Compressing Multisets with Large Alphabets

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Outline

Problem setting

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Given a sequence of i.i.d. symbols $X^n = (X_1, \ldots, X_n)$ with entropy

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we want to losslessly compress the multiset

$$\mathcal{M} = f(X^n) = \{X_1, \dots, X_n\}$$

at rate $H(\mathcal{M}) \leq H(X^n)$.

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Would like efficient, rate-optimal method for any $\mathcal{A}, n$.

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order	queue-like	stack-like

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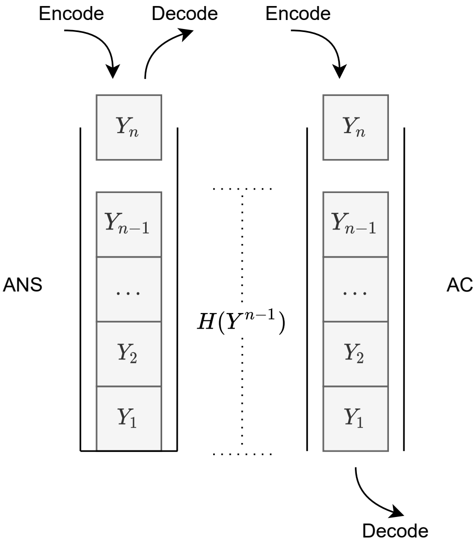
Key difference: ANS decodes in reverse order



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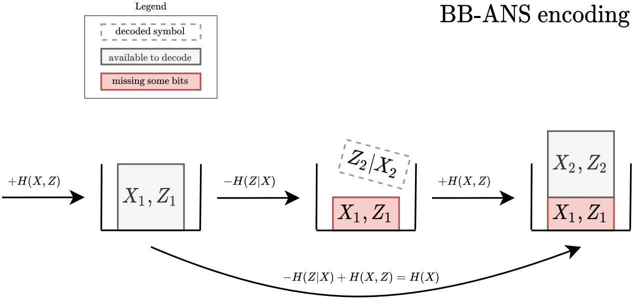

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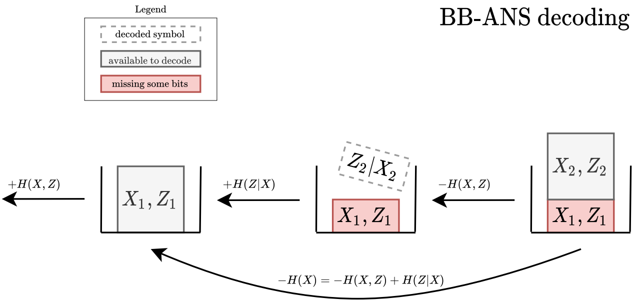

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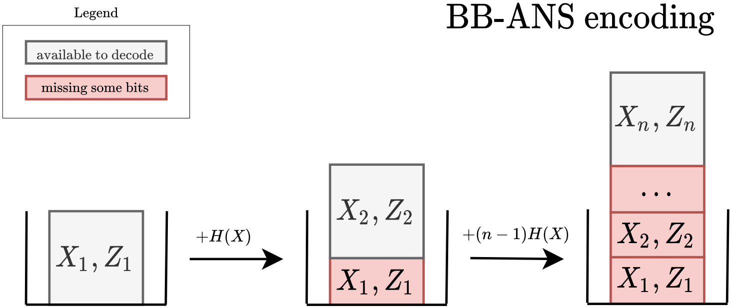
The full picture



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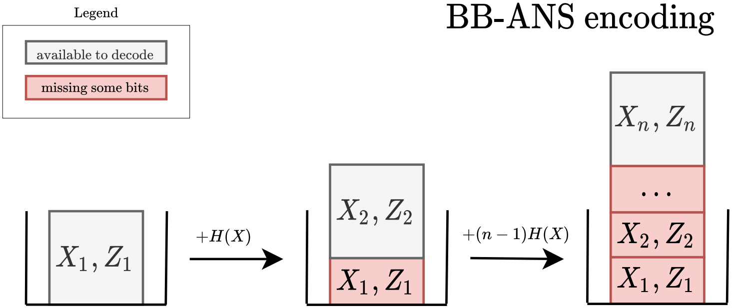

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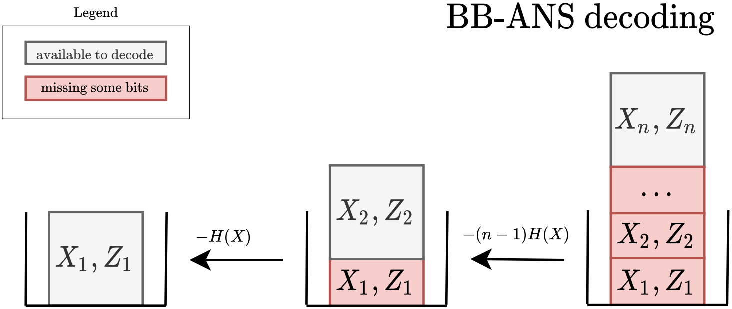

Background: Bits-back with ANS (BB-ANS)

Take-away: BB-ANS gives an operational meaning to the identity

$$H(X) = H(X, Z) - H(Z \mid X) = I(X; Y),$$

where $Y = (X, Z)$.

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It is often called the "order information"

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Can we achieve $H(\mathcal{M})$ on a single multiset $\mathcal{M} = f(X^n)$?

