AUD-37.2

# Spatial Active Noise Control Based on Individual Kernel Interpolation of Primary and Secondary Fields


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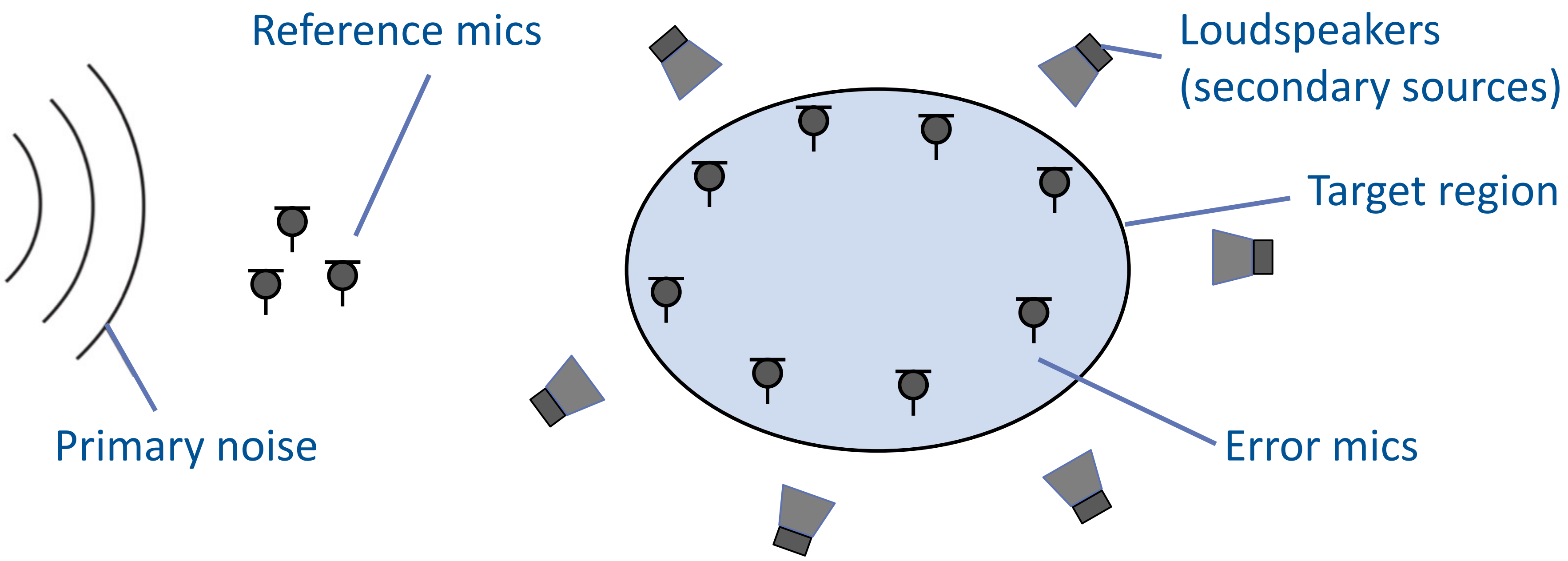
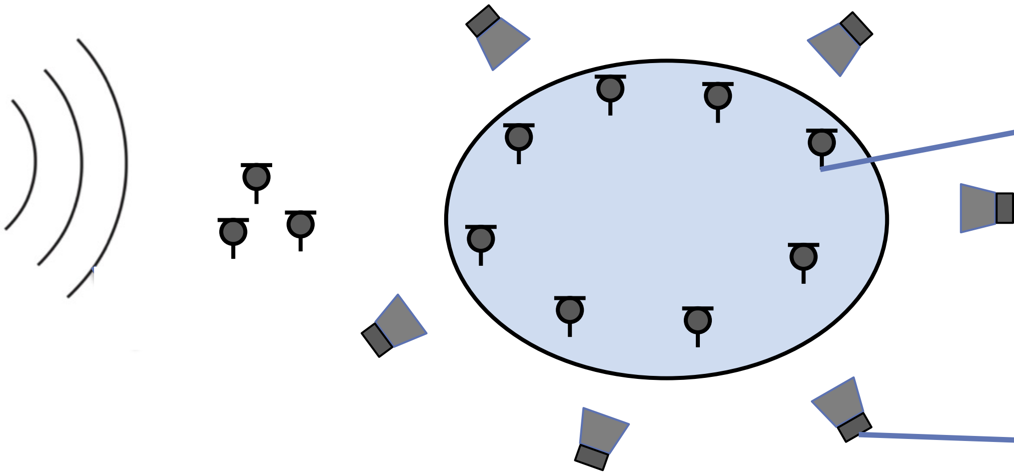
# Spatial Active Noise Control

