Conjugate Augmented Spatial-Temporal Near-Field Sources Localization with Cross Array

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Outline

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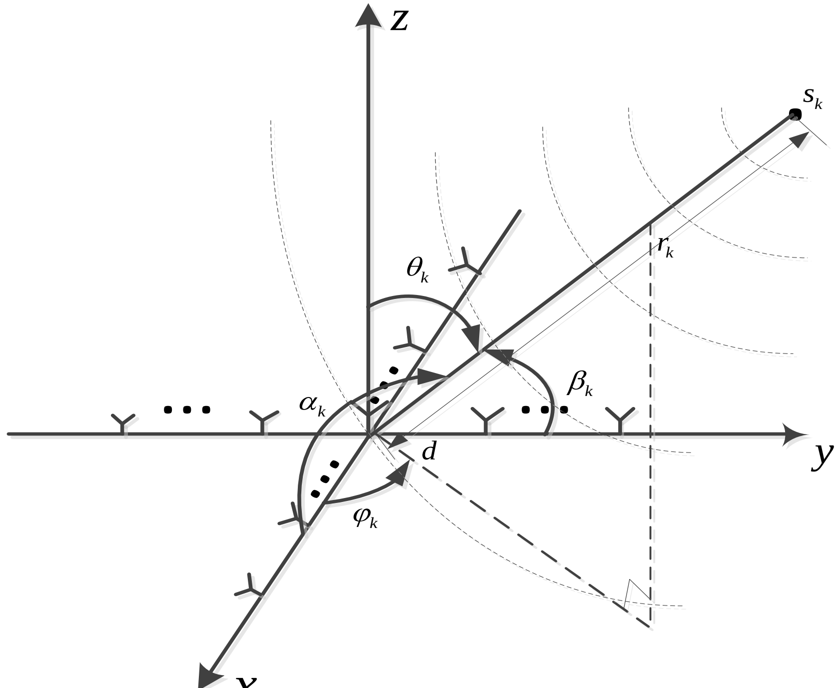
1.Introduction

Array Model



- K narrowband, spatially and temporally uncorrelated sources S_k
- two uniform linear arrays (ULAs) : N_x sensors on x-axis and N_y sensors on y-axis
- the elevation and azimuth angles of the *k-th* signal： θ_k and φ_k
- the angles between the *k-th* signal and the x and y axes： α_k and β_k
- the range of the *k-th* signal: r_k
- If α_k and β_k are determined, θ_k and φ_k can be uniquely identified.

1.Introduction

Data Model

$$x_m(n)=\sum_{k=1}^{K}s_k(n)e^{-j\tau_{m_x,k}}+n_{m_x}(n)$$
$$y_m(n)=\sum_{k=1}^{K}s_k(n)e^{-j\tau_{m_y,k}}+n_{m_y}(n) \quad (1)$$

With the Fresnel approximation, $\tau_{m_x,k}$ and $\tau_{m_y,k}$ can be expressed as:

$$\tau_{m_x,k}=\omega_{xk}m+\phi_{xk}m^2$$
$$\tau_{m_y,k}=\omega_{yk}m+\phi_{yk}m^2 \quad (2)$$

where

$$\omega_{xk}=-\frac{2\pi d}{\lambda}\cos\alpha_k,\qquad \phi_{xk}=\frac{\pi d^2}{\lambda r_k}\sin^2\alpha_k$$
$$\omega_{yk}=-\frac{2\pi d}{\lambda}\cos\beta_k,\qquad \phi_{yk}=\frac{\pi d^2}{\lambda r_k}\sin^2\beta_k \quad (3)$$

1.Introduction

Data Model

$$\mathbf{x}(n) = \mathbf{A}_{\mathbf{x}}\mathbf{s}(n) + \mathbf{n}_{\mathbf{x}}(n) \tag{4}$$

$$\mathbf{y}(n) = \mathbf{A}_{\mathbf{y}}\mathbf{s}(n) + \mathbf{n}_{\mathbf{y}}(n) \tag{5}$$

where

$$\mathbf{A}_{\mathbf{x}} = \left[\mathbf{a}(\omega_{x1},\phi_{x1}),\ldots,\mathbf{a}(\omega_{xk},\phi_{xk})\right]$$

$$\mathbf{a}(\omega_{xk},\phi_{xk}) = \begin{bmatrix} e^{-j[\omega_{xk}(-M_x)+\phi_{xk}(-M_x)^2]} & \cdots & e^{-j(\omega_{xk}M_x+\phi_{xk}M_x^2)} \end{bmatrix}^T$$

$$\mathbf{A}_{y} = \left[\mathbf{a}(\omega_{y1},\phi_{y1}),\ldots,\mathbf{a}(\omega_{yk},\phi_{yk})\right]$$

$$\mathbf{a}(\omega_{yk},\phi_{yk}) = \begin{bmatrix} e^{-j[\omega_{yk}(-M_y)+\phi_{yk}(-M_y)^2]} & \cdots & e^{-j(\omega_{yk}M_y+\phi_{yk}M_y^2)} \end{bmatrix}^T$$

