Efficient Two-Stage Beam Training and Channel Estimation for RIS-aided mmWave Systems via Fast Alternating Least Squares

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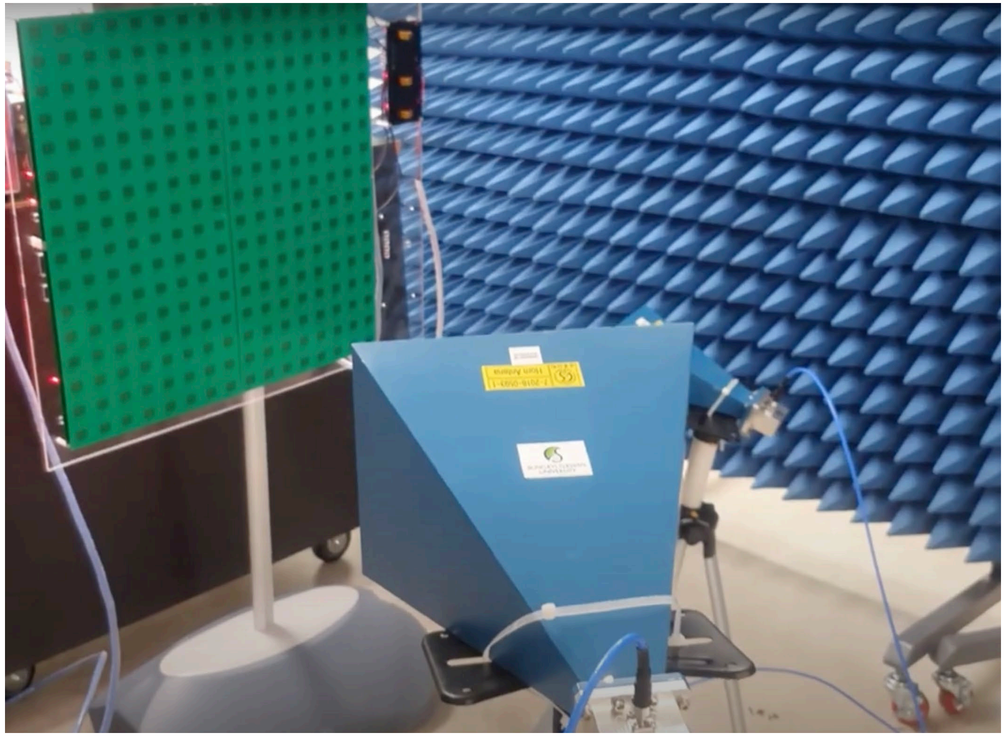
Contents

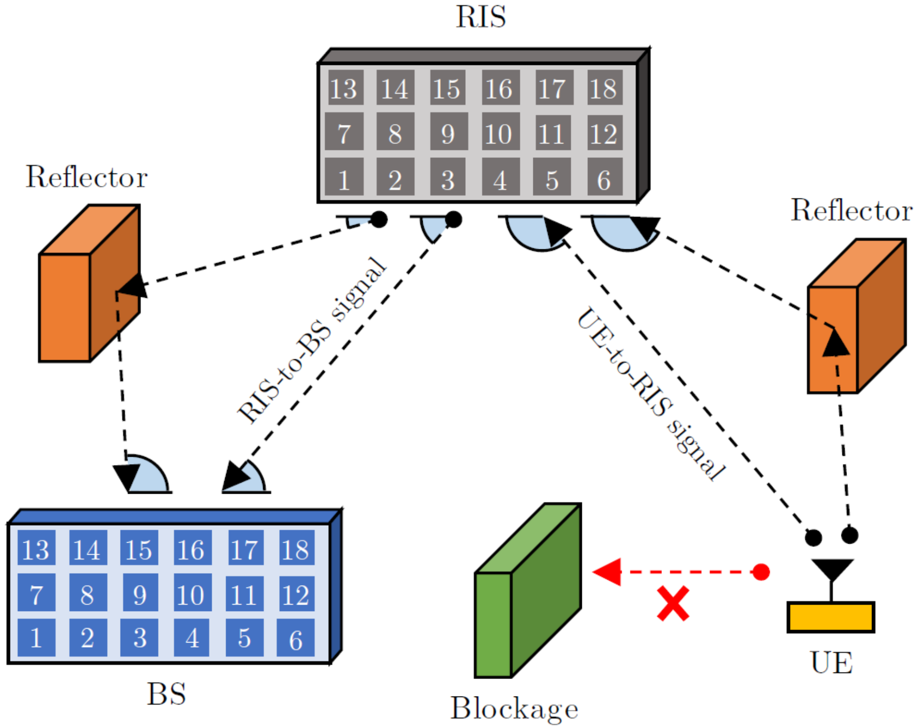
Background & Motivation

RIS (Reconfigurable Intelligent Surface)

- RIS is often called as IRS (Intelligent Reflecting Surface) or LIS (Large Intelligent Surface)
- Each unit element of RIS can programmably change the property of the wireless channel
- In mmWave communications, the RIS can make the signal bypass the blockage by bouncing off the si gnal towards the receiver



