ON THE DETECTION OF NON-STATIONARY SIGNALS IN THE MATCHED SIGNAL TRANSFORM DOMAIN

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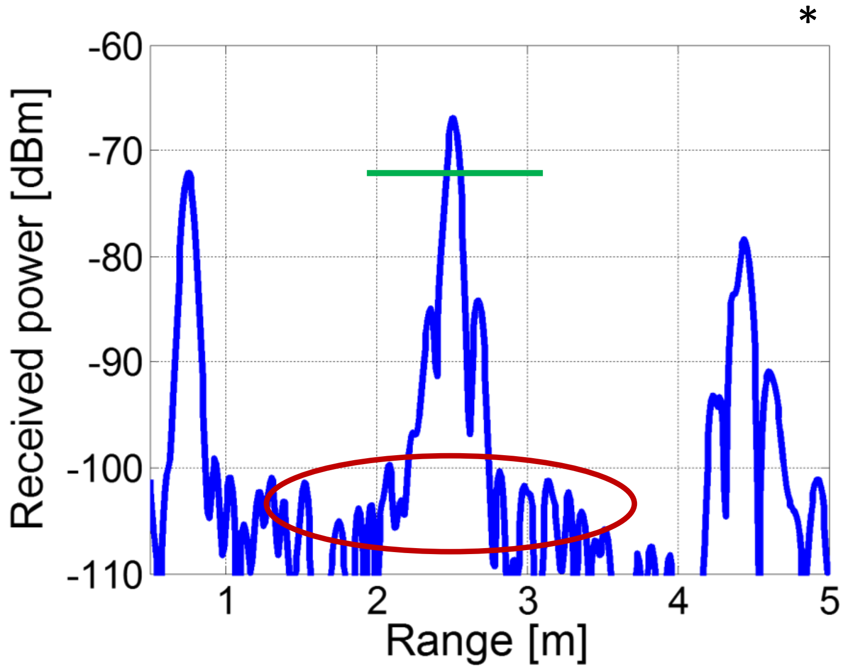
Motivation

Why study a detection problem in the Matched Signal Transform (MST) domain?

The Matched Signal Transform (MST) can be employed to:

- ✓ suppress wide-band time-varying interference,
- ✓ track the Instantaneous Frequency Laws (IFLs) of multi-component non-stationary signals,
- ✓ process nonlinear beat signals provided by a Frequency Modulated Continuous Wave (FMCW) Radar.



- ➢ How can we design a Constant False Alarm Rate (CFAR) detector in the MST domain?
- ➢ What is the probability density function (PDF) of the noise samples in this transformed domain?

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The Matched Signal Transform (MST)

Definition:

$$S(\alpha) = \int_{t\in\mathcal{D}} |\theta'(t)| s(t) e^{-j2\pi\alpha\theta(t)} dt$$

α : modulation rate

$\theta(t)$: characteristic (basis) function

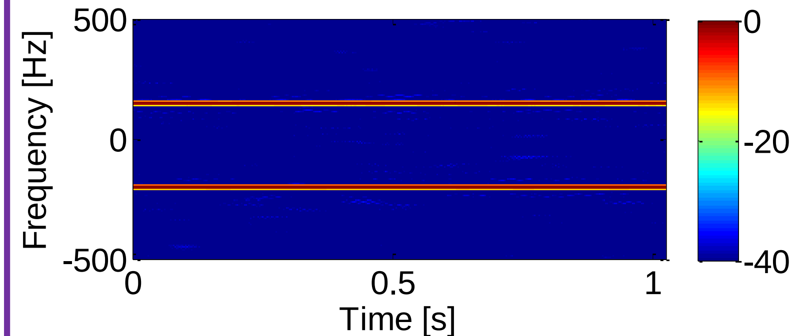
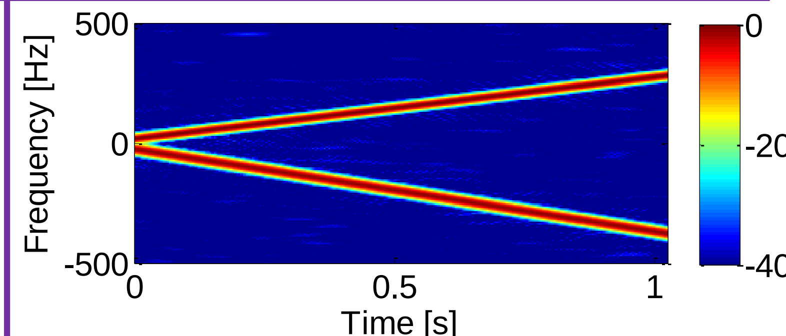
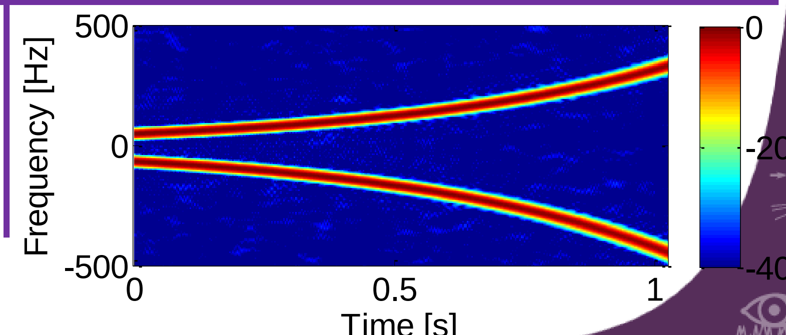
s(t) Time domain → S(α) Modulation rate domain

