香港中文大學(深圳)
The Chinese University of Hong Kong, Shenzhen

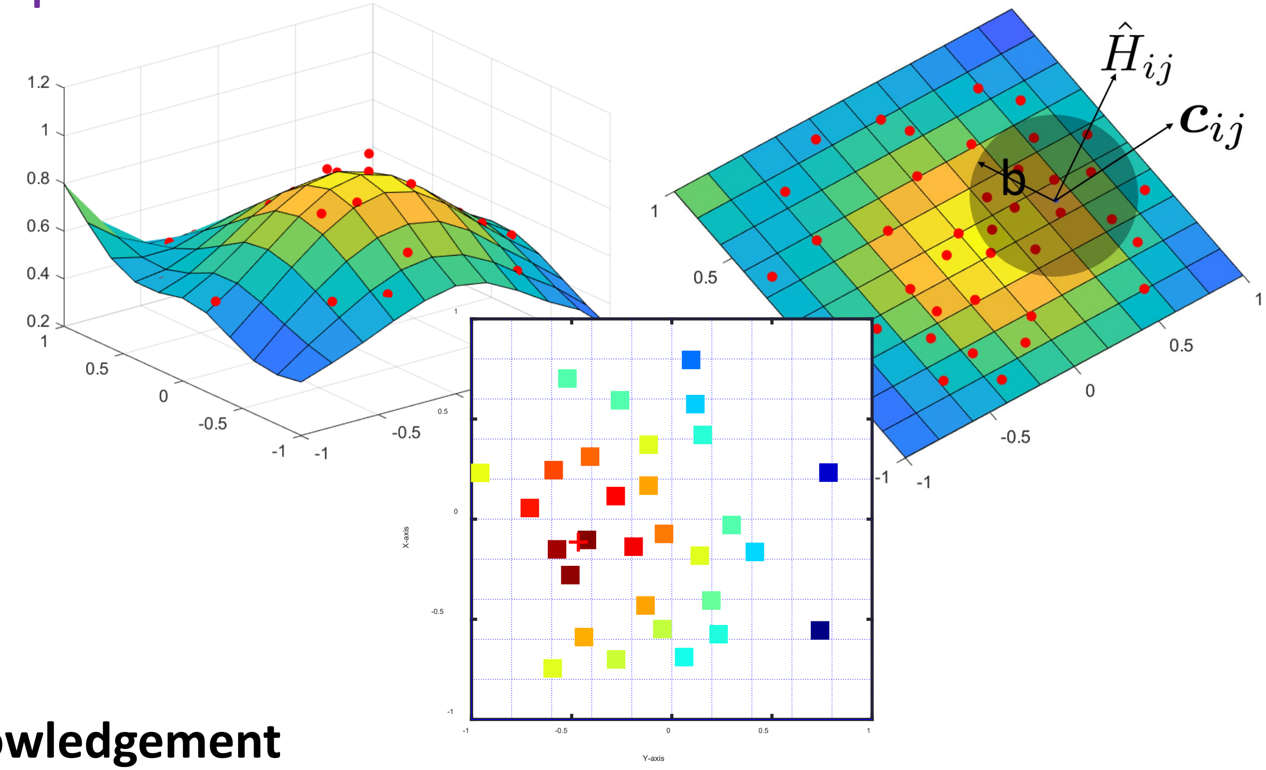
icassp 2022 Singapore

Regression Assisted Matrix Completion for Reconstructing a Propagation Field with Application to Source Localization

Hao Sun, Junting Chen

Future Network of Intelligence Institute (FNii) and
School of Science and Engineering

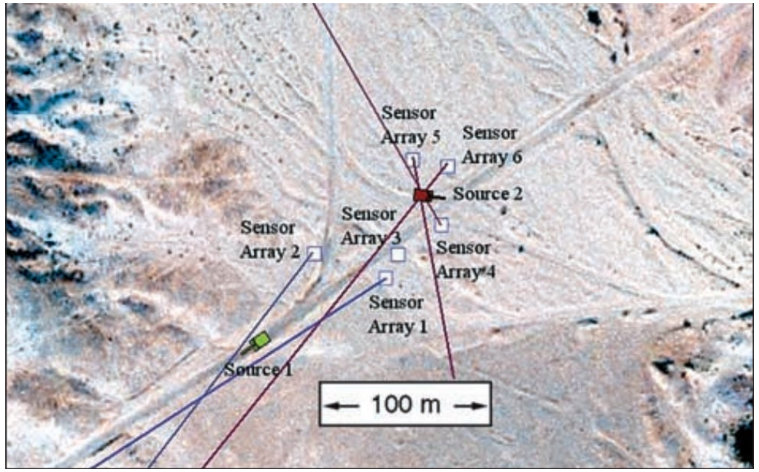
The Chinese University of Hong Kong,
Shenzhen (CUHK-Shenzhen)
Guangdong, China



Acknowledgement

This work was supported in part by the Key Area R&D Program of Guangdong Province with grant No. 2018B030338001, by the National Key R&D Program of China with grant No. 2018YFB1800800, by National Science Foundation of China No. 92067202 and No. 62171398, by Guangdong Research Project No. 2017ZT07X152 and Grant No. 2019QN01X895, and by the Shenzhen Science and Technology Program No. KQTD20200909114730003 and JCYJ20210324134612033.