<Implementation of RIS*>



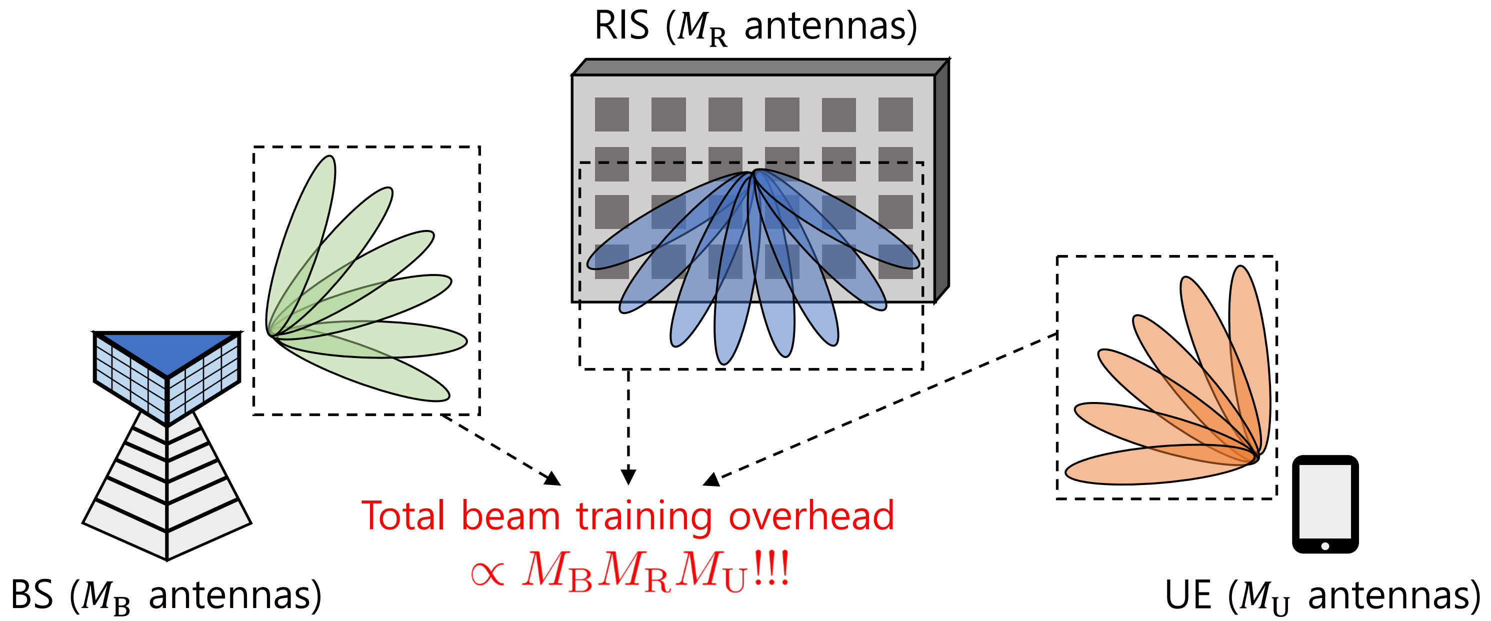
<Bypassing blockage by RIS>

* N. M. Tran et al., "Demo: demonstration of reconfigurable metasurface for wireless communications," IEEE WCNC, 2020.

Background & Motivation

Motivation 1: Excessive Beam Training Overhead

- In RIS-aided systems, RIS also have to perform beam search just like BS and UE. Thus, RIS-aided systems are likely to suffer from excessive beam training overhead



<Beam training in RIS-aided systems>

- Most of RIS channel estimation algorithms have common goal: **Reducing beam training overhead**
- Former works employ techniques such as compressive sensing*, atomic norm minimization (ANM)**, PARAFAC decomposition***

* K. Ardah et al., “TRICE: A channel estimation framework for RIS-aided millimeter-wave MIMO systems,” IEEE SPL, Feb. 2021.
** H. Chung and S. Kim, “Location-aware channel estimation for RIS-aided mmWave MIMO systems via atomic norm minimization”, Arxiv, Jan. 2022.
*** L. Wei et al., "Channel Estimation for RIS-Empowered Multi-User MISO Wireless Communications," IEEE TCOM, 2021.

