Hyperspectral Neutron CT with Material Decomposition

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ERNI (Energy Resolved Neutron Imaging)

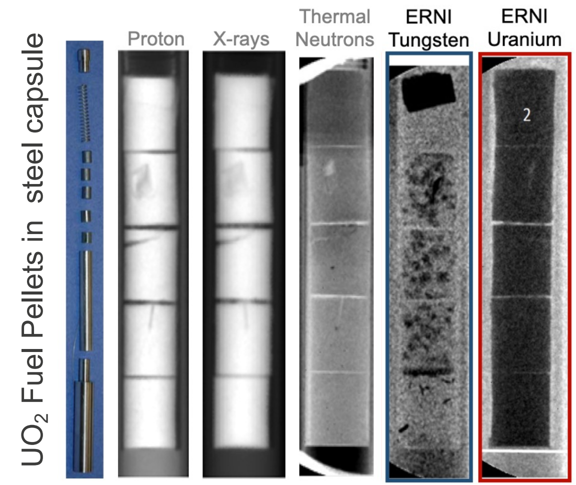
- Neutron radiography fundamentally different contrast mechanism than x-rays or protons:

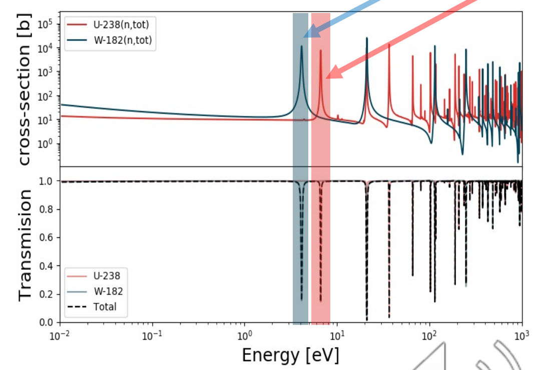
  nuclear interaction vs. electron shell

- Much higher penetration for most materials than typical x-ray sources but sensitivity to hydrogen

- Short pulsed neutron sources allow for energy resolved neutron imaging through the time-of-flight (TOF) technique

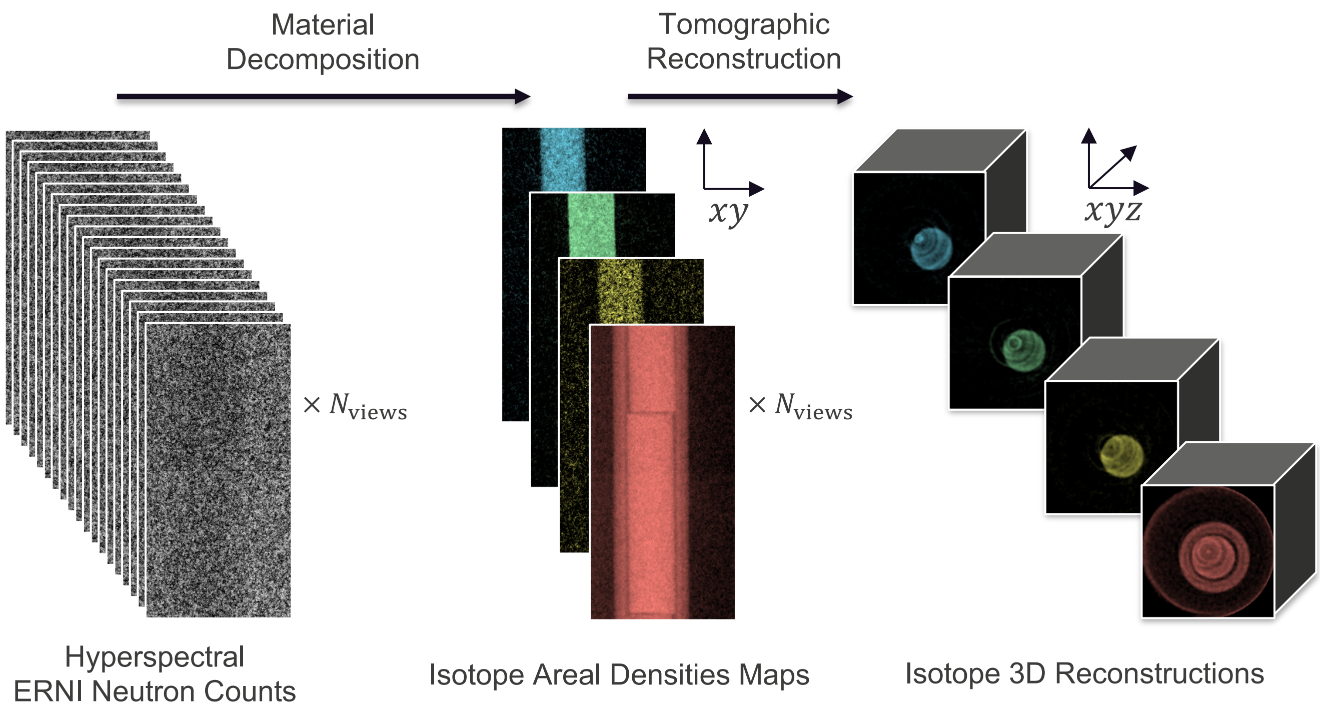
- Cross section resonances in epithermal energy act as ***unique fingerprints***





