}\boldsymbol{A})^T\right] \\ &\quad + N_0\mathrm{tr}\left[\boldsymbol{B}\boldsymbol{B}^H\right] + O\left(2M^{-2}\right)\end{aligned}$$

where α and β are pulse-related quantities and all frequency-depending matrices are evaluated at frequency $\omega = \omega_k$.

Optimal linear decoder and precoder

The optimal decoding and precoding matrices are found by optimizing the MSE formula under the ZF and power normalization constraints in the UL and DL case respectively.
Optimal decoder:

$$\boldsymbol{B} = \frac{1}{\xi}\boldsymbol{H}^\dagger - \frac{1}{\xi}\boldsymbol{H}^\dagger\boldsymbol{H}'\left(\boldsymbol{H}'^H\boldsymbol{P}^\dagger\boldsymbol{H}' + \frac{N_0N_U}{P_T\alpha}\boldsymbol{I}_{N_U}\right)^{-1}\boldsymbol{H}'^H\boldsymbol{P}^\dagger \qquad (1)$$

where $\boldsymbol{H}^\dagger = (\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H$, $\boldsymbol{P}^\dagger = \boldsymbol{I}_N - \boldsymbol{H}(\boldsymbol{H}^H\boldsymbol{H})^{-1}\boldsymbol{H}^H$.
Optimal precoder:

$$\boldsymbol{A} = \frac{1}{\xi}\boldsymbol{H}^+ - \frac{1}{\xi}\boldsymbol{P}^+\boldsymbol{H}'^H\left[\boldsymbol{H}'\boldsymbol{P}^+\boldsymbol{H}'^H + \frac{N_0N_U}{P_T\alpha}\boldsymbol{I}_{N_U}\right]^{-1}\boldsymbol{H}'\boldsymbol{H}^+$$

where $\boldsymbol{H}^+ = \boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{H}^H)^{-1}$, $\boldsymbol{P}^+ = \boldsymbol{I}_N - \boldsymbol{H}^H(\boldsymbol{H}\boldsymbol{H}^H)^{-1}\boldsymbol{H}$.