Motivation 2: We Need to Consider Computational Complexity

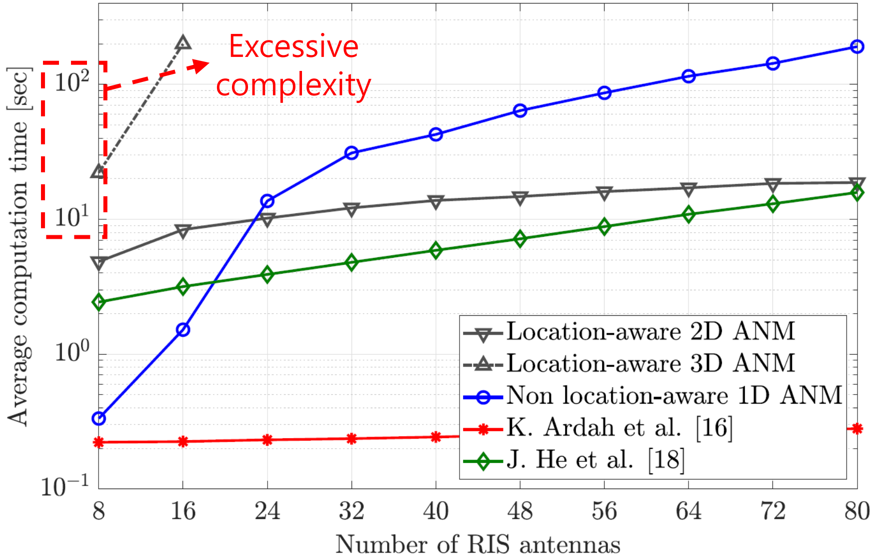
- When we estimate the channel for RIS-aided systems, we normally estimate cascaded effective channel $\mathbf{H}_{\mathrm{eff}}$

$$\mathbf{H}_{\mathrm{eff}} = \mathbf{H}_{\mathrm{RB}}\mathrm{diag}\left(\mathbf{h}_{\mathrm{UR}}\right) \in \mathbb{C}^{M_{\mathrm{B}} \times M_{\mathrm{R}}}$$

$\mathbf{H}_{\mathrm{RB}} \in \mathbb{C}^{M_{\mathrm{B}} \times M_{\mathrm{R}}}$: RIS-to-BS channel

$\mathbf{h}_{\mathrm{UR}} \in \mathbb{C}^{M_{\mathrm{R}} \times 1}$: UE-to-RIS channel

- However, the computing time becomes critical issue since dimension of $\mathbf{H}_{\mathrm{eff}}$ is large
- Techniques such as ANM* and PARAFAC** decomposition can achieve high accuracy, but also has high complexity → **We need to consider both accuracy and complexity**



<Complexity Analysis>

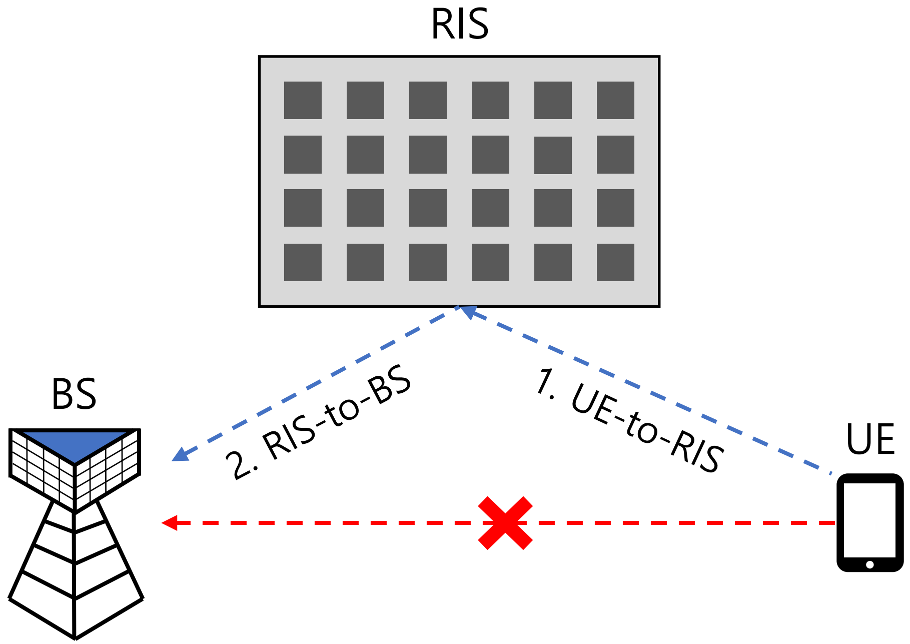
ANM: $\mathcal{O}\left(M_{\mathrm{R}}^4\left(M_{\mathrm{B}}+M_{\mathrm{R}}\right)^{2.5}\right)$

PARAFAC: $\mathcal{O}\left(\kappa\left(8M_{\mathrm{R}}^2 BM_{\mathrm{B}} + 2M_{\mathrm{R}}^3 - 2M_{\mathrm{R}}M_{\mathrm{B}}\right)\right)$

* H. Chung and S. Kim, "Location-aware channel estimation for RIS-aided mmWave MIMO systems via atomic norm minimization", Arxiv, Jan. 2022.
** L. Wei et al., "Channel Estimation for RIS-Empowered Multi-User MISO Wireless Communications," IEEE TCOM, 2021.