Propagation field reconstruction is important in source localization areas.



[1]

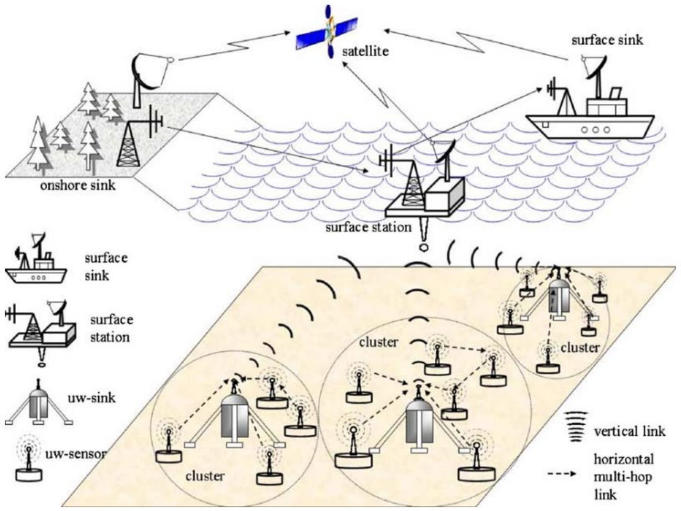
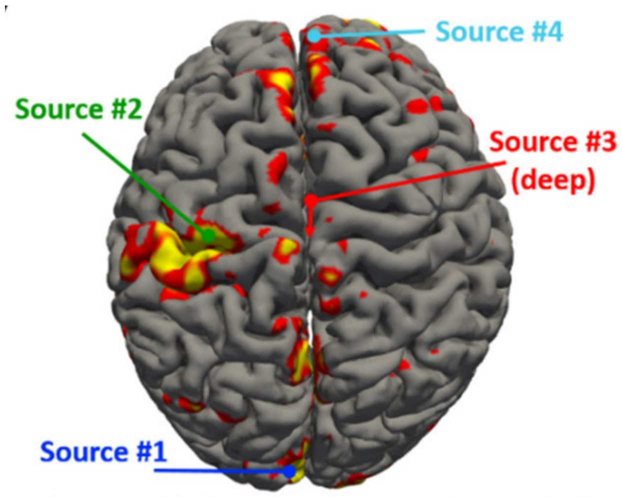


Fig. 1. Architecture for 2D underwater sensor networks.

[2]



[3]

Outdoor source localization:

RSS collected by sensor network: monitor an unknown area.

Underwater source localization:

Pollution monitoring, offshore exploration, disaster prevention, assisted navigation.

Neuroimaging source localization

EEG and fMRI signals :
Locate the active surface in brain cortex.

[1] Chen J C , Yao K , Hudson R E . Source localization and beamforming. Signal Processing Magazine IEEE, 2002.
[2] Akyildiz, I.F., Pompili, D. and Melodia, T.. Underwater acoustic sensor networks: research challenges. Ad hoc networks, 2005.
[3] Thinh, Nguyen, et al. Characterization of dynamic changes of current source localization based on spatiotemporal fMRI constrained EEG source imaging. Journal of neural engineering, 2018.