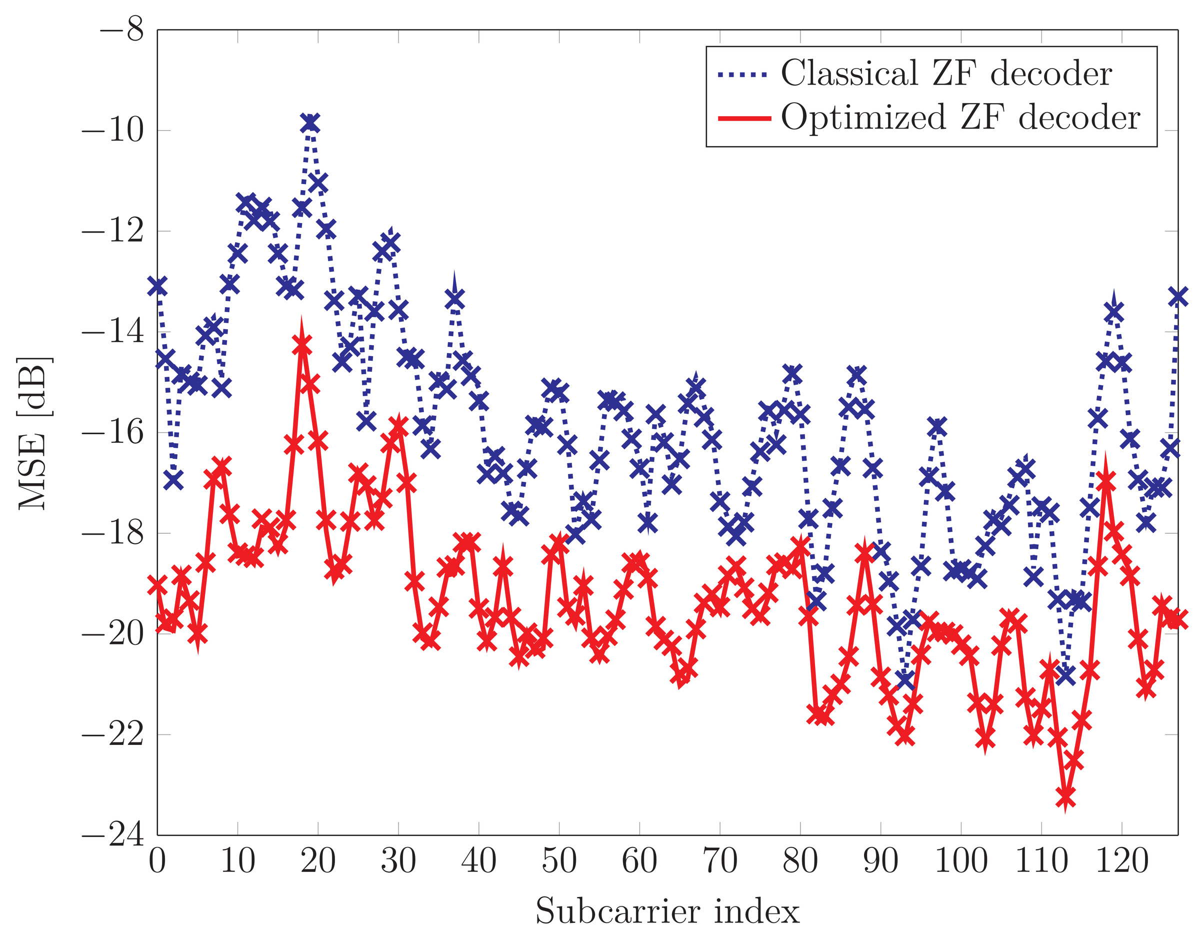
Asymptotic behavior at high SNR

The limit depends on the number of BS antennas N versus the number of users N_U.
$N < 2N_U$: in this case the noise term of the MSE will tend to zero but the distortion will only be partially compensated.
$N \geq 2N_U$: for twice as many antennas as the number of served users, the first order term of the distortion can be fully removed and the MSE will tend to zero at high SNR.

Simulation results

MSE of the classical and optimal decoder

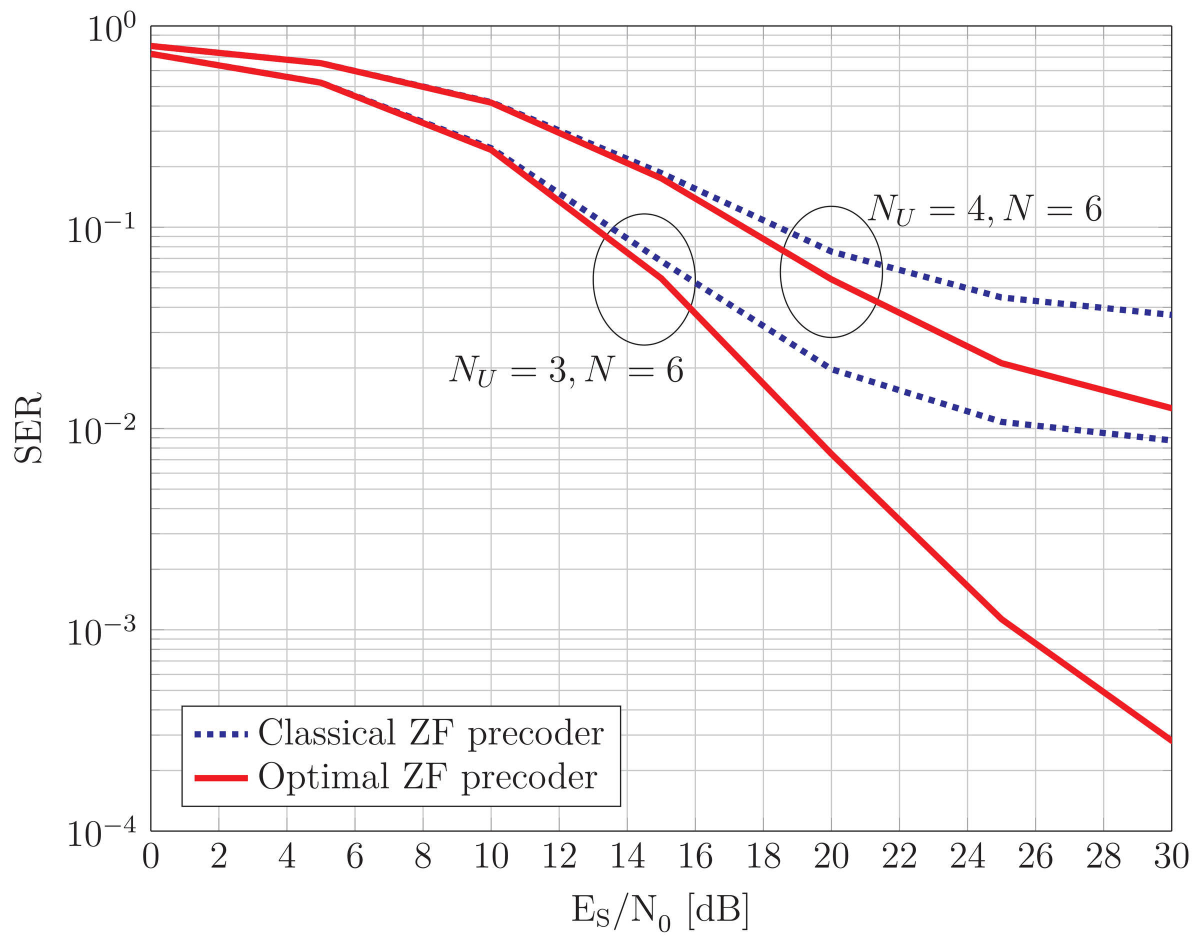
Uplink, 4x2 Veh. B channel, SNR=25dB



The optimal ZF decoder clearly outperforms the classical ZF decoder.

SER of the classical and optimal precoder

Downlink, Veh. B channel, 16-QAM



For a twice bigger number of BS antennas, the first order approximation of the distortion is cancelled and the SER does not saturate.

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