Basic Channel Model for RIS-aided Systems

- ❑ **UE-to-RIS channel and RIS-to-BS channel are cascaded in uplink scenario**
- ❑ **UE-to-RIS channel $\mathbf{h}_{\mathrm{UR}}$ and RIS-to-BS channel $\mathbf{H}_{\mathrm{RB}}$ are given by follows**



$$\mathbf{h}_{\mathrm{UR}} = \sum_{l=1}^{L_{\mathrm{UR}}} \alpha_{\mathrm{UR}}^{l} \mathbf{a}(\phi_{\mathrm{UR}}^{\mathrm{Rx},l}, \theta_{\mathrm{UR}}^{\mathrm{Rx},l}) \in \mathbb{C}^{M_{\mathrm{R}} \times 1}$$

$$\mathbf{H}_{\mathrm{RB}} = \sum_{l=1}^{L_{\mathrm{RB}}} \alpha_{\mathrm{RB}}^{l} \mathbf{a}(\phi_{\mathrm{RB}}^{\mathrm{Rx},l}, \theta_{\mathrm{RB}}^{\mathrm{Rx},l}) \mathbf{a}(\phi_{\mathrm{RB}}^{\mathrm{Tx},l}, \theta_{\mathrm{RB}}^{\mathrm{Tx},l})^{H} \in \mathbb{C}^{M_{\mathrm{B}} \times M_{\mathrm{R}}}$$

L_{UR} : # of signal paths between UE and RIS

L_{RB} : # of signal paths between RIS and BS

$M_{\mathrm{B}}, M_{\mathrm{R}}$: # of antennas in BS and RIS

- ❑ **Cascaded effective channel $\mathbf{H}_{\mathrm{eff}}$ is defined as follows**

$$\mathbf{H}_{\mathrm{eff}} = \mathbf{H}_{\mathrm{RB}} \mathrm{diag}\left(\mathbf{h}_{\mathrm{UR}}\right) \in \mathbb{C}^{M_{\mathrm{B}} \times M_{\mathrm{R}}}$$

System Model & Two-stage Beam Training

RIS Control Matrix & Received Pilot Signal

- RIS control matrix Ω is given by

$$\boldsymbol{\Omega} = \begin{bmatrix} \beta_1 e^{j\vartheta_1} & 0 & \cdots & 0 \\ 0 & \beta_2 e^{j\vartheta_2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \beta_{M_\mathrm{R}} e^{j\vartheta_{M_\mathrm{R}}} \end{bmatrix} \in \mathbb{C}^{M_\mathrm{R} \times M_\mathrm{R}}$$

β_m : RIS reflection coefficient of the m-th RIS antenna. This is 0 when it is deactivated and 1 when it is activated

ϑ_m : Phase shift at the m-th RIS antenna

- RIS control vector ω that simplifies Ω can be given by

$$\boldsymbol{\omega} = \left[\beta_1 e^{j\vartheta_1}, \beta_2 e^{j\vartheta_2}, \ldots, \beta_{M_\mathrm{R}} e^{j\vartheta_{M_\mathrm{R}}}\right]^T \in \mathbb{C}^{M_\mathrm{R} \times 1} \qquad \boldsymbol{\Omega} = \mathrm{diag}\left(\boldsymbol{\omega}\right)$$

- Received pilot received during beam training $\mathbf{Y}$ can be given by

$$\mathbf{Y} = P_\mathrm{Tx} \mathbf{C}^H \mathbf{H}_\mathrm{eff} \mathbf{W} + \mathbf{V} \in \mathbb{C}^{M_\mathrm{B} \times B}$$

P_Tx : Transmission power of UE

$\mathbf{C}$: Combining matrices (= Receive beamforming matrices)

$\mathbf{W}$: A set of RIS control vectors

B : Number of RIS control vectors

Two-stage Beam Training – (1)

- During beam training, we observe part of $\mathbf{H}_{\mathrm{eff}}$, and the unobserved entries are recovered via low rank matrix completion (LRMC)
- However, the unobserved entries cannot be recovered if we obey the rule
- If the partly observed matrix misses some columns or rows, unobserved entries cannot be recovered

$$\text{rank}\,(\mathbf{X}) = 1\;,\;\; \mathbf{X} = \begin{bmatrix} 1 & 2 & 1 & ? \\ 2 & 4 & 2 & ? \\ 3 & 6 & 3 & ? \\ 4 & 8 & 4 & ? \end{bmatrix}$$