An essential property for non-stationary signals:

$$s(t) = \sum_{m=1}^{M} A_m e^{j2\pi\alpha_m\theta(t)} \rightarrow S(\alpha) = \sum_{m=1}^{M} A_m \delta(\alpha - \alpha_m)$$

Localizes non-stationary signals described by $\theta(t)$ at their modulation rates.

Classical MSTs

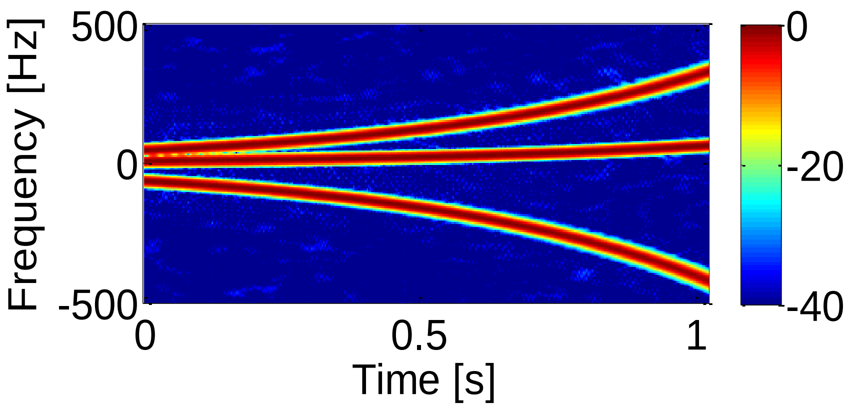
Characteristic function	Transform	Time-Frequency representation of the matched signal
$\theta(t) = t$	Fourier $S(\alpha) = \int_{t\in\mathcal{D}} s(\mathrm{t})e^{-j2\pi\alpha t}dt$	
$\theta(t) = t^2$	Linear MST $S(\alpha) = \int_{t\in\mathcal{D}} \lvert 2t\rvert s(\mathrm{t})e^{-j2\pi\alpha t^2}dt$	
$\theta(t) = e^{kt}$	Exponential $S(\alpha) = \int_{t\in\mathcal{D}} \lvert ke^{kt}\rvert s(\mathrm{t})e^{-j2\pi\alpha e^{kt}}dt$	

MST and Time warping

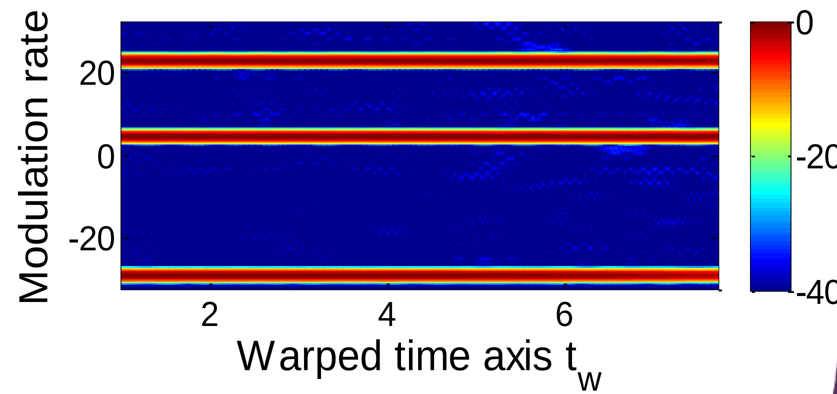
$$s(t) = \sum_{m=1}^{M} A_m e^{j2\pi\alpha_m \theta(t)} \xrightarrow[\text{Warping operator}]{w(t_w) = \theta^{-1}(t_w)} s_{warp}(t_w) = \sum_{m=1}^{M} A_m e^{j2\pi\alpha_m t_w}$$

$$\left\{ \boldsymbol{W} \,\middle|\, \frac{dw}{dt} > 0 \,;\, x(t) \rightarrow \boldsymbol{W}x(t) = x\big(w(t)\big) \right\}$$

All IFLs become simultaneously complex sinusoids (stationary components).