In other words, can we compress $\mathcal{M}$ to $-\log P_{\mathcal{M}}(\mathcal{M})$ bits?

Method: compressing $\mathcal{M}$ to $-\log P_{\mathcal{M}}(\mathcal{M})$ bits

Construct order information $H(X^n \mid \mathcal{M})$ iteratively by "sampling without replacement" from $\mathcal{M}$. Alternate:

1. Decode sample (w.o. replacement) from $\mathcal{M}$
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until $\mathcal{M}$ is depleted.

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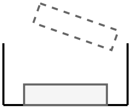

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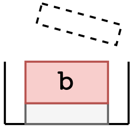

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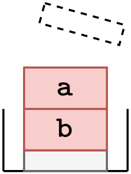

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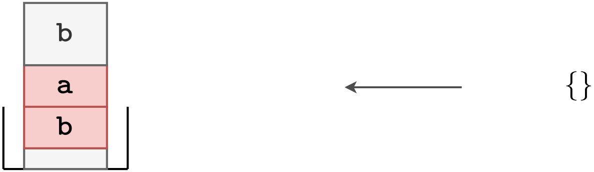

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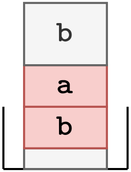

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$$L(\mathcal{M}) = \varepsilon + \log \frac{1}{P_X(\mathtt{b})^2 P_X(\mathtt{a})} - \log \frac{1}{(2/3)(1/2)(1/1)}$$

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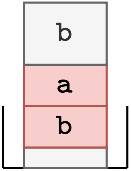

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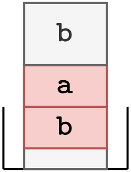

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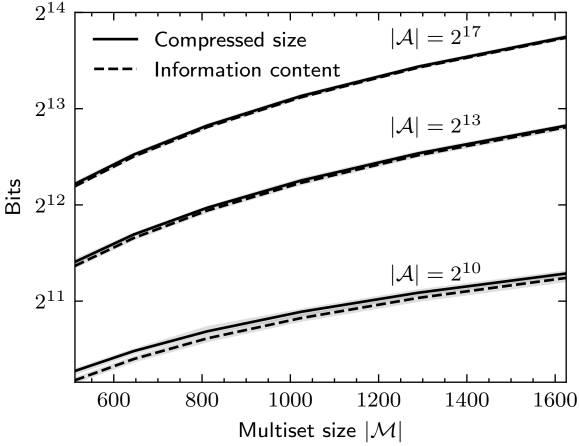
Experiments

Experiments: Synthetic multisets (rate)

Achieves $H(\mathcal{M}) = \mathbb{E}[-\log P_{\mathcal{M}}(\mathcal{M})]$ on single $\mathcal{M}$



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Average complexities

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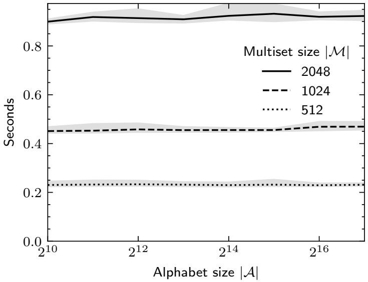

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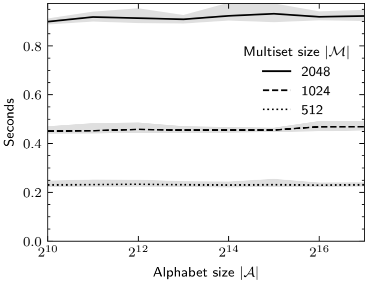

Compute time doesn't scale with $|\mathcal{A}|$, if m is fixed

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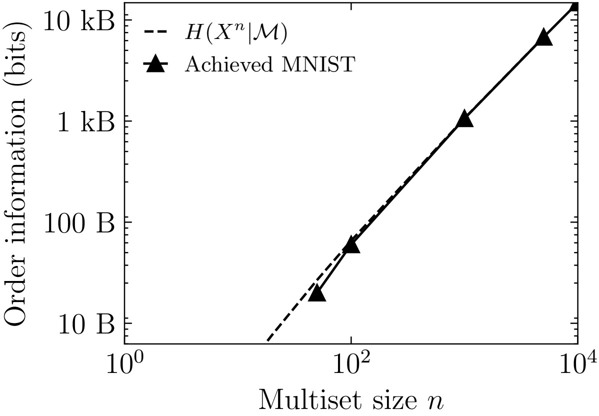

Method removes all order information $H(X^n \mid \mathcal{M})$

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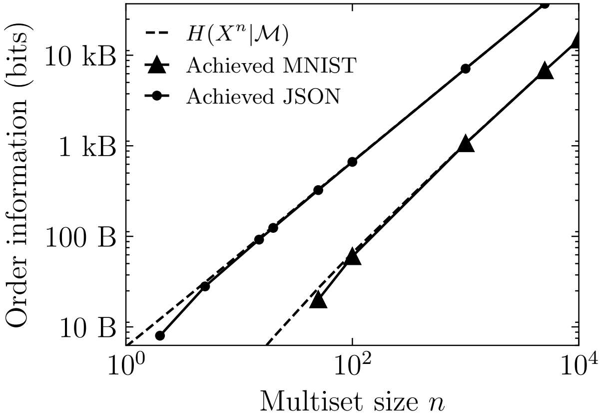

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Thank you!

Meta


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Presented by: dsevero.com and j-towns.github.io

Code: github.com/facebookresearch/multiset-compression