- Spatial active noise control (ANC)
  - Reduce noise in space using multiple loudspeakers
  - Determine loudspeaker signals from reference mic and error mic signals



# ANC in frequency domain

- Formulation of Spatial ANC in frequency domain



1. Measurement equation:

$$\boldsymbol{e} = \underline{\boldsymbol{d}} + \underline{\boldsymbol{G}}\boldsymbol{y}$$

Primary noise — Transfer func matrix

2. Driving signal:

$$\boldsymbol{y} = \underline{\boldsymbol{W}}\,\underline{\boldsymbol{x}}$$

Control filter — Reference mic

3. Update control filter

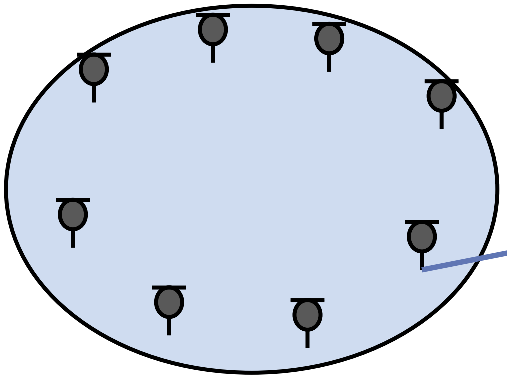
$$\boldsymbol{W} \leftarrow \boldsymbol{W} - \mu \frac{\partial J}{\partial \boldsymbol{W}^*}$$

Cost function

**How to determine cost function $J$ ?**

# Multiple pressure control (MPC)

- Cost function defined as squared sum of error mic signals



Cost function :

$$J = \boldsymbol{e}^{\mathsf{H}} \underline{\boldsymbol{e}}$$

Update rule :

$$\boldsymbol{W}(n+1) = \boldsymbol{W}(\underline{n}) - \mu \boldsymbol{G}^{\mathsf{H}} \boldsymbol{e}(n) \boldsymbol{x}(n)^{\mathsf{H}}$$

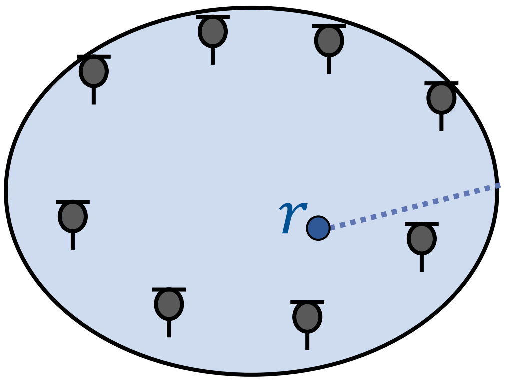
Iteration step

MPC only considers pressure at discrete positions of error mics

➡ Noise reduction in continuous target region is not guaranteed

# Spatial ANC based on kernel interpolation [Ito+ 2020]

- Cost function based on kernel interpolation of sound field




**1. Kernel interpolation of sound field**

$$\hat{u}_{\mathrm{e}}(\boldsymbol{r}) = \left[\left((\boldsymbol{K} + \lambda \boldsymbol{I}_M)^{-1}\right)^{\top} \boldsymbol{\kappa}(\boldsymbol{r})\right]^{\top} \boldsymbol{e}$$

$$= \boldsymbol{z}_{\mathrm{e}}(\boldsymbol{r})^{\top} \boldsymbol{e}$$

$$(\boldsymbol{K})_{mn} = \kappa(\boldsymbol{r}_m, \boldsymbol{r}_n)$$

$$(\boldsymbol{\kappa}(\boldsymbol{r}))_m = \kappa(\boldsymbol{r}, \boldsymbol{r}_m)$$

Position of *m* th error mic

$\kappa(\cdot,\cdot)$ : Kernel function

**2. Cost function based on regional noise power**

$$J = \int_{\Omega} |\hat{u}_{\mathrm{e}}(\boldsymbol{r})|^2 \, d\boldsymbol{r}$$

$$= \boldsymbol{e}^{\mathsf{H}} \boldsymbol{A} \boldsymbol{e}$$

$$\left(\boldsymbol{A} = \int_{\Omega} \boldsymbol{z}_e^*(\boldsymbol{r}) \boldsymbol{z}_e^{\top}(\boldsymbol{r}) d\boldsymbol{r}\right)$$