$\mathbf{n}_{\mathbf{x}}(n)$ and $\mathbf{n}_{\mathbf{y}}(n)$ represent the additive Gaussian noise vectors for the two ULAs, respectively.

2.Proposed Method

1. Frame the received data

$\mathbf{x}(n)$ and $\mathbf{y}(n)$ are divided into L frames according to the principle of maximum overlap in the time domain .The *l-th* (l = 1, 2, · · · , L) frame data can be expressed as:

$$\mathbf{X}_l = \left[\mathbf{x}(l), \mathbf{x}(l+1), \cdots, \mathbf{x}(l+N-L)\right]$$
$$\mathbf{Y}_l = \left[\mathbf{y}(l), \mathbf{y}(l+1), \cdots, \mathbf{y}(l+N-L)\right] \quad (6)$$

2.Proposed Method

2. Construct the conjugate augmented cross-correlation matrix and subspace decomposition

Select one array element from each of the two uniform arrays, then the delay cross-correlation information of the received data of these two array elements is:

$$
\begin{aligned}
r_{m_1,m_2}(l-1+L) &= E\{x_{m_1}(n+l-1)y^*_{m_2}(n)\} \\
&= \sum_{k_1=1}^{K}[a_{x,m_1}(\alpha_{k_1}, r_{k_1})a^*_{y,m_2}(\beta_{k_1}, r_{k_1})\mathbf{R}_{ss}(k_1, l-1+L)] \\
&+\delta(m_1)\bullet\delta(m_2)\bullet\delta(l-1)\bullet\sigma^2_w
\end{aligned}
\tag{7}
$$

where $\mathbf{R}_{ss}$ is the signal delay autocorrelation matrix.

By arranging $r_{m_1,m_2}(l_1)(l_1=1,2,\cdots,2L-1)$, we have

$$
\mathbf{R}_{xy} = [\mathbf{r}_{xy}(1), \mathbf{r}_{xy}(2), \cdots, \mathbf{r}_{xy}(2L-1)] = (\mathbf{A}^*_y(\beta, r)\odot\mathbf{A}_x(\alpha, r))\mathbf{R}_{ss} + \sigma^2_w\cdot\mathbf{R}_{w0} \tag{8}
$$

where $\mathbf{R}_{w0}$ only has one nonzero element, that is, $\mathbf{R}_{w0}(1,L)=1$

2.Proposed Method

Taking advantage of the conjugate symmetry of the autocorrelation function of the signal delay, we have

$$\begin{aligned}\tilde{\mathbf{R}}_{xy} &= \begin{bmatrix} \mathbf{R}_{xy} \\ \mathbf{R}_{xy}^{*}\mathbf{J}_{(2L-1)} \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{A}_y^{*}(\beta,r)\odot\mathbf{A}_x(\alpha,r) \\ \mathbf{A}_y(\beta,r)\odot\mathbf{A}_x^{*}(\alpha,r) \end{bmatrix}\mathbf{R}_{ss} + \sigma_w^2\cdot\begin{bmatrix} \mathbf{R}_{w0} \\ \mathbf{R}_{w0}\mathbf{J}_{(2L-1)} \end{bmatrix} \\ &= \tilde{\mathbf{A}}_{xy}\mathbf{R}_{ss} + \sigma_w^2\cdot\tilde{\mathbf{R}}_{w0}\end{aligned} \tag{9}$$

Calculate the covariance matrix of as:

$$\tilde{\mathbf{R}} = E\left\{\tilde{\mathbf{R}}_{xy}\tilde{\mathbf{R}}_{xy}^{H}\right\} = \tilde{\mathbf{A}}_{xy}\tilde{\mathbf{R}}_{s}\tilde{\mathbf{A}}_{xy}^{H} + \sigma_w^4\tilde{\mathbf{R}}_{w} \tag{10}$$

