RECOVERY OF GRAPH SIGNALS FROM SIGN MEASUREMENTS

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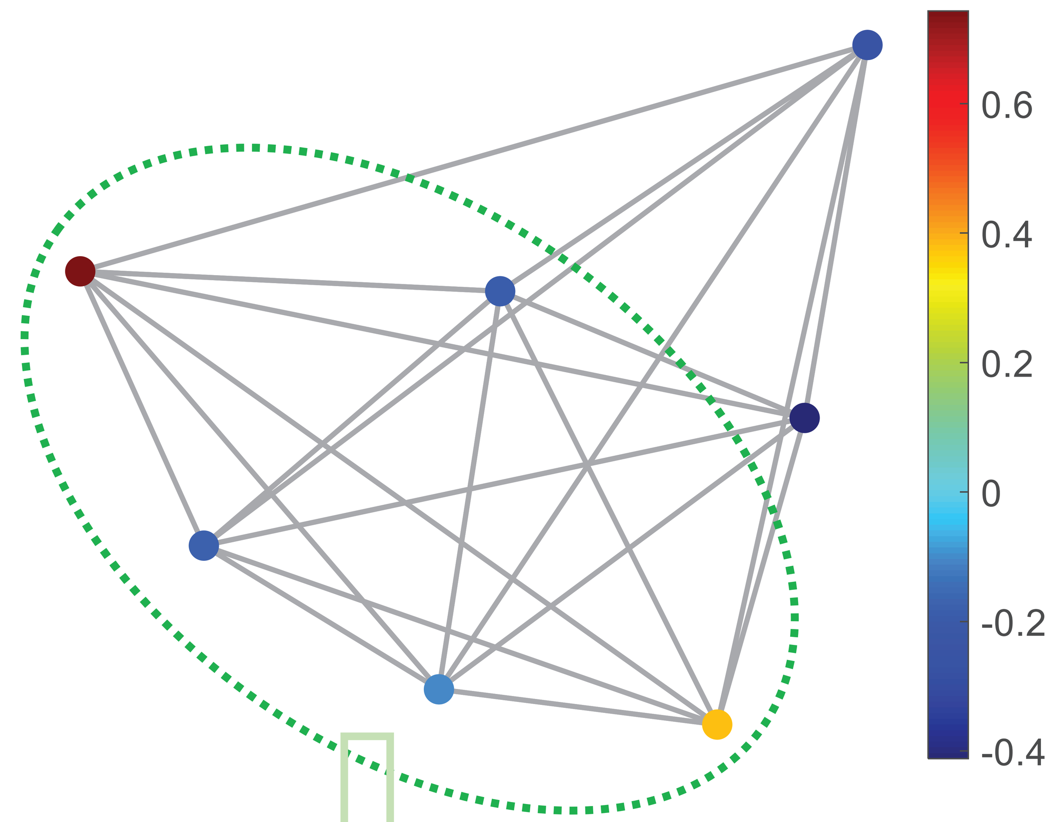
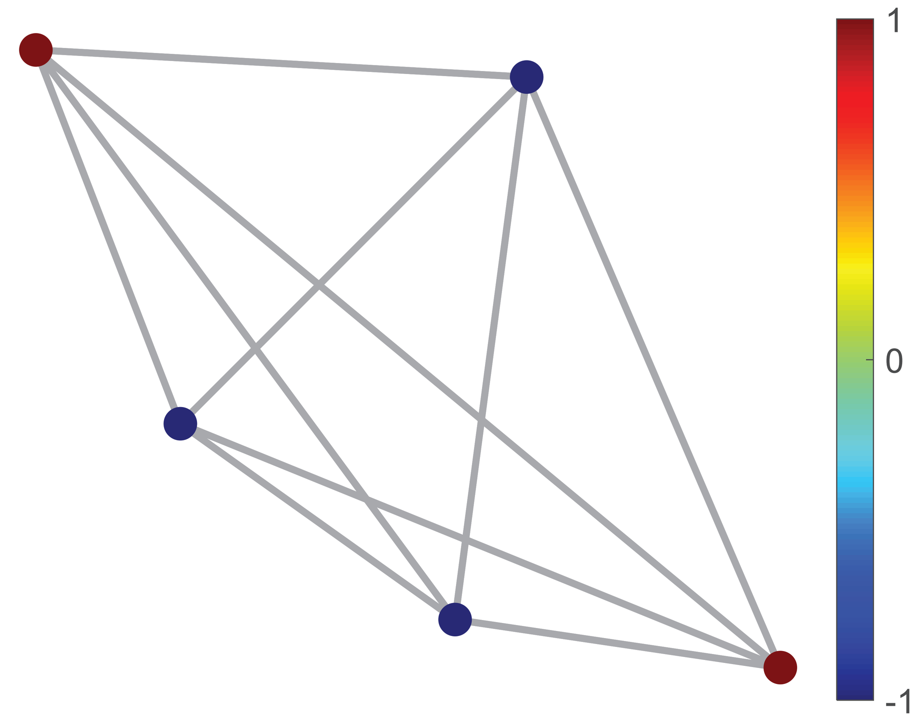
Model