# Kernel function with directional weighting

- Kernel function defined as weighted integral of plane waves

$$\kappa(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{1}{4\pi} \int_{\mathbb{S}_2} \underbrace{\gamma(\boldsymbol{\xi})}_{\text{Weighting function}} \underbrace{\mathrm{e}^{-\mathrm{j}k\boldsymbol{\xi}^\mathsf{T}\boldsymbol{r}_{12}}}_{\text{Plane wave with arrival direction } \boldsymbol{\xi}} \mathrm{d}\boldsymbol{\xi} \quad (\boldsymbol{r}_{12} = \boldsymbol{r}_2 - \boldsymbol{r}_1)$$

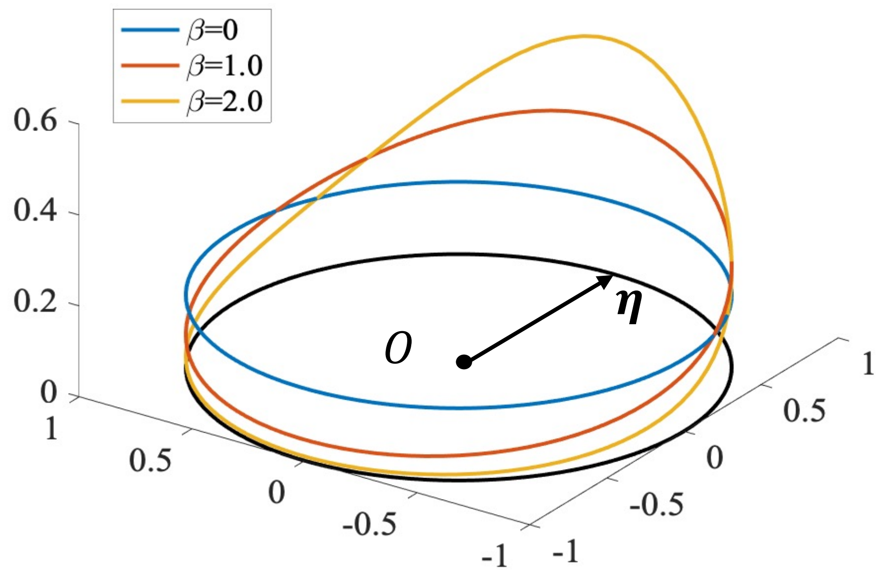
- Weighting function designed to reach maximum at source direction $\boldsymbol{\eta}$

**von Mises-Fisher distribution**

$$\gamma(\boldsymbol{\xi}) = \mathrm{e}^{\beta \boldsymbol{\xi}^\mathsf{T} \boldsymbol{\eta}}$$

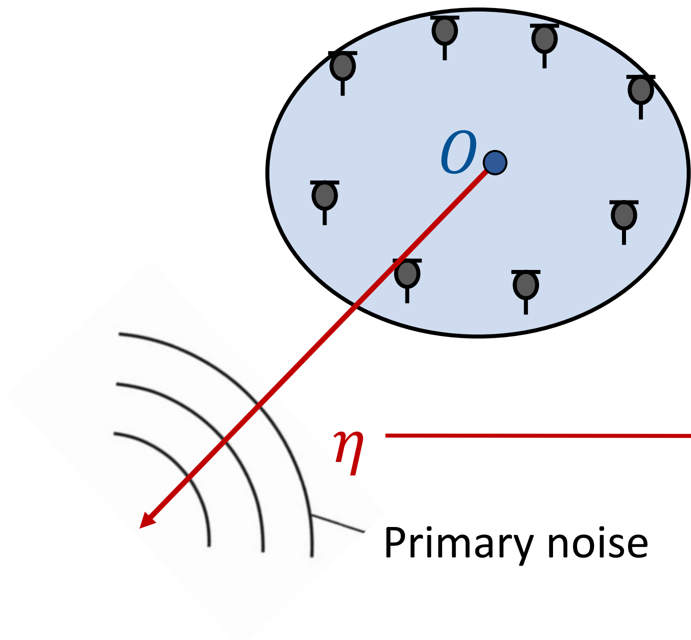
Maximum at $\boldsymbol{\xi} = \boldsymbol{\eta}$