Where $\tilde{\mathbf{R}}_s$ is the covariance matrix of signal delay autocorrelation matrix $\mathbf{R}_{ss}$, $\tilde{\mathbf{R}}_w$ is extremely sparse, with only $\tilde{\mathbf{R}}_w(\frac{N_xN_y+1}{2},\frac{N_xN_y+1}{2}) = 1.$

2.Proposed Method

The eigenvalue decomposition of $\tilde{\mathbf{R}}$

$$\tilde{\mathbf{R}} = \mathbf{U}_s \mathbf{\Sigma}_s \mathbf{U}_s^H + \mathbf{U}_n \mathbf{\Sigma}_n \mathbf{U}_n^H \qquad (11)$$

where

$\mathbf{U}_n$ ⟶ noise subspace

$\mathbf{U}_s$ ⟶ signal subspace

$\mathbf{\Sigma}_s$ ⟶ main eigenvalue matrix

$\mathbf{\Sigma}_n$ ⟶ small eigenvalue matrix

3. Parameter decoupling

According to the relationship between Khatri-Rao product and Kronecker product, the above-constructed delay cross-correlation matrix (formula (8)) can be rewritten as:

$$\mathbf{R}_{xy} = [\mathbf{a}_y^*(\beta_1, r_1) \otimes \mathbf{a}_x(\alpha_1, r_1), \cdots, \mathbf{a}_y^*(\beta_K, r_K) \otimes \mathbf{a}_x(\alpha_K, r_K)]\mathbf{R}_{ss} + \sigma_w^2 \mathbf{R}_{w0} \qquad (12)$$

2.Proposed Method

According to the symmetry of array structure, $\mathbf{a}_y^*(\beta,r)$ can be decomposed into the following form

$$\mathbf{a}_y^*(\beta,r)=\begin{bmatrix} e^{j(-M_y)\omega_y} & & & & & \\ & e^{j(-M_y+1)\omega_y} & & & & \\ & & \ddots & & & \\ & & & 1 & & \\ & & \iddots & & & \\ & e^{j(M_y-1)\omega_y} & & & & \\ e^{jM_y\omega_y} & & & & & \end{bmatrix} \bullet \begin{bmatrix} e^{j(-M_y)^2\phi_y} \\ e^{j(-M_y+1)^2\phi_y} \\ \vdots \\ e^{j(-1)^2\phi_y} \\ 1 \end{bmatrix} = \boldsymbol{\zeta}_y^*(\beta)\boldsymbol{\upsilon}_y^*(\beta,r) \quad (13)$$

Then, $\mathbf{a}_y^*(\beta,r)\otimes\mathbf{a}_x(\alpha,r)$ can be rewritten as:

$$\mathbf{a}_y^*(\beta,r)\otimes\mathbf{a}_x(\alpha,r)=(\boldsymbol{\zeta}_y^*(\beta)\boldsymbol{\upsilon}_y^*(\beta,r))\otimes(\mathbf{I}_{N_x}\mathbf{a}_x(\alpha_k,r_k)) \quad (14)$$

We denote the columns of $\tilde{\mathbf{R}}_{xy}$ as $\tilde{\mathbf{a}}_{xy}$,then

$$\tilde{\mathbf{a}}_{xy}=\begin{bmatrix} (\boldsymbol{\zeta}_y^*(\beta)\boldsymbol{\upsilon}_y^*(\beta,r))\otimes(\mathbf{I}_{N_x}\mathbf{a}_x(\alpha,r)) \\ (\boldsymbol{\zeta}_y(\beta)\boldsymbol{\upsilon}_y(\beta,r))\otimes(\mathbf{I}_{N_x}\mathbf{a}_x^*(\alpha,r)) \end{bmatrix} \quad (15)$$

2.Proposed Method

According to the property of the Kronecker product, we have

$$(\mathbf{A}\otimes\mathbf{B})(\mathbf{C}\otimes\mathbf{D})=\mathbf{AC}\otimes\mathbf{BD} \tag{15}$$