Each pixel has energy inform… through cross section spectra…

Overview



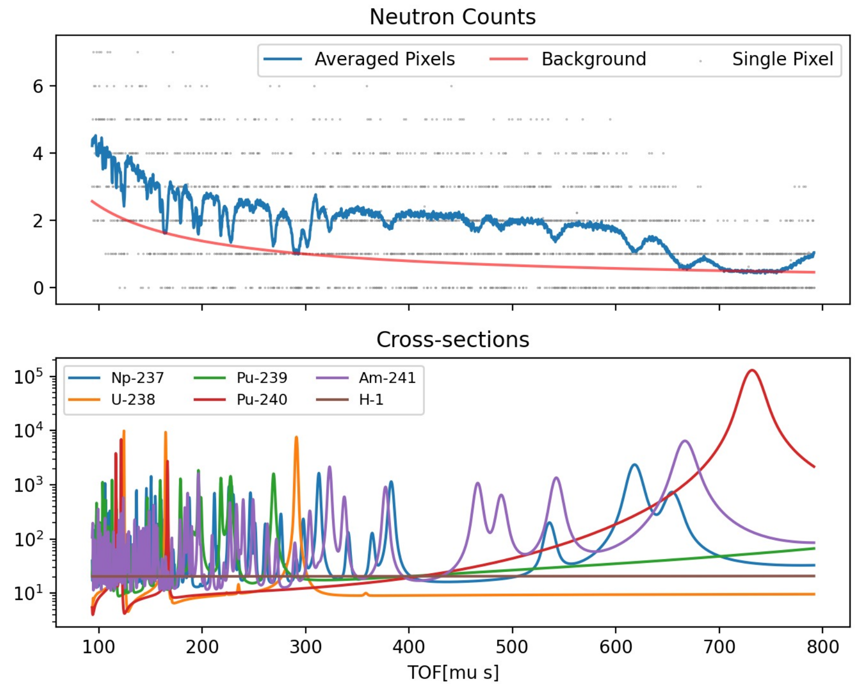
Material
Decomposition
Tomographic
Reconstruction
$\times N_{\text{views}}$
xy
$\times N_{\text{views}}$
xyz
Hyperspectral
ERNI Neutron Counts
Isotope Areal Densities Maps
Isotope 3D Reconstructions

Main Challenges

Extreme Noise
- Very low counts even with months of scan time
- Background counts are significant, random and unknown

Large data size
- Hyperspectral feature adds degree of freedom
  - Easily 100s GB to 100s of TB
- *Several **months** of processing time* with nuclear analysis tool SAMMY (state-of-the-art)

Measurement system partly unknown
- Background counts unknown
- Unknown resolution blurring breaks Beer's Law
- Inaccurate isotope cross section spectra