Unable to recover

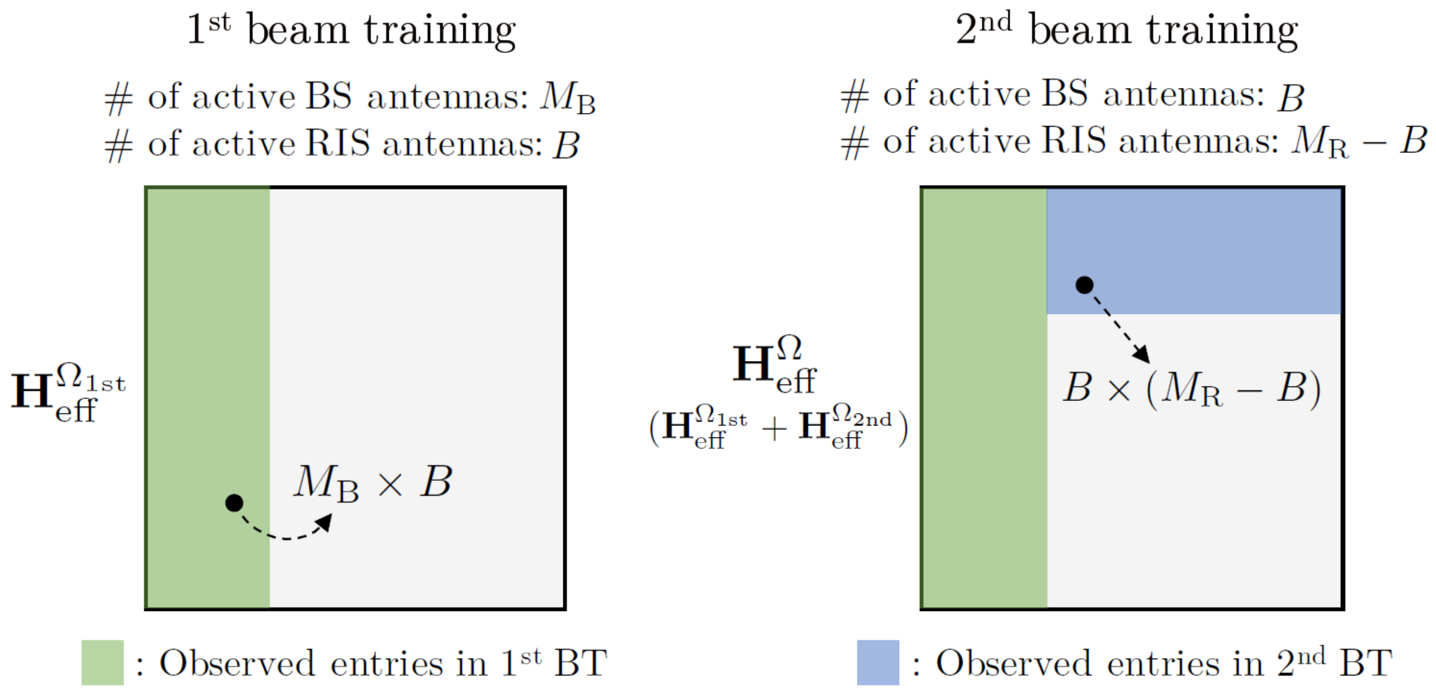
- If there is no missing column or row, then unobserved entries can be recovered

$$\text{rank}\,(\mathbf{X}) = 1\;,\;\; \mathbf{X} = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & ? & ? & ? \\ 3 & ? & ? & ? \\ 4 & ? & ? & ? \end{bmatrix} \Rightarrow \hat{\mathbf{X}} = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 4 & 2 & 6 \\ 3 & 6 & 3 & 9 \\ 4 & 8 & 4 & 12 \end{bmatrix}$$

- Thus, **proposed two-stage beam training aims to observe columns and rows of $\mathbf{H}_{\mathrm{eff}}$**

System Model & Two-stage Beam Training

Two-stage Beam Training – (2)



<Partly observed H_{eff} via two-stage beam training. B columns and B rows are observed>

- **Two-stage beam training can be summarized as follows**
  - Stage 1: BS activates its all array, and RIS only activates its B antennas. Then, B columns of $\mathbf{H}_{\text{eff}}$ can be observed
  - Stage 2: BS activates B antennas, and RIS activates M_{R}-B antennas that were deactivated at the first stage. Then, B rows of $\mathbf{H}_{\text{eff}}$ can be observed
- **Unobserved entries are recovered via LRMC. Details of two-stage beam training can be found in Appendix**

Channel Estimation via FALS

Fast Alternating Least Squares (FALS)

- FALS is one of LRMC techniques, which is proven to be accurate and efficient*
- Since FALS requires the rank of the matrix, we first estimate the rank of $\mathbf{H}_{\text{eff}}$ by counting large singular values of $\mathbf{H}_{\text{eff}}^{\Omega_{1\text{st}}}$
- FALS estimates the column space and the row space of $\mathbf{H}_{\text{eff}}$
- Notations for FALS-based estimation can be given by follows

$\mathbf{H}_{\text{eff}}^{\Omega}$: Partly observed channel

$\boldsymbol{\Omega}, \bar{\boldsymbol{\Omega}}$: Matrix that denotes observed and unobserved entries

$\mathbf{A}_{(i)} \in \mathbb{C}^{M_{\text{B}} \times R}$: Column space at the i-th iteration

$\mathbf{B}_{(i)} \in \mathbb{C}^{M_{\text{R}} \times R}$: Row space at the i-th iteration

$\circ$: Element-wise product

$L_{(i)}$: Objective function at the i-th iteration

$$L_{(i)} = \frac{1}{2}\|\mathbf{H}_{\text{eff}}^{\Omega} - \boldsymbol{\Omega} \circ (\mathbf{A}_{(i)}\mathbf{B}_{(i)}^{H})\|_{\text{F}}^2 + \frac{\lambda}{2}\left(\|\mathbf{A}_{(i)}\|_{\text{F}}^2 + \|\mathbf{B}_{(i)}\|_{\text{F}}^2\right)$$

ϵ : Threshold for convergence

* T. Hastie, R. Mazumder, J. D. Lee, and R. Zadeh, “Matrix completion and low-rank SVD via fast alternating least squares,” J. Mach. Learn. Res., vol. 16, pp. 3367–3402, Dec. 2015.

