Datastructure Lower Bounds for Depth First Search

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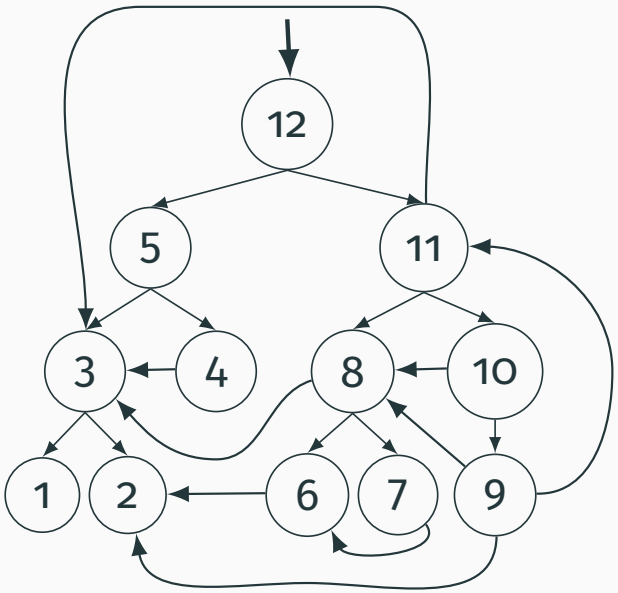
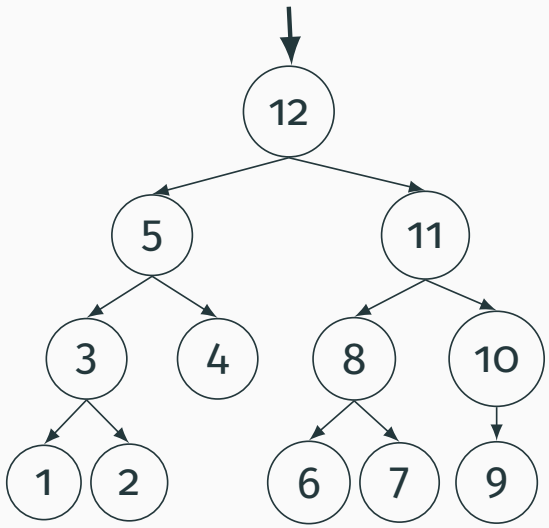
Problem

Problem

LexDFS is the problem of given a graph G and two vertices v_1, v_2, decide if v_1 occurs before v_2 on the DFS order.





- $f(u_9, u_{10}) = 0.$
- $f(u_1, u_2) = 1.$

Known Upper Bounds

w the word size, t number of probes, m number of edges.

Theorem (Chakraborty and Sadakane)

There exist an algorithm producing a datastructure of size

$$(1+\epsilon)n\log n + 2n + o(n).$$

LexDFS *can be solved in constant time and the preprocessing takes* $O(m+n)$.

Theorem (This Work)

- *The datastructure needs size at least* $\Omega(n\log(m/n))$ *bits.*
- *For* t *probes the datastructure needs at least* $n + \Omega(n/w^{O(t)})$ *bits.*
- *A datastructure for answering the degree of a vertex in the DFS tree needs at least* $\Omega(n)$ *bits.*

Communication Complexity and Richness

Transfer

- Alice queries the datastructure.
- Bob has the datastructure and answers.
- Both want to know the answer to a function.
- Alice queries use $t \log s$ bits (s size of the structure, t number of queries).
- Bob answers with tw bits (w word size).

Methods

- Standard Communication Complexity Methods.
- Communication Complexity using Milterson's Richness Method.
- Patrusca and Viola's Probe vs Size Tradeoff Method.