Measurement Model

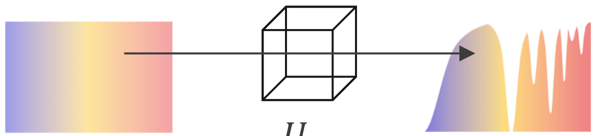
- Neutron count measurements consists of
  - Blank scan α^{blk}
  - Object scan α^{obj},

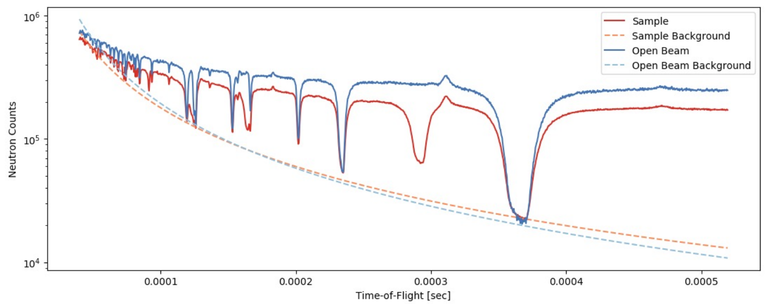
  resolved in energy bins, k, and projection index, i.
- Total counts, α, have significant background, β.
- Each voxel, j, has an attenuation spectrum, $U_{j,*}$

$$\frac{\alpha^{\text{obj}} - \beta^{\text{obj}}}{\alpha^{\text{blk}} - \beta^{\text{blk}}} = T = \exp(-Y) = \exp(-AU) + \text{noise}$$

Transmission measurement　　Attenuation Densities

- Finding hyperspectral densities, U, directly is impractical:
  - Neutron counts extremely noisy
    → Independent CT's impractical
  - Data set extremely large due to 1000s of energy bins
    → Joint CT's impractical
- We are interested in the material composition





T: Transmission Measurement

U: Attenuation Density

A: Tomographic System Matrix

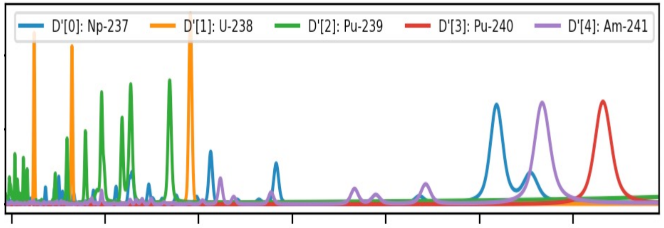
Y: Attenuation Measurement = $-\log(T)$

Reduction of Dimensionality

Isotopes have known attenuation spectra, cross sections, σ
Beer-Lambert law relates transmission with areal densities, ρ_i

$$-\log(T) = Y = \sum_{i=1}^{N_\mathrm{m}} \rho_i \, \sigma_i + \text{noise} = ZD + \text{noise}$$

√Cross Sections = $\sqrt{D}$



Forward Model

$$U = XD \rightarrow \quad Y = AU + \text{noise}$$

$$Y = AXD + \text{noise}$$

Neutron Count Measurements
$\alpha^{\mathrm{obj}}, \alpha^{\mathrm{blk}}$

Solve for "AX" $\Rightarrow \hat{Z}$

Solve for $X \Rightarrow \hat{X}$

0. Background Estimation

$$Y = -\log\left(\frac{\alpha^{\mathrm{obj}} - \beta^{\mathrm{obj}}}{\alpha^{\mathrm{blk}} - \beta^{\mathrm{blk}}}\right)$$