A bandlimited graph signal on $\mathcal{G}$: $\boldsymbol{x}$
(frequency domain: lower bound f_L, upper bound f_U)



Sampling on vertex-domain : $\boldsymbol{\psi}$
+
Sign operator : sign(·)

Sign information of $\boldsymbol{x}$: sign($\boldsymbol{\psi x}$)



Objective:
Find an estimation of $\boldsymbol{x}$ using sign($\boldsymbol{\psi x}$), given the sampling budget.

Tasks:
- Design a recovery algorithm
- Find efficient $\boldsymbol{\psi}$

Recovery

Sign information → Recovery signal: $\boldsymbol{x}^*$

Requirements:
1. Consistency: $\text{sign}(\boldsymbol{\psi x}) = \text{sign}(\boldsymbol{\psi x}^*)$.
2. $\boldsymbol{x}^*$ is bandlimited on $\mathcal{G}$.

Form a constraint space: $\mathcal{C}_v$

Form a constraint space: $\mathcal{C}_b$

Recovery Algorithm

Projecting continuously onto $\mathcal{C}_v$ and $\mathcal{C}_b$ by the following iteration:

$$\boldsymbol{x}_{n+1} = \boldsymbol{P}_b\, \boldsymbol{P}_v\, \boldsymbol{x}_n$$

The operator projecting onto $\mathcal{C}_v$

The operator projecting onto $\mathcal{C}_b$

Convergence Analysis

1. The iterative sequence $\{\boldsymbol{x}_n\}$ converges to some point $\boldsymbol{x}^*$ in the feasible region $\mathcal{C}_v \cap \mathcal{C}_b$.
2. The convergence rate is independent of the selection of the initial point $\boldsymbol{x}_0$.

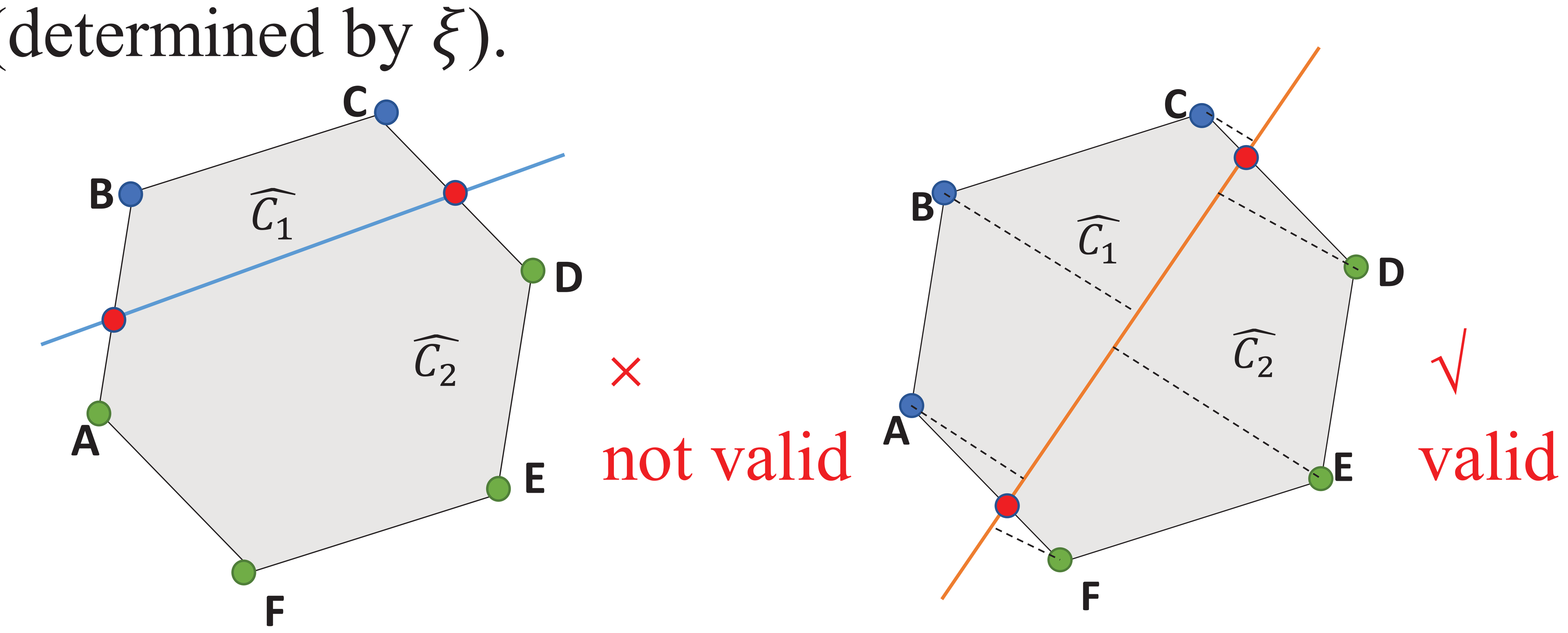
Sampling

Different $\boldsymbol{\psi}$ → Different $\mathcal{C}_v$
Smaller $\mathcal{C}_v \cap \mathcal{C}_b$ → More accurate recovery