Channel Estimation via FALS

Procedure of FALS

Algorithm 1: Channel Estimation via FALS

Input : $\mathbf{A}_{(0)} \in \mathbb{C}^{M_\mathrm{B} \times R}$, $\mathbf{B}_{(0)} \in \mathbb{C}^{M_\mathrm{R} \times R}$, $\mathbf{\Omega}$, $\bar{\mathbf{\Omega}}$, λ, ϵ

Output: $\hat{\mathbf{H}}_{\text{eff}}$

$L_{(0)} \leftarrow \frac{1}{2}\|\mathbf{H}_{\text{eff}}^{\Omega} - \mathbf{\Omega} \circ (\mathbf{A}_{(0)}\mathbf{B}_{(0)}^H)\|_\mathrm{F}^2 + \frac{\lambda}{2}\left(\|\mathbf{A}_{(0)}\|_\mathrm{F}^2 + \|\mathbf{B}_{(0)}\|_\mathrm{F}^2\right);$

$i \leftarrow 0;$

while $L_{(i-1)} - L_{(i)} < \epsilon$ **do** ③

$\quad \mathbf{S} \leftarrow \mathbf{H}_{\text{eff}}^{\Omega} + \bar{\mathbf{\Omega}} \circ \left(\mathbf{A}_{(i)}\mathbf{B}_{(i)}^H\right);$

$\quad \mathbf{A}_{(i+1)} \leftarrow \mathbf{S}\mathbf{B}_{(i)}\left(\mathbf{B}_{(i)}^H\mathbf{B}_{(i)} + \lambda\mathbf{I}_R\right)^{-1};$ ①

$\quad \mathbf{T} \leftarrow \mathbf{H}_{\text{eff}}^{\Omega} + \bar{\mathbf{\Omega}} \circ \left(\mathbf{A}_{(i+1)}\mathbf{B}_{(i)}^H\right);$

$\quad \mathbf{B}_{(i+1)} \leftarrow \mathbf{T}\mathbf{A}_{(i+1)}\left(\mathbf{A}_{(i+1)}^H\mathbf{A}_{(i+1)} + \lambda\mathbf{I}_R\right)^{-1};$ ②

$\quad i \leftarrow i + 1;$

$\quad L_{(i)} \leftarrow \frac{1}{2}\|\mathbf{H}_{\text{eff}}^{\Omega} - \mathbf{\Omega} \circ (\mathbf{A}_{(i)}\mathbf{B}_{(i)}^H)\|_\mathrm{F}^2 + \frac{\lambda}{2}\left(\|\mathbf{A}_{(i)}\|_\mathrm{F}^2 + \|\mathbf{B}_{(i)}\|_\mathrm{F}^2\right);$

end

$\hat{\mathbf{H}}_{\text{eff}} \leftarrow \mathbf{H}_{\text{eff}}^{\Omega} + \bar{\mathbf{\Omega}} \circ \left(\mathbf{A}_{(i)}\mathbf{B}_{(i)}^H\right);$ ④

<Psuedocode of FALS-based channel estimation>

① **While fixing row space (B), find column space (A) that minimizes the objective function**

② **While fixing column space (A), find row space (B) that minimizes the objective function**

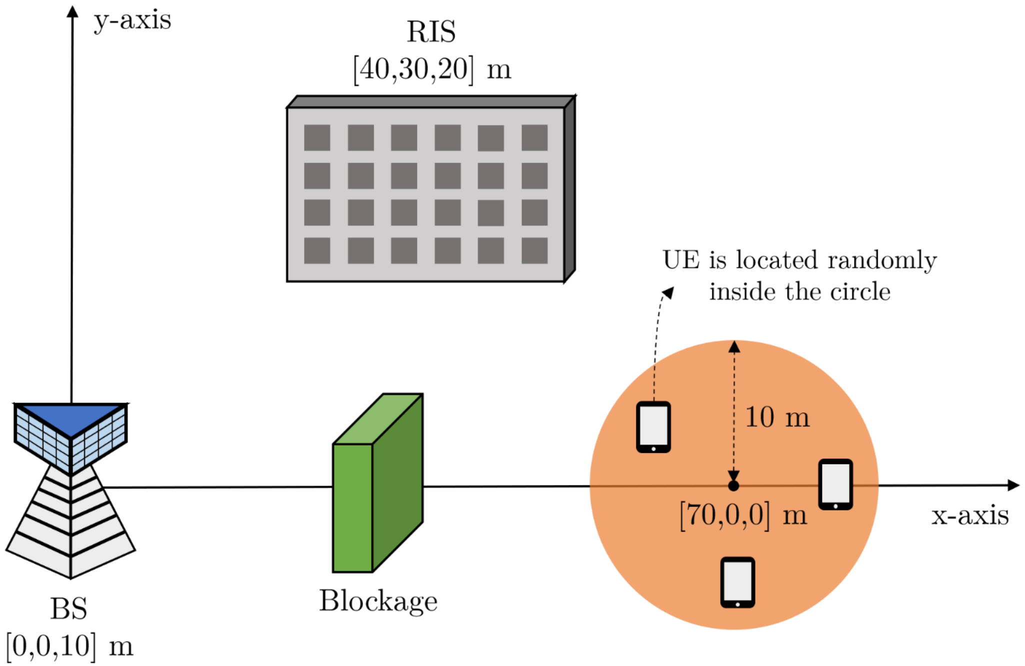
③ **Determine the convergence by comparing the variation of the objective function with the preset threshold**