Y: Attenuation Measurements

→ Y →

1. Material Decomposition
Reduce Dimensionality

$$Y = ZD + \text{noise}$$

Z: Areal Density Maps

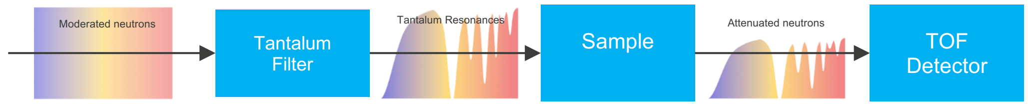
→ $\hat{Z}$ →

2. Tomographic Reconstruction

$$\hat{Z} = AX + \text{noise}'$$

X: Material Decomposed Volume

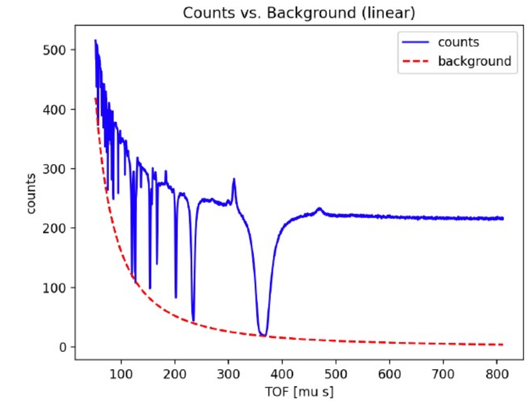
Background Estimation using Opaque Resonances



Helps estimate background through opaque resonances

Background := counts that arrive in an energy bin, E, that are not neutrons of energy, E.

At opaque resonances of the sample, $\alpha = \beta$, so the counts equal the background



Our Approach:
Find background as maximum lower bound of the counts and smooth in log-log.

$$\tilde{t} = \log(t), \qquad \tilde{\alpha} = \log(\alpha), \quad \tilde{\beta} = \log(\beta)$$

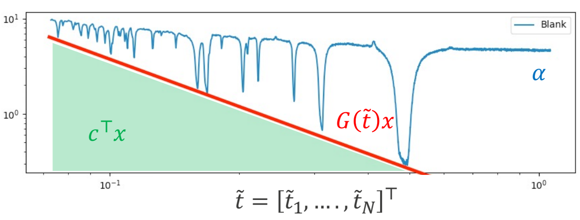
(TOF-vector)

Functional for background: $\sum_{i=0}^{k} \tilde{t}^{i} x_i = G(\tilde{t})x$ (polynomial)

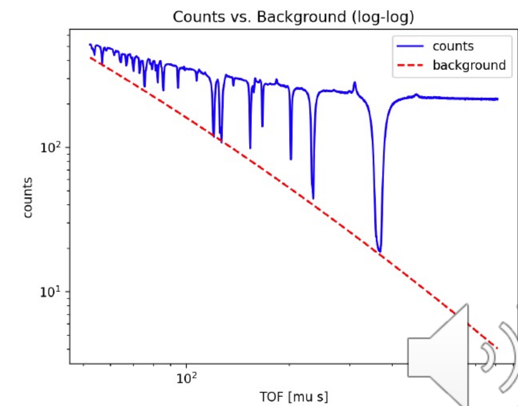
Maximize Area: $\int_{t_1}^{t_N} G(\tilde{t})x \, d\tilde{t} = \sum_{i=0}^{k} x_i \frac{1}{i+1}\left(\tilde{t}_N^{i+1} - \tilde{t}_1^{i+1}\right) = -c^{\top}x$

Constraint: $G(\tilde{t})x \leq \tilde{\alpha}$

Linear Programming formulation: $\hat{x} = \arg\min_{\substack{x \\ G(\tilde{t})x \leq \tilde{\alpha}}} \{c^{\top}x\}, \qquad \beta = \exp(G(\tilde{t})\hat{x})$



For individual pixels with high noise counts, background is determined as scaled version of reference background using transparent energy region as reference.



