

# PARTIAL FACE RECOGNITION: A SPARSE REPRESENTATION-BASED APPROACH

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## Motivation

**Task:** Recognition with partial observations.



(a) Training: holistic data

(b) Testing: partial data

**Problem Statement**

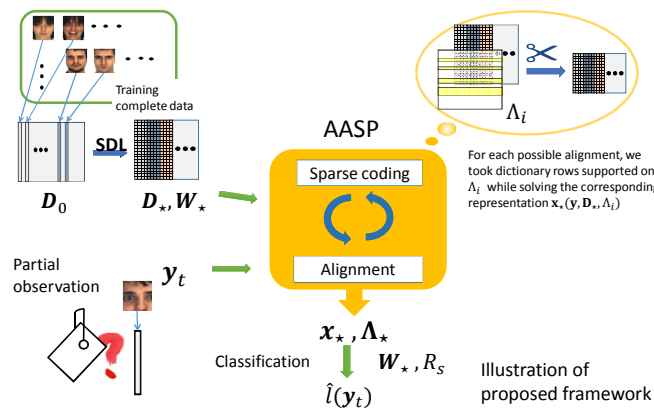
### Challenges:

- Alignment info. is unknown
- The test sample has limited information
- Major distinguished features are missing
- There is No hope for recovery

### Solution:

- Learn a better representation using supervised dictionary learning (SDL) framework [1]
- Propose simultaneous alignment and sparse coding procedure Here: matching with sparse penalty, In [2]: a more complicated method.
- Study the performance limit

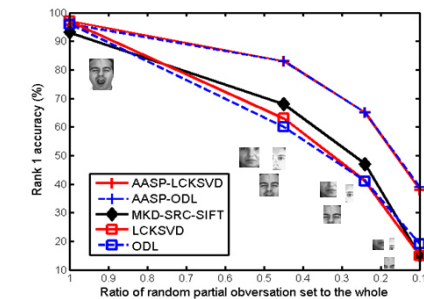
## Illustration



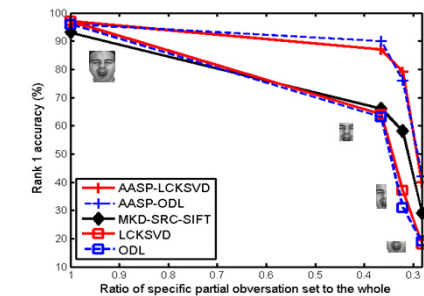
## Experiments

**Dataset:** AR dataset

**Comparisons:** MKD-SRC (feature based [4]); SDL with the same initialization.



Rank 1 accuracy on 3 random partial patterns



Rank 1 accuracy on 3 specific partial patterns

The larger  $\Lambda$  be, the higher classification accuracy obtains!

## Methods

### Supervised Dictionary Learning

Given training data  $(\mathbf{y}_i, l_i) \in \mathbb{R}^m \times \mathbb{R}, i \in T_1$ , find the optimal dictionary  $\mathbf{D}_*$  and classifier parameter  $\mathbf{W}_*$  that yield a good representation & low regression loss  $R_s$

$$(\mathbf{X}_*, \mathbf{D}_*, \mathbf{W}_*) = \arg \min_{\mathbf{x}_i, \mathbf{D}, \mathbf{W}} \sum_{i \in T_1} R_s(l_i, f(\mathbf{x}_i(\mathbf{y}_i, \mathbf{D}), \mathbf{W})) + \beta \|\mathbf{y}_i - \mathbf{D}\mathbf{x}_i\|_2^2 + \rho \text{sparsity}(\mathbf{x}_i), \beta > 0.$$

### Alternating Alignment and Sparse Coding Procedure (AASP)

For test data  $\mathbf{y}_t$ , seek the best alignment  $\Lambda_*$  among the set of all hypothesis  $S$  as well as sparse representation  $\mathbf{x}_*(\mathbf{y}_t, \mathbf{D}_*)$

$$(\mathbf{x}_*(\mathbf{y}_t, \mathbf{D}_*), \Lambda_*) = \arg \min_{\mathbf{x} \in \mathbb{R}^n, \Lambda_i \in S} \frac{1}{2} \|\mathbf{y}_t - (\mathbf{D}_*)_{\Lambda_i} \mathbf{x}\|_2^2 + \rho \text{sparsity}(\mathbf{x}).$$

### Classification:

Find a label among  $L$  that minimizes the regression loss

$$\hat{l}(\mathbf{y}_t) = \arg \min_{l_i \in L} R_s(l_i, f(\mathbf{x}_*(\mathbf{y}_t, \mathbf{D}), \mathbf{W}_*))$$

## Analysis

### Data Model

Assume that  $\exists \mathbf{D}, \mathbf{x}, \|\mathbf{x}\|_0 = s < n$  is sparse such that  $\mathbf{y} = \mathbf{D}\mathbf{x} + \mathbf{z}$  ( $\mathbf{z}$  is some noise process and  $\mathbb{E}(\mathbf{z}^T \mathbf{z}) < \infty$ ) and  $\Lambda$  is the observation set.

• We also assume that  $\mathbf{D}, \Lambda, \mathbf{x}$  can be obtained from some algorithms if unknown.

**Q:** Can classification based on  $\mathbf{y}_\Lambda$  give the same result as on  $\mathbf{y}$  ?

### The noiseless case

$|\Lambda| \geq \text{rank}(\mathbf{D})$  in general cases;  
 $|\Lambda| \geq \mathcal{O}(s \log n)$  the lower bound will become sharper if Compressed Sensing assumptions in [3] is satisfied.

### The noisy case

Lemma: Under squared loss,  $\mathbf{x}_*^p = \arg \min 1/2 \|\mathbf{y}_\Lambda - \mathbf{D}_\Lambda \mathbf{x}\|_2^2 + \rho \|\mathbf{x}\|_1$ , if  $\mathbf{x}_*^p$  also solves  $\min \|\mathbf{y}_\Lambda - \mathbf{D}_\Lambda \mathbf{x}\|_2^2$ , then classification from partially observed data  $\mathbf{y}_\Lambda$  is just as accurate as with complete data.

## Conclusions & References

- Performance limit is analyzed based on exact recovery.
- Our method is robust especially in the case of severe information missing.

### References

- [1] J. Mairal, J. Ponce, G. Sapiro, A. Zisserman, and F. R. Bach, "Supervised dictionary learning," NIPS2009
- [2] X. Sun, N. Nasrabadi, and T. Tran, "Sparse coding with fast image alignment via large displacement optical flow," ICASSP, 2016
- [3] E.J. Candes, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," Info. Theory 2006
- [4] S. Liao and A. K. Jain, "Partial face recognition: An alignment free approach," in Biometrics ICB 2011