Direct MST



Time warping-based MST
(Fourier transform in the warped time axis)

$$S(\alpha) = \int_{t\in\mathcal{D}} |\theta'(t)| s(t) e^{-j2\pi\alpha\theta(t)} dt$$

$$S(\alpha) = \int_{t_w\in\mathcal{D}_w} s_{warp}(t_w) e^{-j2\pi\alpha t_w} dt_w$$

MST implementations

Analog MSTs

$$S(\alpha) = \int_{t \in \mathcal{D}} |\theta'(t)| \mathrm{s(t)} e^{-j2\pi\alpha\theta(t)} dt$$

$$S(\alpha) = \int_{t_w \in \mathcal{D}_w} s_{warp}(t_w) e^{-j2\pi\alpha t_w} dt_w$$

Discretization

$s[n] = s(t_n), \quad t_n = nT_S, \quad n = \overline{0, N-1}$

$\alpha_k, k = \overline{0, K-1}$

Discrete MSTs

$$S_{MST}[k] = \frac{1}{\Theta} \sum_{n=0}^{N-1} |\theta'(t_n)| s[n] \, e^{-j2\pi\alpha_k\theta(t_n)}$$

$\Theta = \sum_{n=0}^{N-1} |\theta'(t_n)|$: amplitude normalization

Direct MST: summation for each α_k

$$S_{RS}[k] = \frac{1}{N} \sum_{n=0}^{N-1} s_w[n] e^{-j2\pi\alpha_k t_{w,n}}$$

$s_w[n]$: resampled version of $s[n]$ at the moments $t_{w,n} = nT_{S,w}$

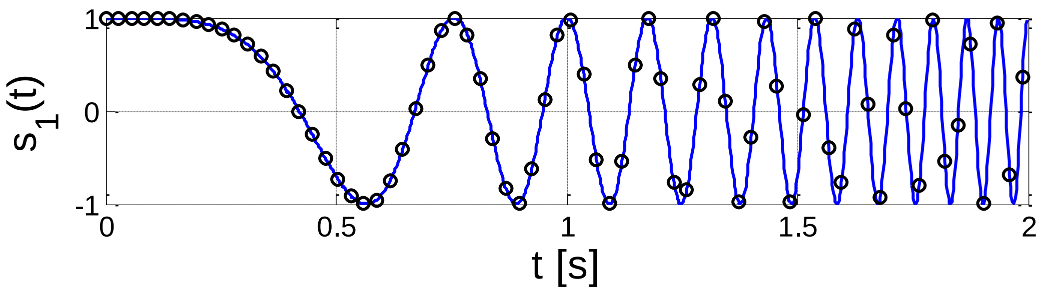
Time Resampling
+
Fast Fourier Transform (FFT)

Time resampling

$$s_1(t) = A_1 e^{j2\pi\alpha_1\theta(t)}$$

e.g.: $\theta(t) = t^2 + t^3$

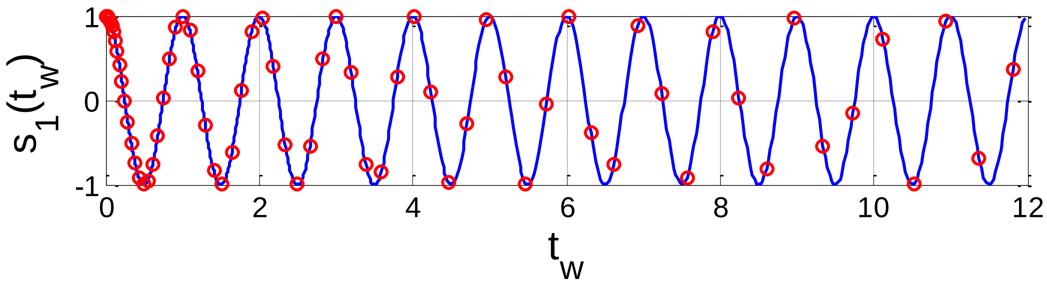
s_1 in the initial time axis (uniform sampling)