Material Decomposition: Solving $Y = ZD + \text{noise}$

Noise Modeling:
Attenuation, Y, random with unconventional distribution

$$Y = -\log\left(\frac{\alpha^{\text{obj}} - \beta^{\text{obj}}}{\alpha^{\text{blk}} - \beta^{\text{blk}}}\right),$$

α's and β's are Poisson

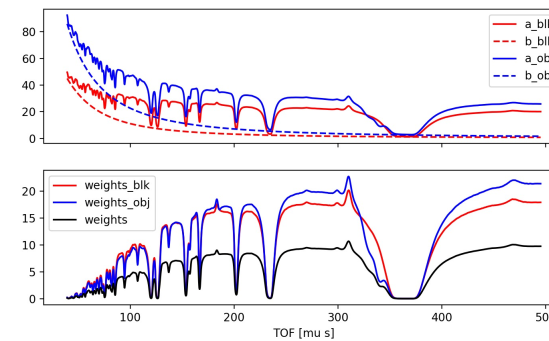
$$\mathbb{V}\text{ar}(Y|Z) \approx \frac{\alpha^{\text{obj}} + \beta^{\text{obj}}}{(\alpha^{\text{obj}} - \beta^{\text{obj}})^2} + \frac{\alpha^{\text{blk}} + \beta^{\text{blk}}}{(\alpha^{\text{blk}} - \beta^{\text{blk}})^2} = V^{-1}$$

Y: Hyperspectral attenuation measurement
Z: Areal density maps
D: Dictionary of cross sections

Gaussian approximation for negative log-likelihood:

$$-\log \mathbb{P}(Y|Z) \approx \frac{1}{2}\|D^\top Z^\top - Y\|_V^2$$

Counts and inverse variances (weights)



$\alpha^{blk}, \beta^{blk}$

$\alpha^{obj}, \beta^{obj}$

V

Reconstruction:
Compute decomposed sinogram, Z, as maximum a posteriori (MAP) estimate using Basis Pursuit Denoising (BPDN):

$$\hat{Z} = \arg\min_{Z \geq 0} \left\{\frac{1}{2}\|D^\top Z^\top - Y^\top\|_V^2 + \rho\|Z^\top\|_1\right\}.$$

The objective is separable by projections, so we solve it one view at a time!

Tomographic Reconstruction: Solving $\hat{Z} = AX + \text{noise}'$

$$\hat{Z} = AX + \text{noise}'$$

$\hat{Z}^{(m)}$: sinogram of material, m
$X^{(m)}$: CT volume of material, m

Hyperspectal sinogram:

$\mathbb{C}\text{ov}\left(Y_{i,*}|Z\right)^{-1} = V_i$ $\quad$ $(N_{\text{energy}} \times N_{\text{energy}}$ diagonal)

Decomposed sinogram:

$\mathbb{C}\text{ov}\left(\hat{Z}_{i,*}|X\right)^{-1} \approx (D^+)^\top V_i D^+ = W_i$ $\quad$ $(N_{\text{material}} \times N_{\text{material}}$ dense)

$$-\log \mathbb{P}\left(\hat{Z}|X\right) = \frac{1}{2}\sum_i \left\|\hat{Z}_{i,*} - A_{i,*}X\right\|_{W_i}^2$$

Impractical since W_i's are dense precisions matrices

Since the cross sections, D, are approximately orthogonal, use diagonal approximation: $\left(W^{(m)}\right)_{i,i} = (W_i)_{m,m}$

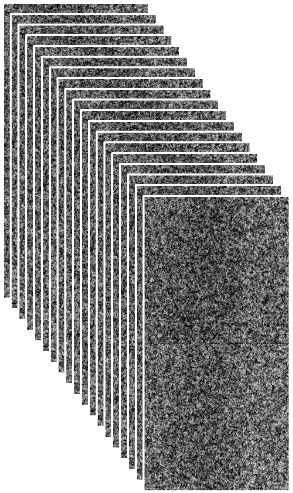
$$-\log \mathbb{P}(Z|X) \approx \frac{1}{2}\sum_{m=1}^{N_{\text{material}}} \left\|\hat{Z}^{(m)} - AX^{(m)}\right\|_{W^{(m)}}^2$$

Independent, separable reconstructions using SV-MBIR [1] with q-GGMRF prior model

[1] Xiao Wang, et. al. "High performance model based image reconstruction," SIGPLAN Not., vol. 51, no. 8, Feb. 2016.