Select the vertices that minimize the feasible region to form the sampling set

Greedy Sampling

1. Check whether an unsampled vertex ξ is valid.
e.g. Vectors A and D contribute the most to the size of the feasible region.
Valid: A and D lie on opposite sides of the hyperplane (determined by ξ).

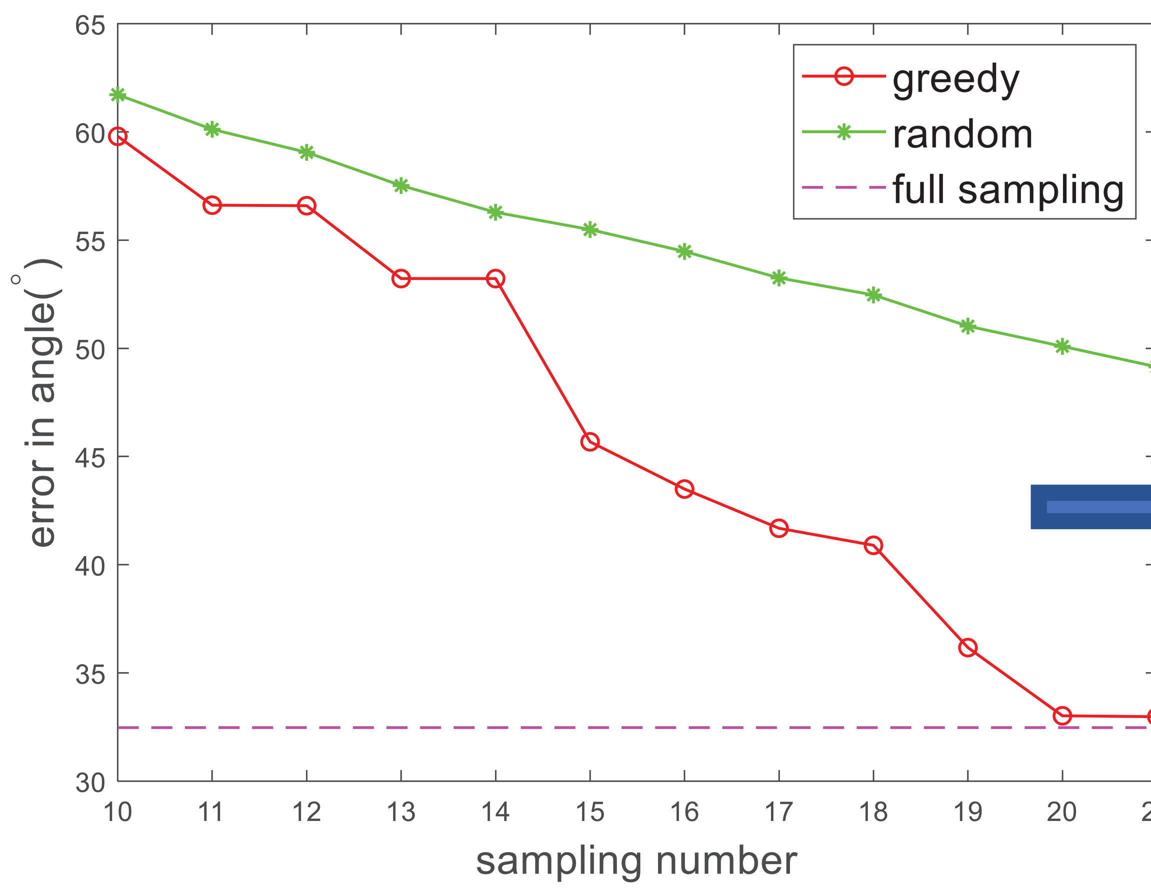


2. Find the vertex that most closely separates the feasible region in half among all the valid vertices.

Experiments

Performance on a sensor graph

40 vertices, 153 edges, $f_L = 29$, $f_U = 35$
Recovery error: $\arccos\langle \boldsymbol{x}, \boldsymbol{x}^* \rangle$ ($\|\boldsymbol{x}\| = \|\boldsymbol{x}^*\| = 1$)



Average error for 50 initial signals

Greedy: our proposed algorithm
Random: 50 times random sampling
Full sampling: all the vertices are sampled