$s_1(t)$ $s_1[n], t_n = nT_S$

Warping with $t_w = \theta(t)$

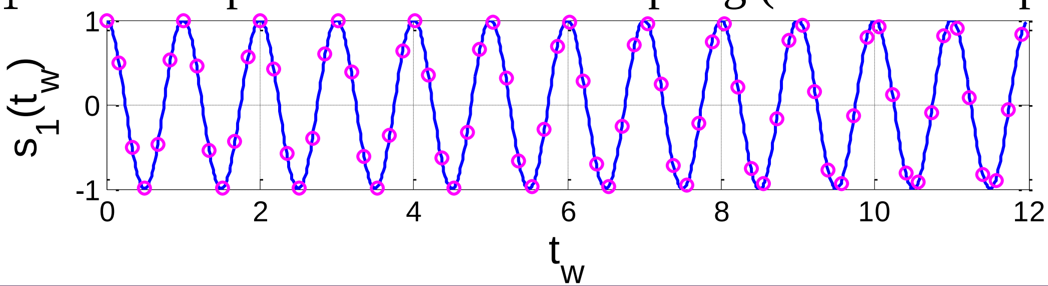
s_1 in the warped time axis (non-uniform sampling)



$s_1(t_w)$ $s_1[n], \theta(t_n)$

Resampling (Interpolation)

s_1 in the warped time axis + resampling (uniform sampling)



$s_1(t_w)$ $s_{w,1}[n], t_{w,n} = nT_{S,w}$

The MST of noisy signals

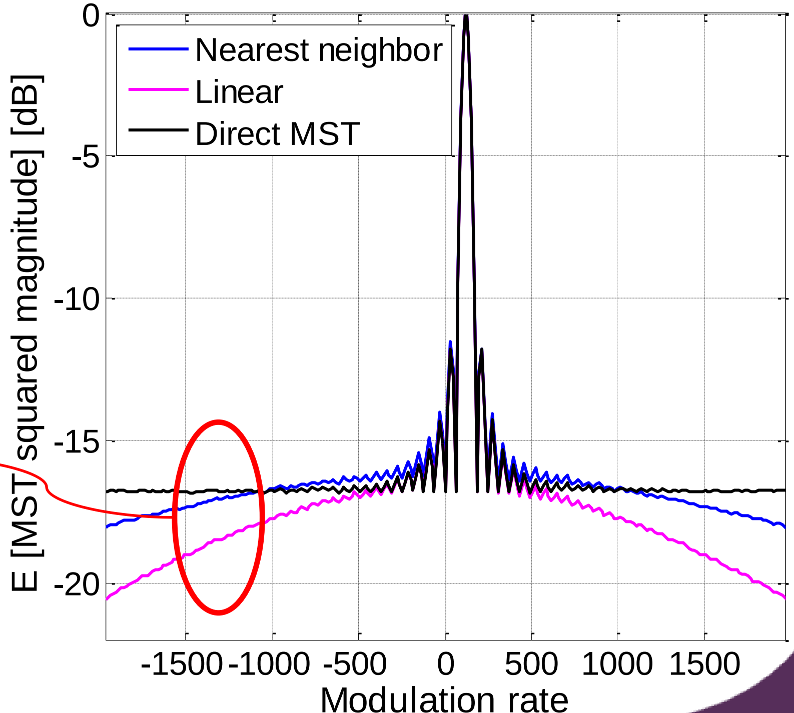
$$x[n] = \sum_{m=1}^{M} A_m e^{j2\pi\alpha_m\theta(nT_S)} + w_R[n] + jw_I[n] = s[n] + w[n]$$

Matched signal + Complex Circular White Gaussian Noise

$$\begin{cases} E\{w_{R,I}[n]\} = 0 \\ E\left\{\left|w_{R,I}[n]\right|^2\right\} = \sigma^2 \end{cases}$$

Expectation of the MST's squared magnitude

- ✓ $E\{|X_{RS}[k]|^2\}$
  - Nearest neighbor interpolation
  - Linear interpolation
- ✓ $E\{|X_{MST}[k]|^2\}$



The noise floor in the MST domain depends on:
Implementation
&
Modulation rate.

The MST of noise samples

$$w[n] = w_R[n] + jw_I[n]$$

Complex Circular White Gaussian Noise

$$\begin{cases} E\{w_{R,I}[n]\} = 0 \\ E\left\{\left|w_{R,I}[n]\right|^2\right\} = \sigma^2 \end{cases}$$

Probability Density Function (PDF)

$$f_1(u) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{u^2}{2\sigma^2}} \longleftrightarrow F_1(v) = e^{-\frac{\sigma^2 v^2}{2}}$$

Characteristic Function (CF)

What is the PDF of the noise samples in the MST domain?