Our goal is to reconstruct the propagation field from sparse measurements

Information available:
Sensor location and energy measurement pairs

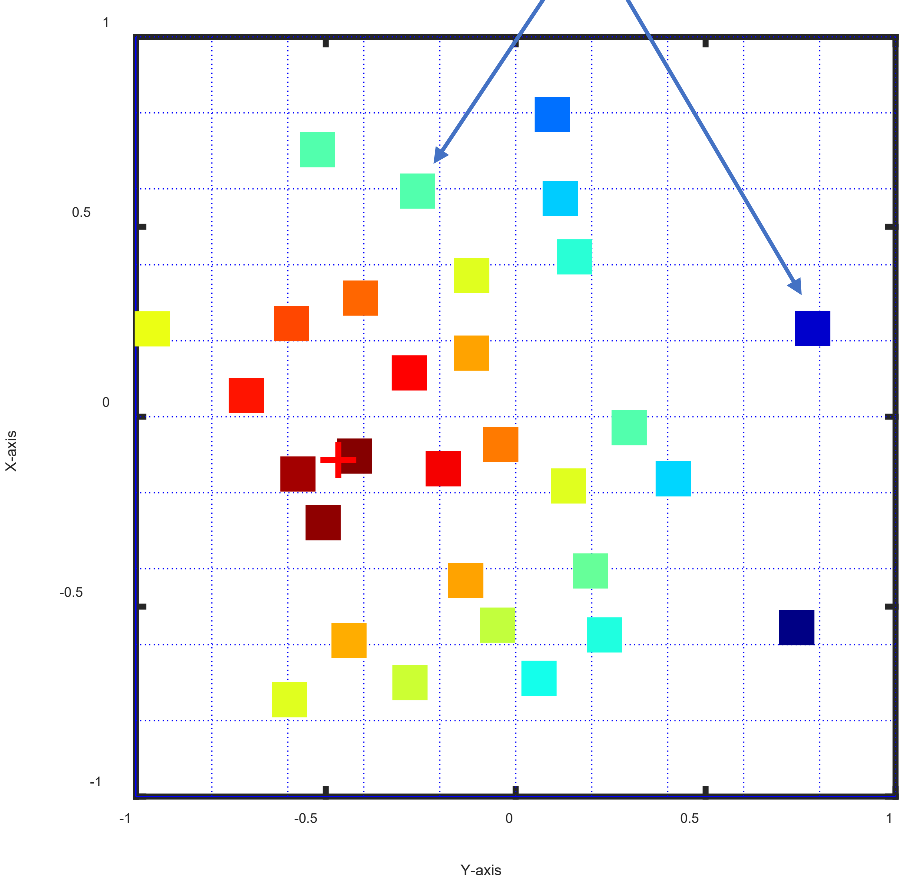
$$\{(\boldsymbol{z}_m, \gamma_m)\}$$

$m = 1,2,\ldots,M.$

$$\gamma_m = g(d(\boldsymbol{s}, \boldsymbol{z}_m)) + \epsilon_m$$

$g(d)$ is an *unknown* non-increasing function in distance d, ϵ_m is a random variable with zero mean and variance σ^2

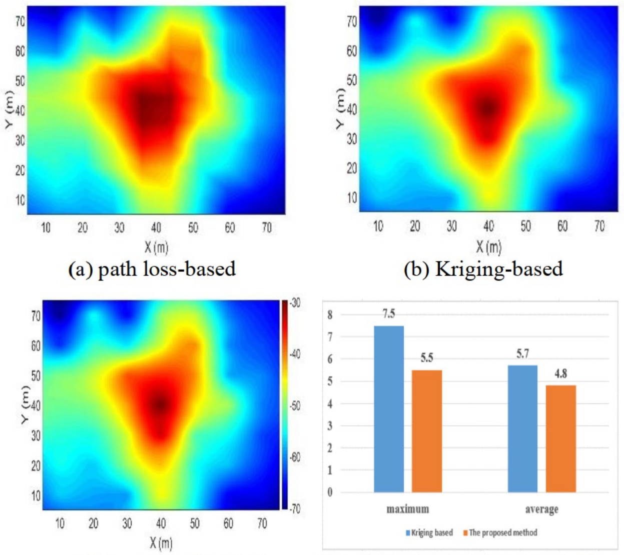
Sensor measurements



Challenge:
- Sparse information
- No information on the power decay law

Existing methods for propagation field reconstruction

Kriging Interpolation [4]

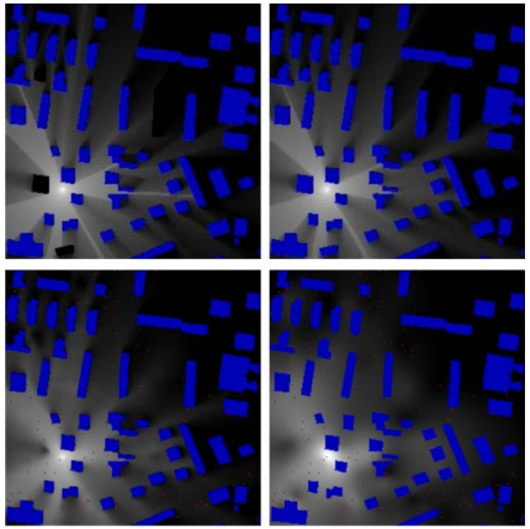


(a) path loss-based (b) Kriging-based

(c) the proposed method (d) Kriging standard deviation (dB)

$$P(x_j) = \sum_{i=1}^{n} w_i P(x_i)$$

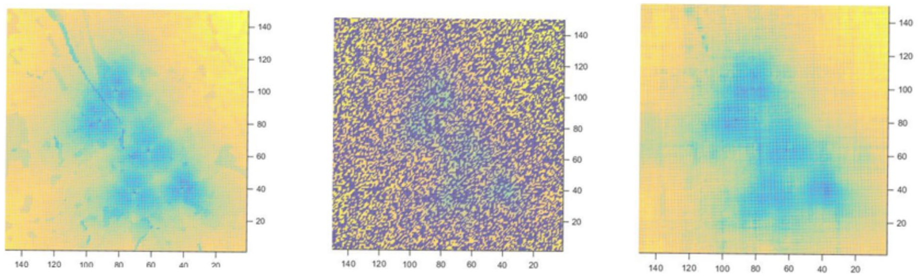
RadioUNet [5]



From top-left to bottom-right. 1:Ground truth radio map. 2:RadioUNet$_{c}$ with all buildings. 3:RadioUNet$_{s}$ with missing buildings. 4:RBF.

Loss function: $\mathcal{L}(\mathbf{p}) = \frac{1}{K} \sum_{k} \|\mathbf{g}_k - U_{\mathbf{p}}(\mathbf{f}_k)\|$

Matrix Completion [6]