$\beta(\geq 0)$ : Sharpness parameter



# Kernel function with directional weighting

- Kernel function weighted on primary noise direction




$$\kappa(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{1}{4\pi} \int_{\mathbb{S}_2} \gamma(\boldsymbol{\xi}) \mathrm{e}^{-\mathrm{j}k\boldsymbol{\xi}^{\mathsf{T}}\boldsymbol{r}_{12}} \, \mathrm{d}\boldsymbol{\xi}$$

$$\gamma(\boldsymbol{\xi}) = \mathrm{e}^{\beta\boldsymbol{\xi}^{\mathsf{T}}\boldsymbol{\eta}}$$

$$\kappa(\boldsymbol{r}_1, \boldsymbol{r}_2) = j_0\left(\sqrt{(\mathrm{j}\beta\boldsymbol{\eta} - k\boldsymbol{r}_{12})^{\mathsf{T}}(\mathrm{j}\beta\boldsymbol{\eta} - k\boldsymbol{r}_{12})}\right)$$

Primary noise direction is inserted into kernel function

- Benefits of kernel-interpolation-based ANC
  - Can consider pressure in the entire target region
  - Can utilize prior information of primary noise source direction

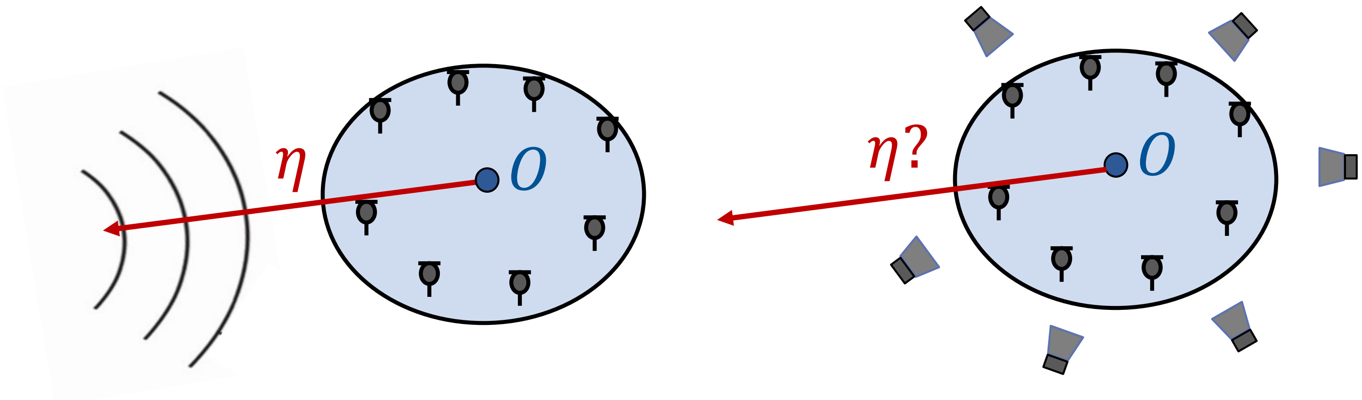
# Issue of kernel interpolation in previous study

- Sound field inside target region
  = superposition of primary noise and secondary sources field

$$\hat{u}_{\mathrm{e}}(\boldsymbol{r}) = \boldsymbol{z}_{\mathrm{e}}(\boldsymbol{r})^{\top}\boldsymbol{e}$$

$$= \underbrace{\boldsymbol{z}_{\mathrm{e}}(\boldsymbol{r})^{\top}\boldsymbol{d}}_{\text{Primary noise field}} + \underbrace{\boldsymbol{z}_{\mathrm{e}}(\boldsymbol{r})^{\top}\boldsymbol{G}\boldsymbol{y}}_{\text{Secondary sources field}}$$

Weighted on primary noise direction $\boldsymbol{\eta}$



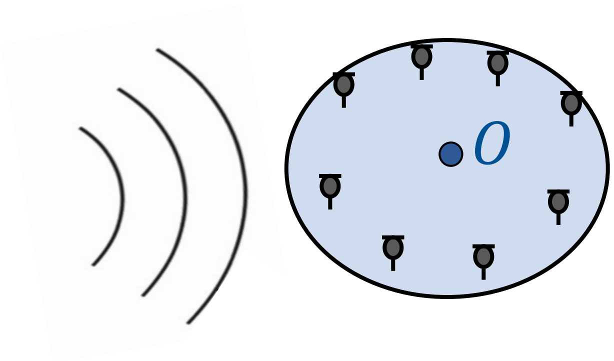

Secondary sources field is interpolated with directional weighting on primary noise direction