Direct implementation

$$W_{MST}[k] = \frac{1}{\Theta} \sum_{n=0}^{N-1} |\theta'(t_n)| w[n] \, e^{-j2\pi\alpha_k \theta(t_n)}$$

$Re\{W_{MST}[k]\}$ $Im\{W_{MST}[k]\}$

Time resampling implementation

$$W_{RS}[k] = \frac{1}{N} \sum_{n=0}^{N-1} w[n] e^{-j2\pi\alpha_k t_{w,n}}$$

$Re\{W_{RS}[k]\}$ $Im\{W_{RS}[k]\}$

The MST of noise samples
Direct implementation

$$Re\{W_{MST}[k]\} = \sum_{n=0}^{N-1} w_R[n] \frac{1}{\Theta} |\theta'(t_n)| \cos\big(2\pi\alpha_k\theta(t_n)\big) + \sum_{n=0}^{N-1} w_I[n] \frac{1}{\Theta} |\theta'(t_n)| \sin\big(2\pi\alpha_k\theta(t_n)\big)$$

$$Im\{W_{MST}[k]\} = -\sum_{n=0}^{N-1} w_R[n] \frac{1}{\Theta} |\theta'(t_n)| \sin\big(2\pi\alpha_k\theta(t_n)\big) + \sum_{n=0}^{N-1} w_I[n] \frac{1}{\Theta} |\theta'(t_n)| \cos\big(2\pi\alpha_k\theta(t_n)\big)$$

✓ The real and imaginary parts of a sample $W_{MST}[k]$ are a weighted sum of the initial noise samples. The PDF of $Re/Im\{W_{MST}[k]\}$ can be computed using classical results of random variables theory.

❖ If $x_0, x_1, \ldots, x_{N-1}$ are random variables having the CFs $F_{x_0}(v), F_{x_1}(v), \ldots, F_{x_{N-1}}(v)$, then the CF of

$$S = \sum_{n=0}^{N-1} a_n x_n \quad \text{is} \quad F_S(v) = \prod_{n=0}^{N-1} F_{x_n}(a_n v)\,.$$

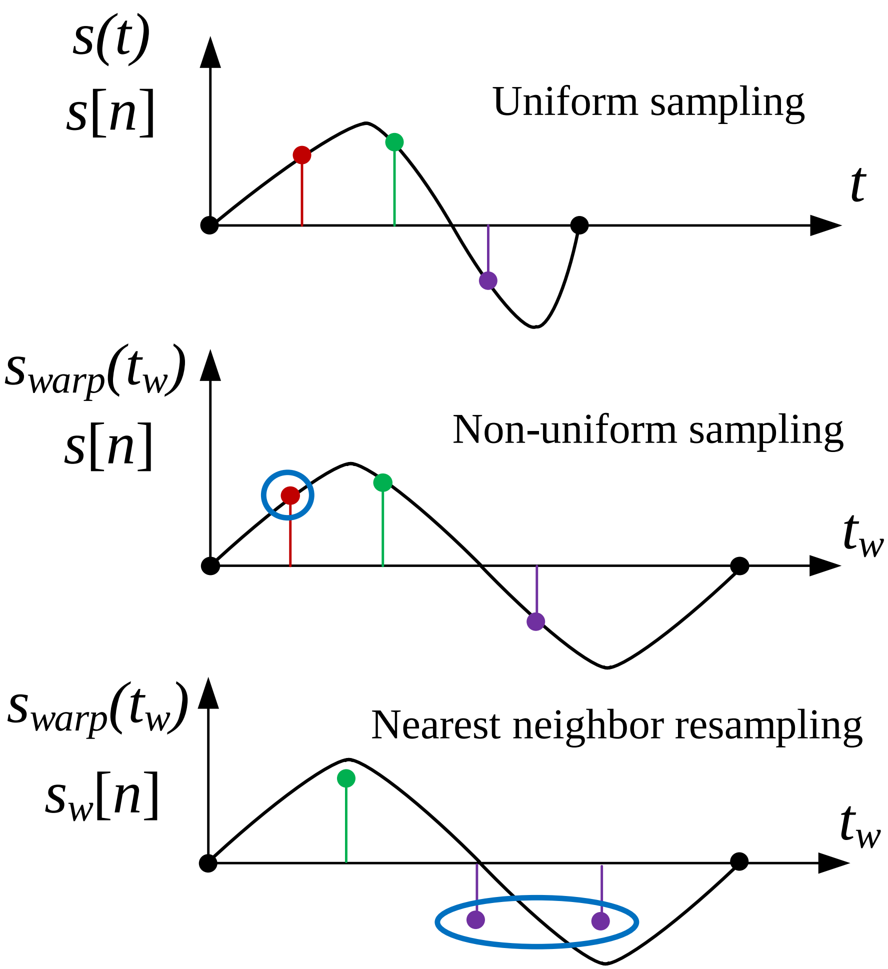
$$F_{Re/Im\{W_{MST}[k]\}}(v,k) = \exp\left\{-\left(\frac{\sigma^2}{\Theta^2}\sum_{n=0}^{N-1}|\theta'(t_n)|^2\right)\frac{v^2}{2}\right\}$$

The CF of a Gaussian noise, independent of α_k

$$\underbrace{\left(\frac{\sigma^2}{\Theta^2}\sum_{n=0}^{N-1}|\theta'(t_n)|^2\right)}_{\sigma_W^2}$$

The MST of noise samples
Resampling-based implementation (1)

Interpolation method: Nearest neighbor



After nearest neighbor resampling:

- Some samples may not appear anymore,
- While others may be repeated.