$\tilde{\mathbf{a}}_{xy}$ can be rewritten as follows:

$$\begin{aligned}\tilde{\mathbf{a}}_{xy}&=\begin{bmatrix}(\boldsymbol{\zeta}_y^*(\beta)\boldsymbol{\upsilon}_y^*(\beta,r))\otimes(\mathbf{I}_{N_x}\mathbf{a}_x(\alpha,r))\\(\boldsymbol{\zeta}_y(\beta)\boldsymbol{\upsilon}_y(\beta,r))\otimes(\mathbf{I}_{N_x}\mathbf{a}_x^*(\alpha,r))\end{bmatrix}\\&=\begin{bmatrix}(\boldsymbol{\zeta}_y^*(\beta)\otimes\mathbf{I}_{N_x})(\boldsymbol{\upsilon}_y^*(\beta,r)\otimes\mathbf{a}_x(\alpha,r))\\(\boldsymbol{\zeta}_y(\beta)\otimes\mathbf{I}_{N_x})(\boldsymbol{\upsilon}_y(\beta,r)\otimes\mathbf{a}_x^*(\alpha,r))\end{bmatrix}\\&=\begin{bmatrix}\mathbf{C}_1(\beta)(\boldsymbol{\upsilon}_y^*(\beta,r)\otimes\mathbf{a}_x(\alpha,r))\\\mathbf{C}_2(\beta)(\boldsymbol{\upsilon}_y(\beta,r)\otimes\mathbf{a}_x^*(\alpha,r))\end{bmatrix}\\&=\begin{bmatrix}\mathbf{C}_1(\beta)&\\&\mathbf{C}_2(\beta)\end{bmatrix}\begin{bmatrix}(\boldsymbol{\upsilon}_y^*(\beta,r)\otimes\mathbf{a}_x(\alpha,r))\\(\boldsymbol{\upsilon}_y(\beta,r)\otimes\mathbf{a}_x^*(\alpha,r))\end{bmatrix}\end{aligned} \tag{16}$$

2.Proposed Method

We define $\mathbf{C}(\beta)$ that is only related to β :

$$\mathbf{C}(\beta)=\begin{bmatrix}\mathbf{C}_1(\beta) & \\ & \mathbf{C}_2(\beta)\end{bmatrix} \tag{17}$$

Based on the rank reduction principle ,we can construct the MUSIC spectral function related to angle β :

$$\mathbf{D}(\beta)=\mathbf{C}^H(\beta)\mathbf{U}_n\mathbf{U}_n^H\mathbf{C}(\beta) \tag{18}$$

2.Proposed Method

With the same operation as (16), we have

$$\begin{aligned}
\tilde{\mathbf{a}}_{xy} &= \begin{bmatrix} \mathbf{C}_1(\beta) & \\ & \mathbf{C}_2(\beta) \end{bmatrix} \begin{bmatrix} (\mathbf{\upsilon}_y^*(\beta,r) \otimes \mathbf{a}_x(\alpha,r)) \\ (\mathbf{\upsilon}_y(\beta,r) \otimes \mathbf{a}_x^*(\alpha,r)) \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{C}_1(\beta) & \\ & \mathbf{C}_2(\beta) \end{bmatrix} \begin{bmatrix} (\mathbf{I}_{N_y} \mathbf{\upsilon}_y^*(\beta,r) \otimes \boldsymbol{\zeta}_x(\alpha)\mathbf{\upsilon}_x(\alpha,r)) \\ (\mathbf{I}_{N_y} \mathbf{\upsilon}_y(\beta,r) \otimes \boldsymbol{\zeta}_x^*(\alpha)\mathbf{\upsilon}_x^*(\alpha,r)) \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{C}_1(\beta) & \\ & \mathbf{C}_2(\beta) \end{bmatrix} \begin{bmatrix} (\mathbf{I}_{N_y} \otimes \boldsymbol{\zeta}_x(\alpha))(\mathbf{\upsilon}_y^*(\beta,r) \otimes \mathbf{\upsilon}_x(\alpha,r)) \\ (\mathbf{I}_{N_y} \otimes \boldsymbol{\zeta}_x^*(\alpha))(\mathbf{\upsilon}_y(\beta,r) \otimes \mathbf{\upsilon}_x^*(\alpha,r)) \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{C}_1(\beta) & \\ & \mathbf{C}_2(\beta) \end{bmatrix} \begin{bmatrix} \mathbf{E}_1(\alpha)(\mathbf{\upsilon}_y^*(\beta,r) \otimes \mathbf{\upsilon}_x(\alpha,r)) \\ \mathbf{E}_2(\alpha)(\mathbf{\upsilon}_y(\beta,r) \otimes \mathbf{\upsilon}_x^*(\alpha,r)) \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{C}_1(\beta) & \\ & \mathbf{C}_2(\beta) \end{bmatrix} \begin{bmatrix} \mathbf{E}_1(\alpha) & \\ & \mathbf{E}_2(\alpha) \end{bmatrix} \begin{bmatrix} (\mathbf{\upsilon}_y^*(\beta,r) \otimes \mathbf{\upsilon}_x(\alpha,r)) \\ (\mathbf{\upsilon}_y(\beta,r) \otimes \mathbf{\upsilon}_x^*(\alpha,r)) \end{bmatrix}
\end{aligned} \tag{18}$$