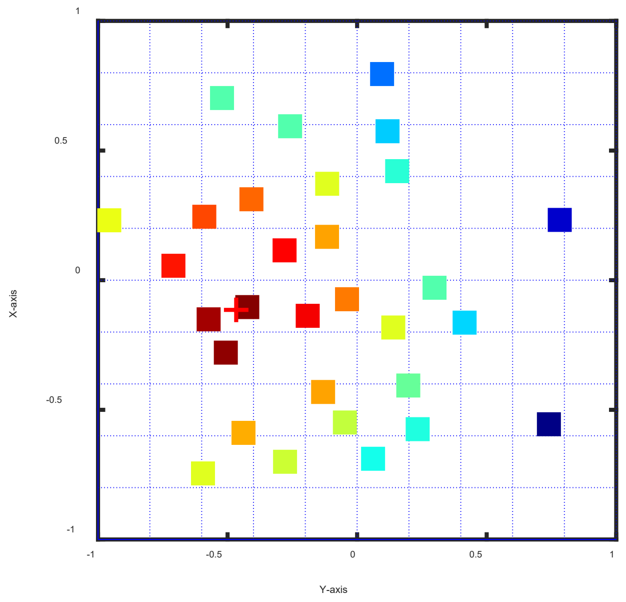
a) the original pathloss map, b) the sparse one with the missing entries, c) the reconstructed one

$$\begin{aligned} \underset{\boldsymbol{X} \in \mathbb{R}^{N \times N}}{\text{minimize}} \quad & \|\boldsymbol{X}\|_* \\ \text{subject to} \quad & |X_{ij} - H_{ij}| \leq \epsilon, \qquad \forall (i,j) \in \Omega \end{aligned}$$

[4] D. Mao, W. Shao, Z. Qian, H. Xue, X. Lu and H. Wu. Constructing accurate radio environment maps with Kriging interpolation in cognitive radio networks. CSQRWC, 2018.
[5] R. Levie, et al, RadioUNet: Fast Radio Map Estimation With Convolutional Neural Networks. IEEE TWC 2021
[6] Chouvardas, Symeon, et al. A method to reconstruct coverage loss maps based on matrix completion and adaptive sampling. ICASSP, 2016.

Recall matrix modeling and processing

Matrix observation $\boldsymbol{H}$ on $N \times N$ grid points



Constant Uncertainty:

γ_m

$$\underset{\boldsymbol{X}\in\mathbb{R}^{N\times N}}{\text{minimize}} \quad \|\boldsymbol{X}\|_*$$

$$\text{subject to} \quad |X_{ij} - H_{ij}| \leq \epsilon, \qquad \forall (i,j) \in \Omega$$

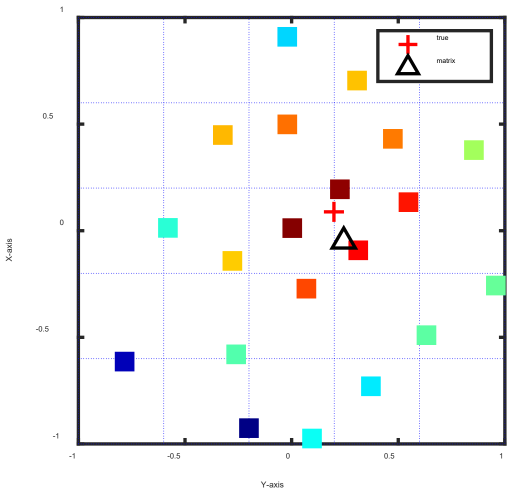
$$\bar{\boldsymbol{H}} = \sigma_1 u_1 v_1 + \varepsilon$$

Assume low rank, only first singular value and vectors matters and other term serves as noise.
The peak of the first singular vectors represent the location with the unimodality and symmetry property

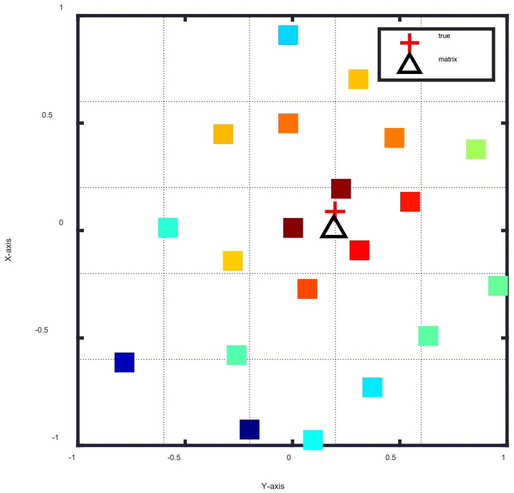
Uncertainty ϵ influence the localization accuracy

The simulation is under different uncertainty ϵ based on same topology of sensor distribution

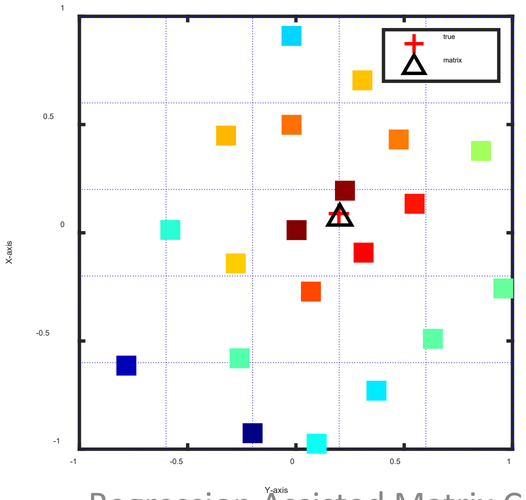
ϵ=0.00



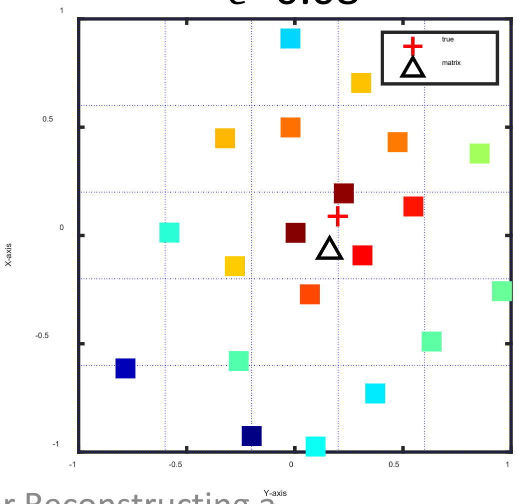
ϵ=0.02



ϵ=0.04



ϵ=0.08



Refined matrix model

Constant uncertainty

Old model:

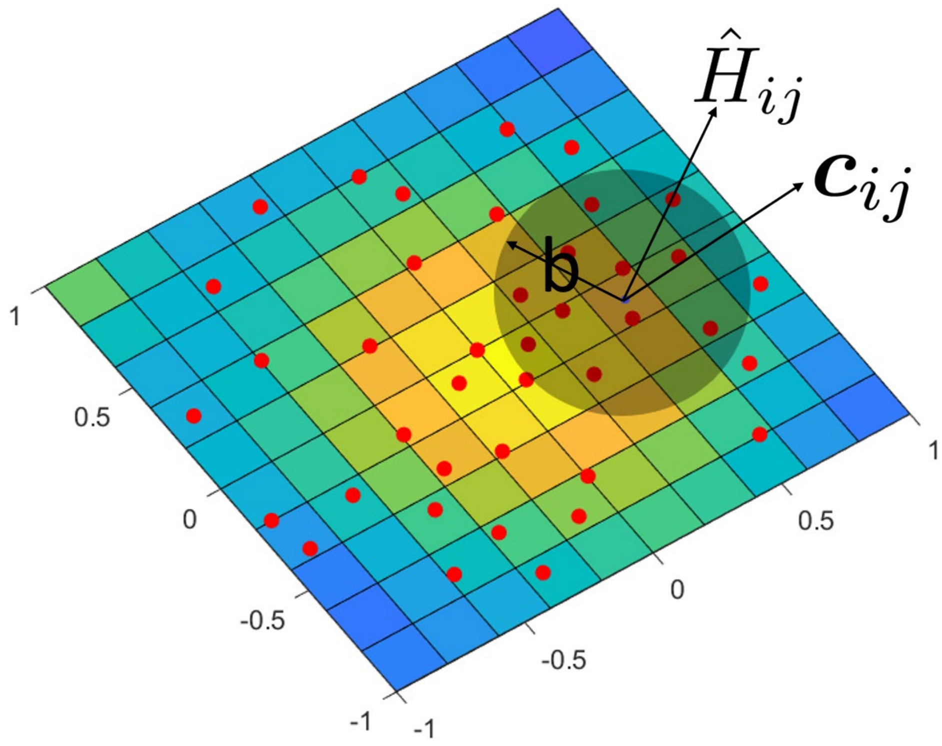
$$\underset{\boldsymbol{X}\in\mathbb{R}^{N\times N}}{\text{minimize}} \quad \|\boldsymbol{X}\|_*$$
$$\text{subject to} \quad |X_{ij} - H_{ij}| \leq \epsilon, \qquad \forall (i,j)\in\Omega$$

Refined model:

$$\underset{\boldsymbol{X}\in\mathbb{R}^{N\times N}}{\text{minimize}} \quad \|\boldsymbol{X}\|_*$$
$$\text{subject to} \quad |X_{ij} - \hat{H}_{ij}| \leq \bar{\epsilon}_{ij}, \qquad \forall (i,j)\in\Omega$$

$\hat{H}_{ij}$ represent the estimated value at the (i , j) th grid cell center. The difference $\bar{\epsilon}_{ij}$ represents the uncertainty of $\hat{H}_{ij}$.

Obtain $\hat{H}_{ij}$



Estimation matrix $\hat{H}_{ij}$ construction

Define $\rho(\boldsymbol{z}) = g(d(\boldsymbol{s}, \boldsymbol{z}))$. Estimating $\rho(\boldsymbol{z})$ in the neighborhood of c with

Zero-th order model :

$$\hat{\rho}(\boldsymbol{z};\boldsymbol{c}) = \alpha(\boldsymbol{c})$$

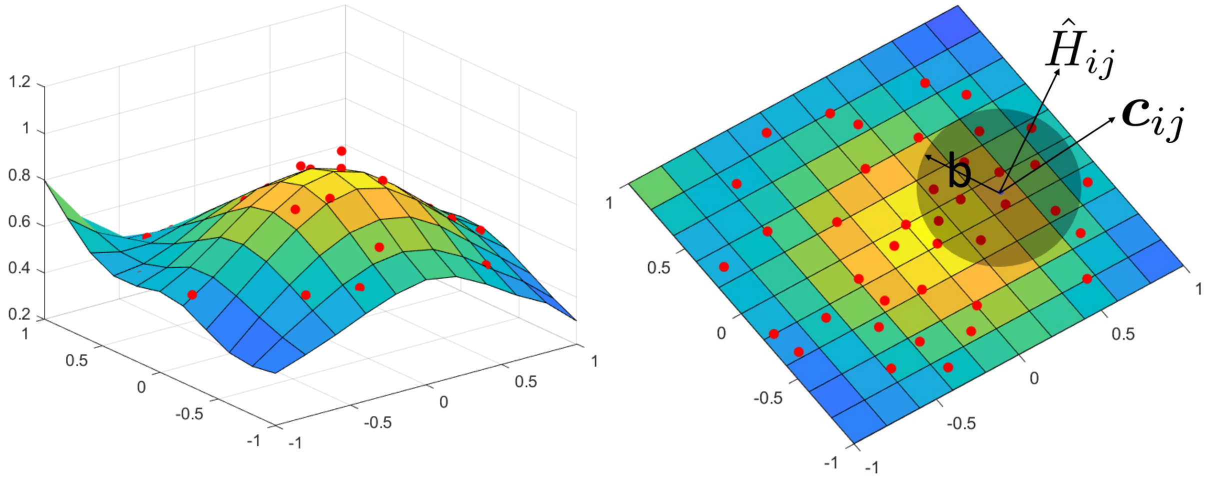
First order model :