We can define an index function

$$\beta[n, l]$$

to link N samples from the initial signal to N samples of the resampled one, in the following way:

Sample n from the initial signal appears in the resampled signal at the indices:

$$\beta[n, 1], \beta[n, 2], \ldots, \beta[n, \nu(n)],$$

where $\nu(n)$ is the number of repetitions of sample n.

The MST of noise samples
Resampling-based implementation (2)

Interpolation method: Nearest neighbor

$$Re\{W_{RS}[k]\} = \sum_{n=0}^{N-1} w_R[n] \frac{1}{N} \sum_{l=1}^{\nu(l)} \cos\left(2\pi\alpha_k t_{w,\beta[n,l]}\right) + \sum_{n=0}^{N-1} w_I[n] \frac{1}{N} \sum_{l=1}^{\nu(l)} \sin\left(2\pi\alpha_k t_{w,\beta[n,l]}\right)$$

$$Im\{W_{RS}[k]\} = -\sum_{n=0}^{N-1} w_R[n] \frac{1}{N} \sum_{l=1}^{\nu(l)} \sin\left(2\pi\alpha_k t_{w,\beta[n,l]}\right) + \sum_{n=0}^{N-1} w_I[n] \frac{1}{N} \sum_{l=1}^{\nu(l)} \cos\left(2\pi\alpha_k t_{w,\beta[n,l]}\right)$$

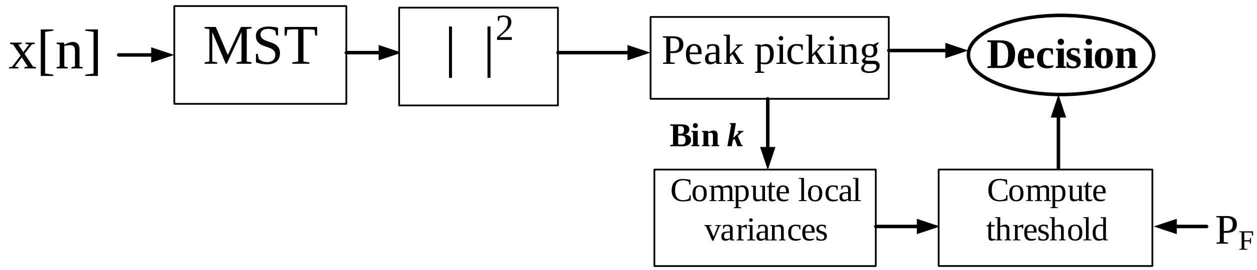
✓ The real and imaginary parts of a sample $W_{RS}[k]$ are a weighted sum of the initial noise samples that takes into account the index function $\beta[n,l]$.

$$F_{Re/Im\{W_{RS}[k]\}}(v,k) = \exp\left\{-\left(\frac{\sigma^2}{N^2}\sum_{n=0}^{N-1}\left(\left(\sum_{l=1}^{\nu(l)}\cos\left(2\pi\alpha_k t_{w,\beta[n,l]}\right)\right)^2 + \left(\frac{1}{N}\sum_{l=1}^{\nu(l)}\sin\left(2\pi\alpha_k t_{w,\beta[n,l]}\right)\right)^2\right)\right)\frac{v^2}{2}\right\}$$

The CF of a Gaussian noise, whose variance $\sigma_W^2[k]$ depends on:

- α_k : the modulation rate,
- $\beta[n,l]$: the actual linking between the initial signal and the resampled one.

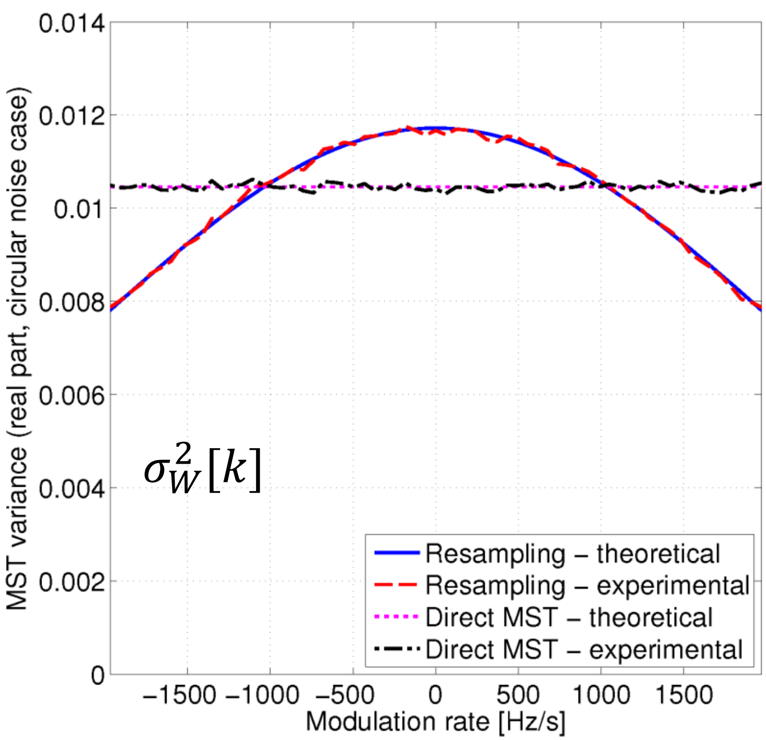
Detection scheme in the MST domain (1)