④ **Fill the unobserved entires with estimated column space and row space**

Simulation Results & Analysis

Simulation Environment

- ❑ **Position of BS, RIS, and UE are set as figure below**
- ❑ **Simulation parameters are summarized in the table. To evaluate the channel estimation accuracy, we calculate normalized mean squared error (NMSE)**



<Position of BS, RIS, and UE>

Parameter setting

Parameter	Value	Parameter	Value
$M_{\mathrm{B}}, M_{\mathrm{R}}$	64 (8 x 8 UPA)	Tx power	20 dBm
Noise power	-100 dBm	Monte Carlo trials	500

<Definition of NMSE>

$$\mathrm{NMSE} = \frac{1}{Q}\sum_{q=1}^{Q}\frac{\|\hat{\mathbf{H}}_{\mathrm{eff}}^{q} - \mathbf{H}_{\mathrm{eff}}^{q}\|_{\mathrm{F}}^{2}}{\|\mathbf{H}_{\mathrm{eff}}^{q}\|_{\mathrm{F}}^{2}}$$

$\hat{\mathbf{H}}_{\mathrm{eff}}^{q}$: Estimated cascaded effective channel at q-th trial

$\mathbf{H}_{\mathrm{eff}}^{q}$: Actual cascaded effective channel at q-th trial

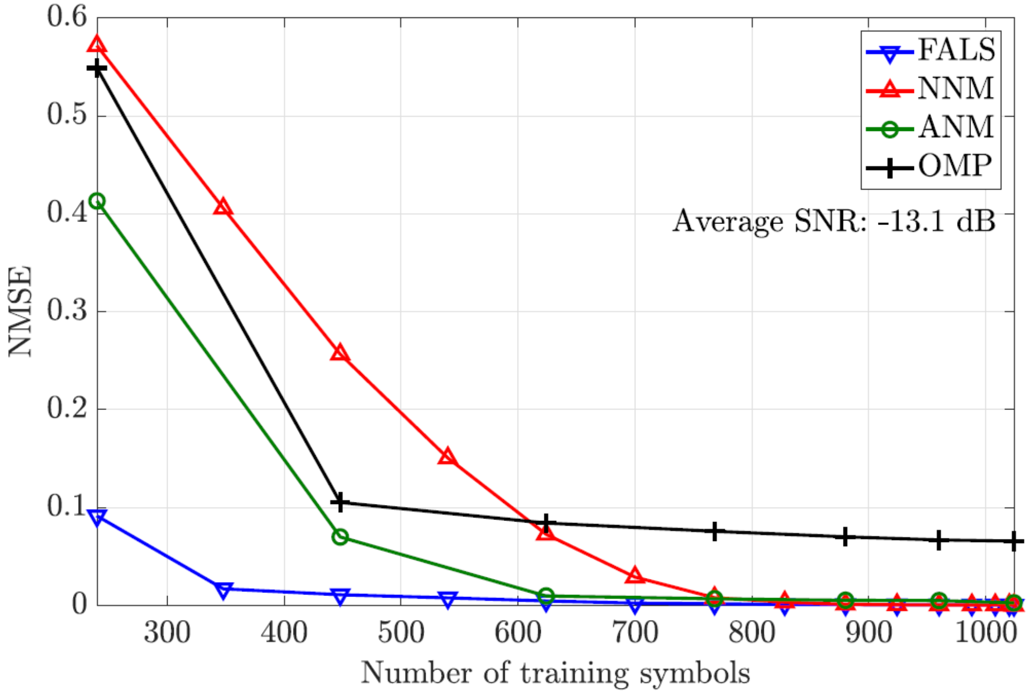
- ❑ **UE-to-RIS and RIS-to-BS channels are modelled based on statistical 28 GHz channel model***

* M. K. Samimi and T. S. Rappaport, “3-D millimeter-wave statistical channel model for 5G wireless system design,” IEEE TMTT, 2016.