$$\hat{\rho}(\boldsymbol{z};\boldsymbol{c}) = \alpha(\boldsymbol{c}) + \boldsymbol{\beta}^{\mathrm{T}}(\boldsymbol{c})(\boldsymbol{z}-\boldsymbol{c})$$

Distance weighted regression problem:

$$\underset{\boldsymbol{\theta}}{\text{minimize}} \sum_{m=1}^{M} w_m(\boldsymbol{c})\big(\gamma_m - \hat{\rho}(\boldsymbol{z}_m;\boldsymbol{c})\big)^2$$

$$\boldsymbol{\theta} = \{\alpha(\boldsymbol{c}), \boldsymbol{\beta}(\boldsymbol{c}), \ldots\} \quad w_m(\boldsymbol{c}) = K\left(\frac{||\boldsymbol{z}_m - \boldsymbol{c}||}{b}\right)$$



The solution of zero-th order problem serves as $\hat{H}_{ij}$

Solution to zero-th order model

$$\hat{\alpha}(\boldsymbol{c}) = \frac{\sum_{m=1}^{M} w_m(\boldsymbol{c})\gamma_m}{\sum_{m=1}^{M} w_m(\boldsymbol{c})}.$$

Solution to first order model

$$\begin{bmatrix} \hat{\alpha}(\boldsymbol{c}) \\ \hat{\boldsymbol{\beta}}(\boldsymbol{c}) \end{bmatrix} = (\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{Z})^{-1}\boldsymbol{Z}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{\gamma}$$

$$\boldsymbol{Z} = \begin{bmatrix} 1 & (\boldsymbol{z}_1 - \boldsymbol{c})^T \\ \vdots & \vdots \\ 1 & (\boldsymbol{z}_M - \boldsymbol{c})^T \end{bmatrix}, \boldsymbol{\gamma} = \begin{bmatrix} \gamma_1 \\ \vdots \\ \gamma_M \end{bmatrix}, \boldsymbol{W} = diag\{w_m(\boldsymbol{c})\}.$$

The zero-th order estimator $\hat{\alpha}(\boldsymbol{c})$ resembles the inverse distance weighting interpolation for ρ(c) where the first order estimator is used to analyze the estimation error

Therefore, $\hat{H}_{ij} = \hat{\alpha}(\boldsymbol{c}_{ij})$

Estimation error analysis

Define estimation error $\xi_{ij} \triangleq \hat{\alpha}(\boldsymbol{c}_{ij}) - \rho(\boldsymbol{c}_{ij})$

Theorem 1: The bias and variance of estimation error ξ_{ij} under zero-th order local polynomial regression are

$$\mathbb{E}\{\xi_{ij}|\boldsymbol{z}_1, \cdots, \boldsymbol{z}_M\} = \frac{\nabla\rho(\boldsymbol{c}_{ij}) \sum_{m=1}^{M} (\boldsymbol{z}_m - \boldsymbol{c}_{ij})\, w_m(\boldsymbol{c}_{ij})}{\sum_{m=1}^{M} w_m(\boldsymbol{c}_{ij})} + o(b)$$

$$\mathbb{V}\{\xi_{ij}|\boldsymbol{z}_1, \cdots, \boldsymbol{z}_M\} = \frac{\sum_{m=1}^{M} w_m^2(\boldsymbol{c}_{ij})\sigma^2}{\sum_{m=1}^{M} w_m(\boldsymbol{c}_{ij}) \sum_{m=1}^{M} w_m(\boldsymbol{c}_{ij})}$$

$\nabla\rho(\boldsymbol{c}_{ij}) = [\frac{d\rho(\boldsymbol{c}_{ij})}{dx} \; \frac{d\rho(\boldsymbol{c}_{ij})}{dy}]$ which represents the slope of the propagation field can be estimated using the first order model solution $\hat{\boldsymbol{\beta}}^{\mathrm{T}}(\boldsymbol{c}_{ij})$.