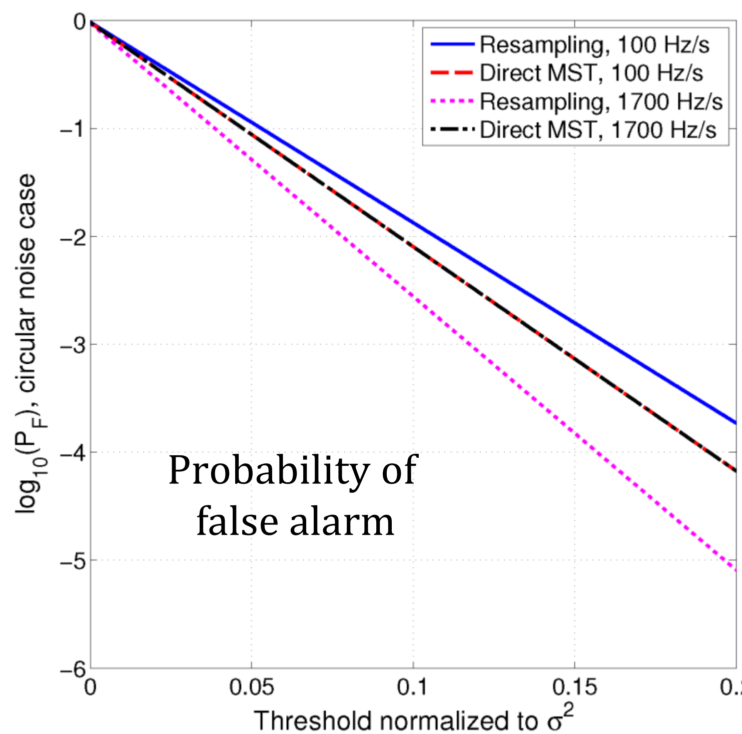


$|W[k]|^2 = Re\{W[k]\}^2 + Im\{W[k]\}^2$

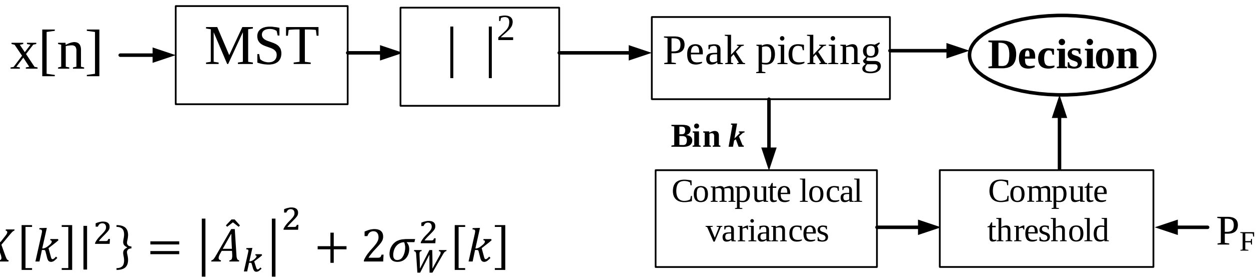
Sum of two squared independent Gaussian variables.

$$PDF_{|W[k]|^2}(u) = \frac{1}{2\sigma_W^2[k]} \exp\left(-\frac{u}{2\sigma_W^2[k]}\right) \text{ and } P_F = \int_{\gamma}^{\infty} PDF_{|W[k]|^2}(u)\, du$$