Define $\mathbf{E}(\alpha)$ that is only related to α :

$$\mathbf{E}(\alpha) = \begin{bmatrix} \mathbf{E}_1(\alpha) & \\ & \mathbf{E}_2(\alpha) \end{bmatrix} \tag{19}$$

Based on (19), we can construct another MUSIC spectral function for angle α :

$$\mathbf{F}(\hat{\beta},\alpha) = \mathbf{E}^H(\alpha)\mathbf{C}^H(\hat{\beta})\mathbf{U}_n\mathbf{U}_n^H\mathbf{C}(\hat{\beta})\mathbf{E}(\alpha) \tag{20}$$

4. Spectral peak search

We separately perform three 1-D searches to obtain estimates of the three parameters:

For β :

$$\hat{\beta} = \arg\max_{\beta} \frac{1}{\det[\mathbf{D}(\beta)]} \tag{21}$$

For α :

$$\hat{\alpha} = \arg\max_{\alpha} \frac{1}{\det[\mathbf{F}(\hat{\beta},\alpha)]} \tag{22}$$

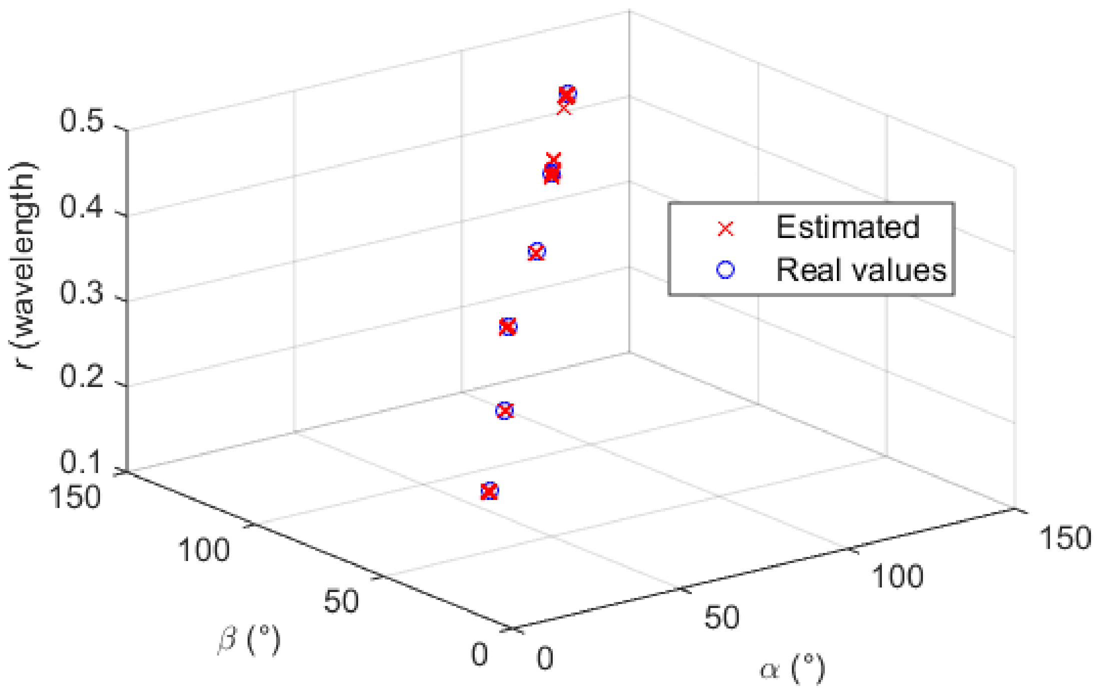
For r :

$$\hat{r}_k = \arg\max_{r} f(\hat{\alpha}_k, \hat{\beta}_k, r) = \frac{1}{\tilde{\mathbf{a}}_{xy}^H(\hat{\alpha},\hat{\beta},r)\mathbf{U}_n\mathbf{U}_n^H\tilde{\mathbf{a}}_{xy}(\hat{\alpha},\hat{\beta},r)} \tag{23}$$