Uncertainty Characterization

Define $\mu_{ij} = \frac{\hat{\boldsymbol{\beta}}^{\mathrm{T}}(\boldsymbol{c}_{ij}) \sum_{m=1}^{M} (\boldsymbol{z}_m - \boldsymbol{c}_{ij}) \, w_m(\boldsymbol{c}_{ij})}{\sum_{m=1}^{M} w_m(\boldsymbol{c}_{ij})}$ and $\nu_{ij}^2 = \frac{\sum_{m=1}^{M} w_m^2(\boldsymbol{c}_{ij}) \sigma^2}{\sum_{m=1}^{M} w_m(\boldsymbol{c}_{ij}) \sum_{m=1}^{M} w_m(\boldsymbol{c}_{ij})}$

Assume all possible value for ξ_{ij} is in Gaussian distribution $\mathcal{N}(\mu_{ij}, \nu_{ij}^2)$

Construct a confidence interval for the ξ_{ij} of the probability $1 - \delta$

$$P(-\eta_\delta \leq \frac{\xi_{ij} - \mu_{ij}}{\nu_{ij}} \leq \eta_\delta) = 1 - \delta$$

The $1 - \delta$ confidence interval of ξ_{ij} is

$$(\mu_{ij} - \eta_\delta \nu_{ij}, \; \mu_{ij} + \eta_\delta \nu_{ij})$$

As a result, we propose to choose the uncertainty parameter as

$$\bar{\epsilon}_{ij} = \max(|\mu_{ij} - \eta_\delta \nu_{ij}|, \; |\mu_{ij} + \eta_\delta \nu_{ij}|)$$

Numerical results:

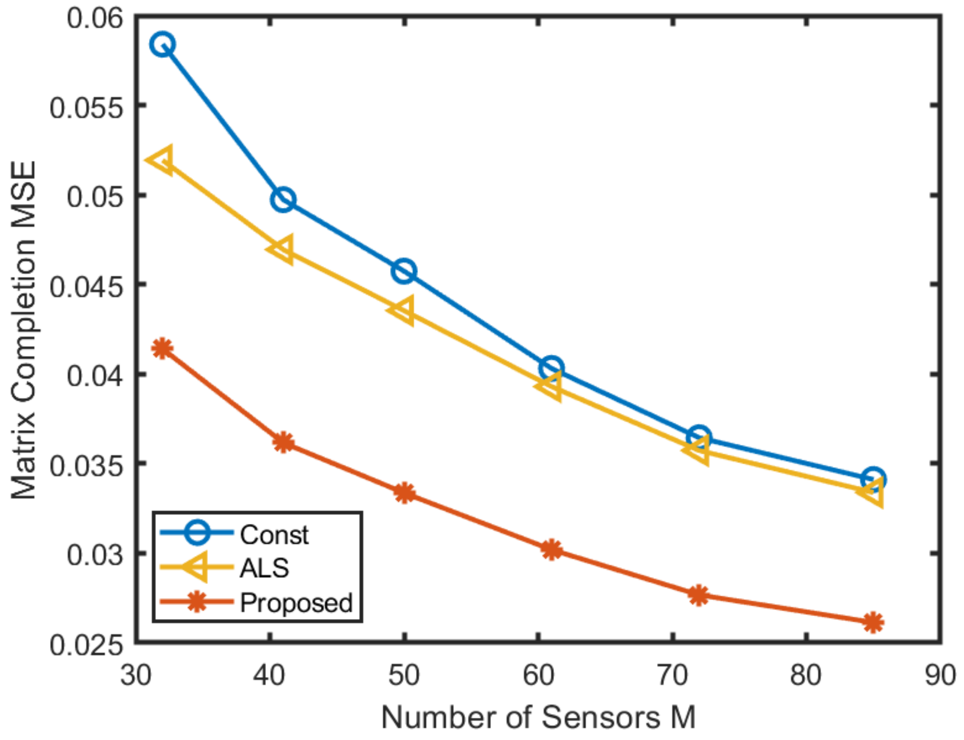
Underwater Scenario Model:

$$\gamma = (1 + d^{1.5} A(f)^d)^{-1} + \xi$$

where f=5kHz, d is the distance, and ξ is to model the noise.

Baseline: Alternating least square method minimizes $||\boldsymbol{y} - \mathcal{A}(\boldsymbol{X})||$, $\boldsymbol{X} = \boldsymbol{LR}$

Baseline: Constant Uncertainty.
The noise for each observation fixed, i.e., $\bar{\epsilon}_{ij}$ is fixed.