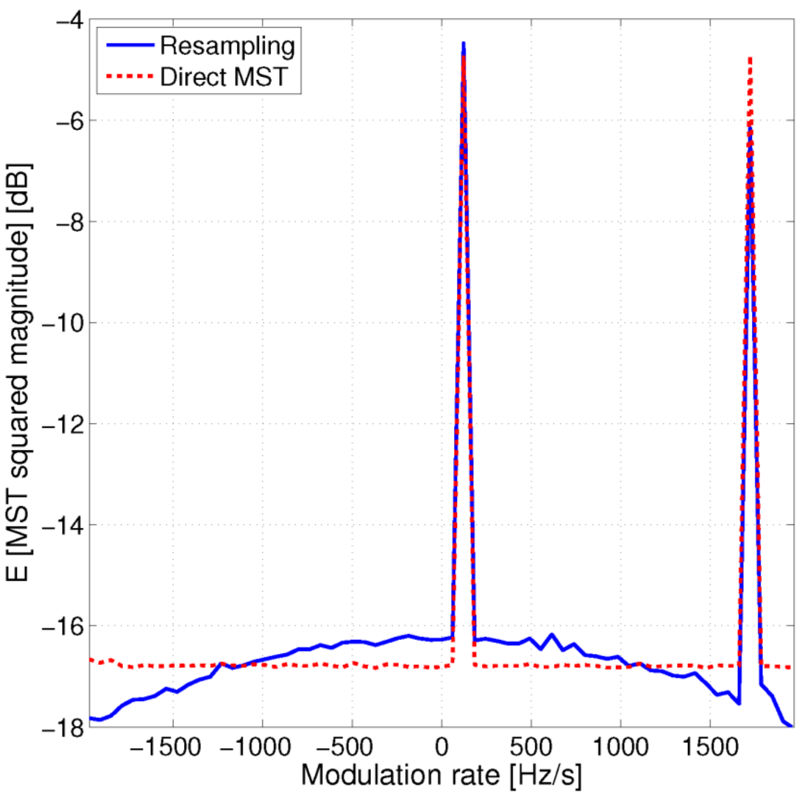


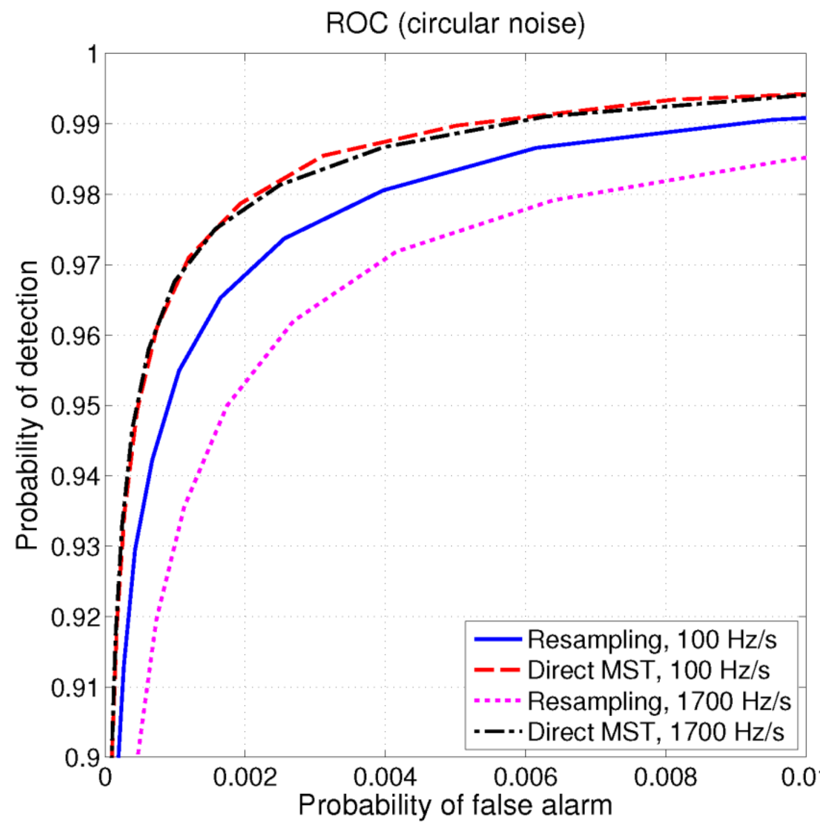


Detection scheme in the MST domain (2)



$$\begin{cases} H_1:\ E\{|X[k]|^2\} = \left|\hat{A}_k\right|^2 + 2\sigma_W^2[k] \\ H_0:\ E\{|X[k]|^2\} = 2\sigma_W^2[k] \end{cases}$$





Conclusions & Perspectives

Contributions:

- We point out the characteristics of white Gaussian noise in the MST domain, and
- Propose a detection scheme for non-stationary signals processed with two implementations of the discrete MST.

In future work:

- The theoretical parameters of the noise in the MST domain will be determined for other interpolation methods.
- The results will be applied to radar and ultrasound applications.

Thanks for your attention !!!