Simulation Results & Analysis

NMSE vs Number of Training Symbols, Complexity Analysis

- Beam training overhead is proportional to number of training symbols, which can be defined as $B(M_B+M_R-B)/N_B$. Here, N_B denotes the number of RF chains at BS
- NMSE when spending less overhead: Proposed << ANM < OMP < Nuclear norm minimization (NNM)
- Computational complexity: OMP < Proposed << NNM << ANM



<NMSE versus number of training symbols>

Algorithm	Computational complexity	Elapsed time
Proposed	$\mathcal{O}\left(\eta\left(M_B M_R + R\left(M_B + M_R\right) + R^2\right)\right)$	0.27 secs
NNM*	$\mathcal{O}\left(\left(M_B + M_R\right)^{3.5}\right)$	7.3 secs
ANM**	$\mathcal{O}\left(M_R^4\left(M_B + M_R\right)^{2.5}\right)$	277 secs
OMP***	$\mathcal{O}\left(R M_B B G\right)$	0.14 secs

η : Iterations for FALS-based channel estimation

G : Size of discretized grid in OMP

$$G > \eta \gg M_B, M_R > B \gg R$$

* E. J. Candes and B. Recht, "Exact matrix completion via convex optimization," Found. Comput. Math., vol. 9, pp. 717–772, Apr. 2009.
** H. Chung and S. Kim, "Location-aware channel estimation for RIS-aided mmWave MIMO systems via atomic norm minimization", Arxiv, Jan. 2022.
*** K. Ardah et al., "TRICE: A channel estimation framework for RIS-aided millimeter-wave MIMO systems," IEEE SPL, Feb. 2021.

Conclusions

- **We proposed two-stage beam training and FALS-based channel estimation to achieve following goals**
  - Reduce beam training overhead in RIS-aided systems
  - Reduce computational complexity
- **To make the unobserved entries of cascaded effective channel recoverable, the columns and the rows of the channel are observed via two-stage beam training**
- **Then, FALS recovers the unobserved entries by estimating column space and row space**
- **Simulation results show that the proposed algorithm is accurate when spending less beam training overhead. Also, its complexity is lower than other time-consuming algorithms**

Thank you

Q / A

Equations for Two-stage Beam Training

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한양 1939

❑ Received pilot signals, RIS control vector, and partial observation at the first stage beam training

$$\mathbf{Y}_{1\mathrm{st}} = P_{\mathrm{Tx}}\mathbf{C}_{1\mathrm{st}}^{H}\mathbf{H}_{\mathrm{eff}}\mathbf{W}_{1\mathrm{st}} + \mathbf{V} \in \mathbb{C}^{M_{\mathrm{B}}\times B}$$

$$\mathbf{W}_{1\mathrm{st}} = \left[\mathbf{\Psi}_{B}^{T}, \mathbf{O}_{B,M_{\mathrm{R}}-B}\right]^{T} \in \mathbb{C}^{M_{\mathrm{R}}\times B}$$

$$\mathbf{H}_{\mathrm{eff}}^{\Omega_{1\mathrm{st}}} = \frac{\left(\mathbf{C}_{1\mathrm{st}}^{H}\right)^{-1}\mathbf{Y}_{1\mathrm{st}}\mathbf{W}_{1\mathrm{st}}^{H}}{BP_{\mathrm{Tx}}} \in \mathbb{C}^{M_{\mathrm{B}}\times M_{\mathrm{R}}}$$

$\mathbf{C}_{1\mathrm{st}} \in \mathbb{C}^{M_{\mathrm{B}}\times M_{\mathrm{B}}}$: Full rank combining matrix

$\mathbf{\Psi}_{N}$: N x N DFT matrix

$\mathbf{O}_{M,N}$: M x N zero matrix

❑ Received pilot signals, RIS control vector, and partial observation at the second stage beam training

$$\mathbf{Y}_{2\mathrm{nd}} = P_{\mathrm{Tx}}\mathbf{C}_{2\mathrm{nd}}^{H}\mathbf{H}_{\mathrm{eff}}\mathbf{W}_{2\mathrm{nd}} + \mathbf{U} \in \mathbb{C}^{B\times(M_{\mathrm{R}}-B)}$$

$$\mathbf{W}_{2\mathrm{nd}} = \left[\mathbf{O}_{(M_{\mathrm{R}}-B),B}, \mathbf{\Psi}_{M_{\mathrm{R}}-B}^{T}\right]^{T} \in \mathbb{C}^{M_{\mathrm{R}}\times(M_{\mathrm{R}}-B)}$$

$$\mathbf{H}_{\mathrm{eff}}^{\Omega_{2\mathrm{nd}}} = \frac{\mathbf{C}_{2\mathrm{nd}}\mathbf{Y}_{2\mathrm{nd}}\mathbf{W}_{2\mathrm{nd}}^{H}}{B(M_{\mathrm{R}}-B)P_{\mathrm{Tx}}} \in \mathbb{C}^{M_{\mathrm{B}}\times M_{\mathrm{R}}}$$

Final observation of cascaded effective channel

$$\mathbf{H}_{\mathrm{eff}}^{\Omega} = \mathbf{H}_{\mathrm{eff}}^{\Omega_{1\mathrm{st}}} + \mathbf{H}_{\mathrm{eff}}^{\Omega_{2\mathrm{nd}}} \in \mathbb{C}^{M_{\mathrm{B}}\times M_{\mathrm{R}}}$$