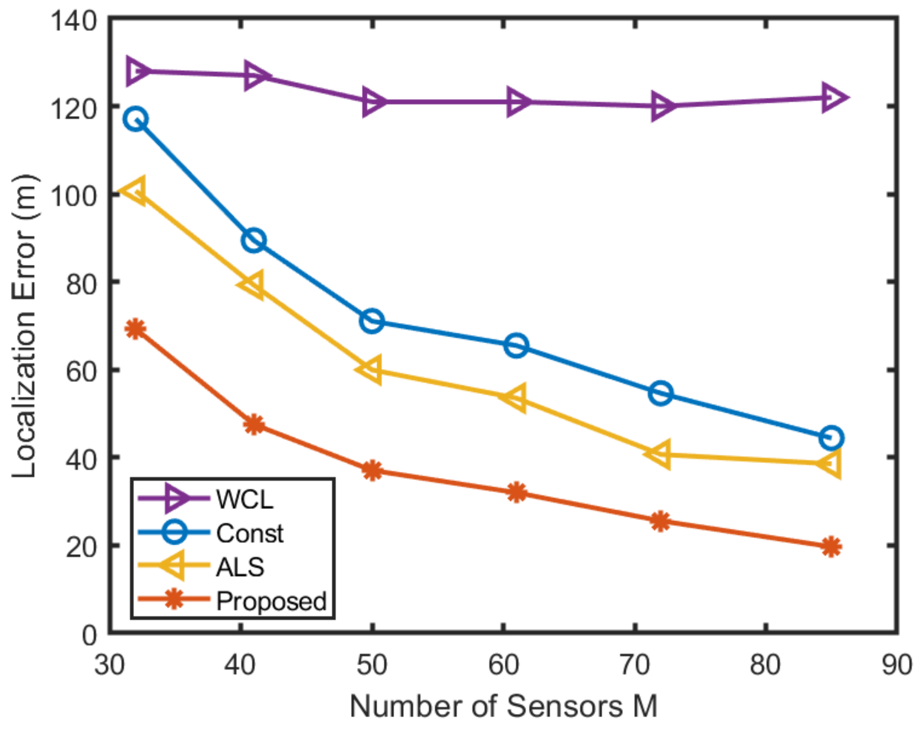


Numerical results:

Baseline: Weighted centroid localization, $\hat{\boldsymbol{s}}_{\mathrm{WCL}} = \sum_{m=1}^{M} w_m \boldsymbol{z}_m / \sum_{m=1}^{M} w_m$, where $w_m = \gamma_m$ serves as the weight.

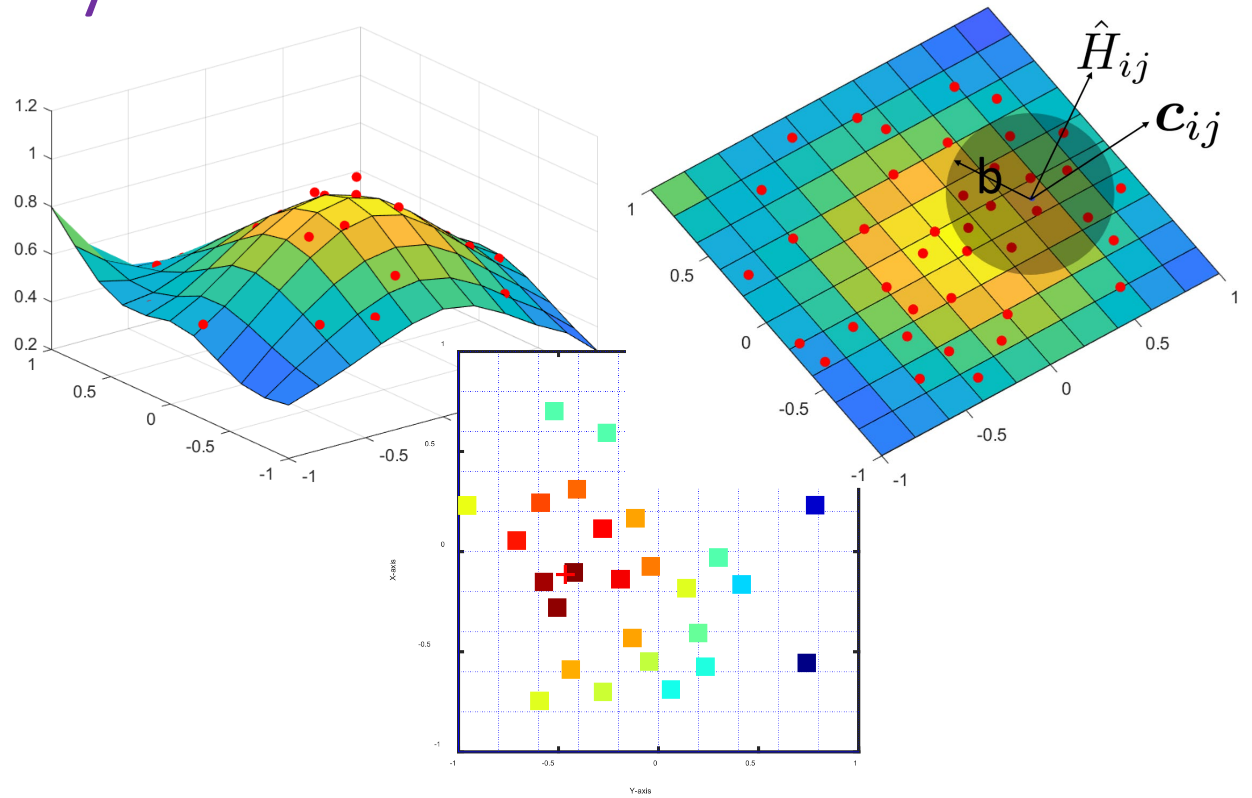
Baseline: Alternating least square method, minimizes $||\boldsymbol{y} - \mathcal{A}(\boldsymbol{X})||$, $\boldsymbol{X} = \boldsymbol{L}\boldsymbol{R}$

Baseline: Constant Uncertainty.
The noise for each observation fixed, i.e., $\bar{\epsilon}_{ij}$ is fixed.



Conclusion: Regression Assisted Matrix Completion improves localization accuracy

- Proposed method improves the matrix completion and localization accuracy.
- Key idea: Quantify the construction error of each matrix grid→ estimate the regression error
- Key techniques:
  - Local polynomial regression for matrix construction
  - Uncertainty construction
  - Matrix completion



香港中文大學(深圳)
The Chinese University of Hong Kong, Shenzhen

Thank you & Questions

haosun1@link.cuhk.edu.cn