3. Simulation Results

Performance test

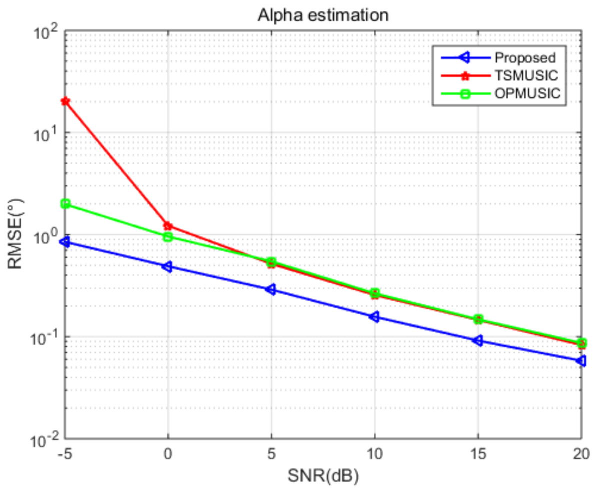
There are six uncorrelated NF sources from $\{20^\circ, 35^\circ, 0.2\lambda\}$, $\{40^\circ, 55^\circ, 0.25\lambda\}$, $\{60^\circ, 80^\circ, 0.3\lambda\}$, $\{80^\circ, 95^\circ, 0.35\lambda\}$, $\{100^\circ, 115^\circ, 0.4\lambda\}$ and $\{120^\circ, 135^\circ, 0.45\lambda\}$ impinging onto a symmetric cross array with $N_x = N_y = 3$, i.e., the total number of elements of the cross array is 5.

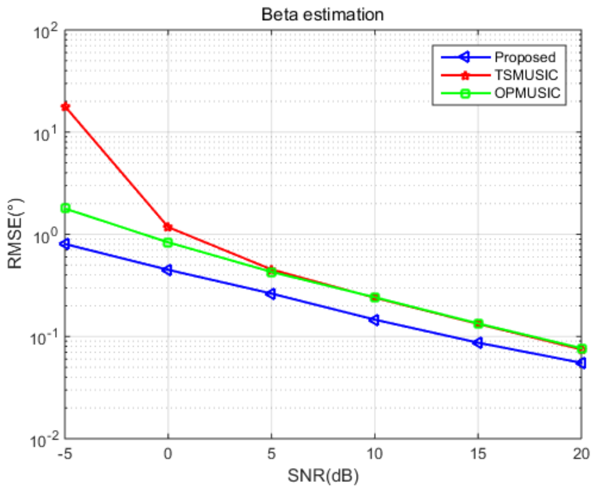


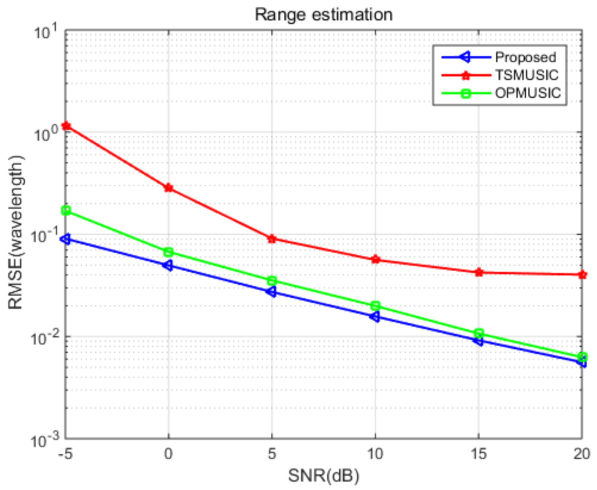
3. Simulation Results

Performance versus SNR

There are three uncorrelated NF sources impinging onto a symmetric cross array with $N_x = N_y = 5$, the number of snapshots, frames and Monte Carlo trials are set to be 1000 ,100 and 500, respectively.







4. Conclusions

- **A new localization method for NF sources has been proposed using a cross array in this paper.**
- **Make full use of the space-time two-dimensional characteristics of the signal to improve the degree of freedom of the algorithm, and use mathematical knowledge and dimensionality reduction technology to reduce the complexity of the algorithm.**
- **This algorithm can realize direction finding under the condition of underdetermination, and can automatically match, achieving a good compromise between accuracy and complexity.**

Thanks for your attention!