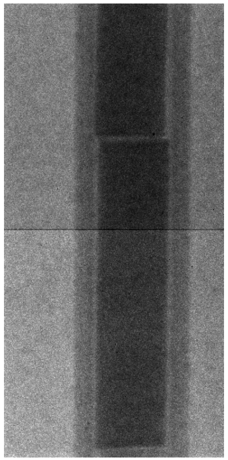
Demonstrating CT on U-Pu-Zr Dataset

- ERNI CT: U-Pu-Zr Fuel Pellet in Steel Container
- Low correlation of cross section spectra allows for independent CT's



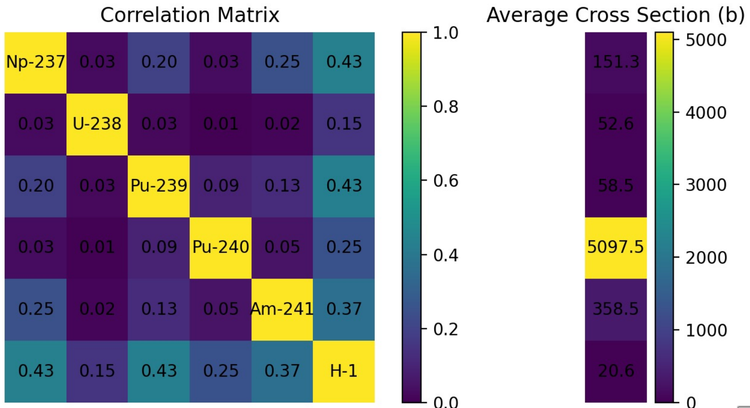
ERNI Data
100 views, 2290 TOF bins
each with bin width of 320 ns
1eV to 60 eV

Acquired at LANCE



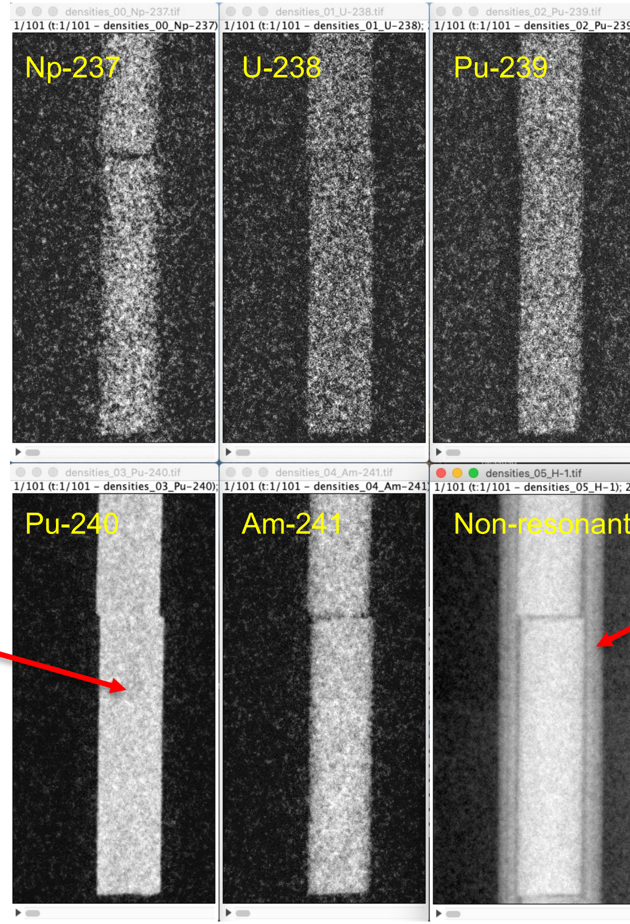
Single View
Averaged over Energy

Cross-section Dictionary



Material Decomposition Results

Low noise areal density maps, $\hat{Z}$, (from high noise ERNI measurement)



Saturation effects from linear model insufficiencies

Steel container and zirconium core component (non-resonant)

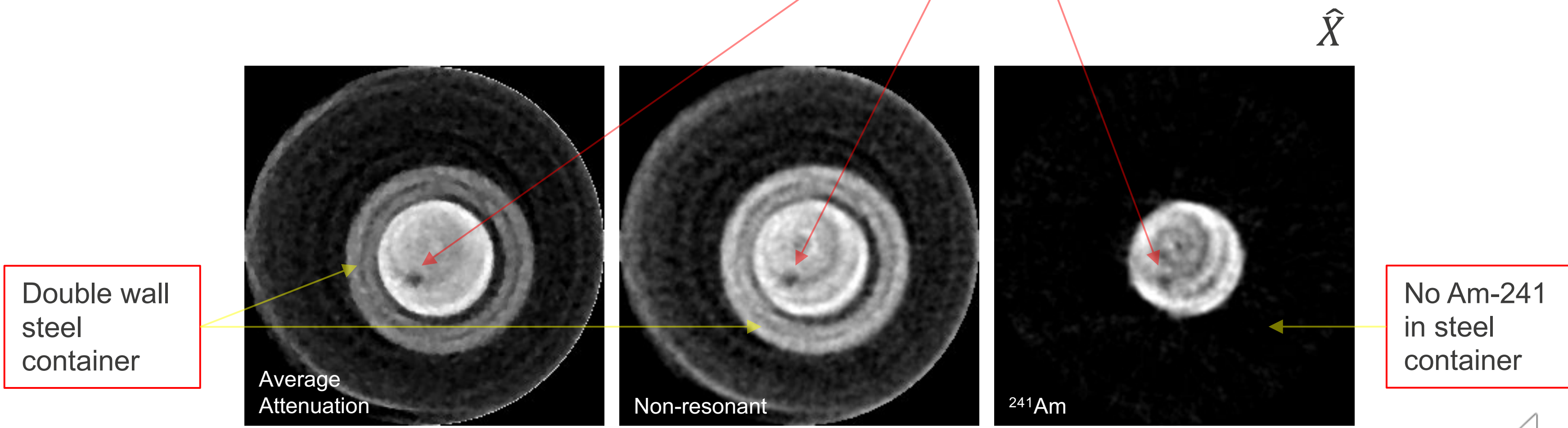
Hydrogen (H-1) class combines detection of all isotopes without distinct resonances

ERNI-CT Results

Current CT results accomplish to
- Give semi-qualitative measures of isotope distribution
- Allow determining features

Common feature detected, guiding destructive evaluation

Density Maps of 3D CT Reconstruction



Comparison to SAMMY

Table 1. Computation time comparison of the material decomposition. (*: interpolated, **: extrapolated)

Compuation Time	SAMMY	Proposed
Time per pixel spectrum	1.72 s	2.18 ms*
Time per CT data set	248.8 days**	7.58 h

Table 2. Estimated Areal densities for a single spectrum. (* SAMMY did not converge using ^{1}H)

Areal Densities [atoms / 1000 barn]	SAMMY	Proposed
^{237}Np	0.471	0.577
^{238}U	10.665	1.163
^{239}Pu	1.930	2.284
^{240}Pu	0.540	0.001
^{241}Am	0.493	0.727
^{1}H equivalent	N/A*	0.031

About 800X speedup:
≈ **8 months vs 8 hours**

Similar density estimates as SAMMY, with some outliers due to saturation effect.

In future, ground truth data or simulated data necessary to verify accuracy.

Conclusion and Future Work

- **Conclusion:**
  - First full material decomposed neutron TOF CT reconstruction pipeline
  - Significant speedup compared to processing with SAMMY
  - Robust background estimation
  - Material decomposed radiographs possible when CT not available

Future/Current Work:

- Quantitative accuracy not verified
  - Use well defined ground truth phantom, simulated data with Geant4
- Model struggles with saturation effects, noise and finite pulse width
  - Use non-linearized model
  - model Poissons directly
  - model resolution function due to finite pulse width of neutron beam

Thank you

More questions?

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