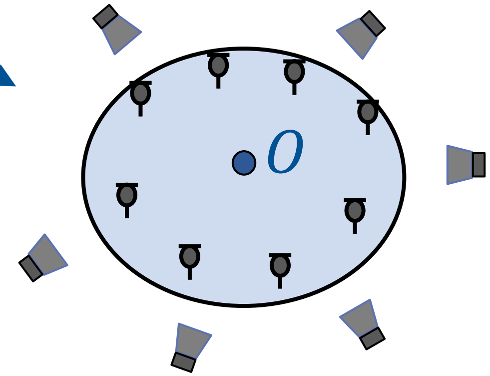
➡ Interpolation accuracy is limited

# Proposed method: individual kernel interpolation

1. Decompose error mic signals into primary noise and secondary sources components

$$\boldsymbol{e} = \boldsymbol{d} + \boldsymbol{G}\boldsymbol{y}$$




$$\hat{\boldsymbol{d}} = \boldsymbol{e} - \boldsymbol{G}\boldsymbol{y}$$

$$\boldsymbol{s} = \boldsymbol{G}\boldsymbol{y}$$

2. Individual kernel interpolation

$$\hat{u}_{\mathrm{p}}(\boldsymbol{r}) = \boldsymbol{z}_d(\boldsymbol{r})^{\mathsf{T}}\hat{\boldsymbol{d}}$$

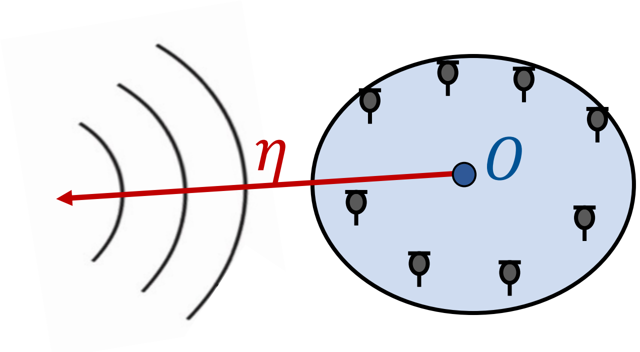
$$\hat{u}_{\mathrm{s}}(\boldsymbol{r}) = \boldsymbol{\zeta}_y(\boldsymbol{r})^{\mathsf{T}}\boldsymbol{y}$$

3. Sound field estimation

$$\hat{u}_{\mathrm{e}}(\boldsymbol{r}) = \hat{u}_{\mathrm{p}}(\boldsymbol{r}) + \hat{u}_{\mathrm{s}}(\boldsymbol{r})$$

# Proposed method: individual kernel interpolation

- Primary noise field interpolation

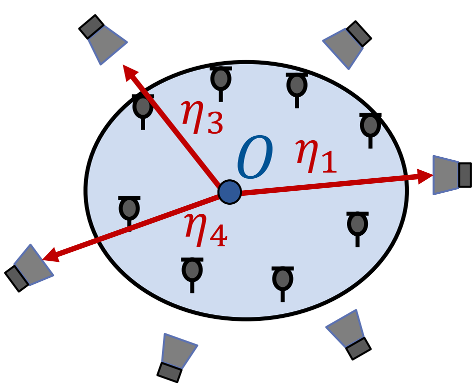



$$\hat{u}_{\mathrm{p}}(\boldsymbol{r}) = \left[\left(\left(\boldsymbol{K} + \lambda \boldsymbol{I}_M\right)^{-1}\right)^{\top} \boldsymbol{\kappa}(\boldsymbol{r})\right]^{\top} \hat{\boldsymbol{d}}$$

$$= \boldsymbol{z}_d(\boldsymbol{r})^{\top} \hat{\boldsymbol{d}}$$

Weighted on primary noise direction

- Secondary sources field interpolation




$$\hat{u}_{\mathrm{s}}(\boldsymbol{r}) = \sum_{l=1}^{L} \boldsymbol{z}_{y,l}(\boldsymbol{r})^{\mathsf{T}} \boldsymbol{G}_l y_l$$

Weighted on *l* th secondary source direction

$$= \boldsymbol{\zeta}_y(\boldsymbol{r})^{\mathsf{T}} \boldsymbol{y}$$

Utilize both primary noise and secondary sources directions

➡ Interpolation accuracy can be improved

# Individual-kernel-interpolation-based ANC

- Cost function based on regional noise power

$$J = \int_{\Omega} \left| \hat{u}_{\mathrm{p}}(\boldsymbol{r}) + \hat{u}_{\mathrm{s}}(\boldsymbol{r}) \right|^2 d\boldsymbol{r}$$
$$= \hat{\boldsymbol{d}}^{\mathsf{H}} \boldsymbol{A}_{dd} \hat{\boldsymbol{d}} + \boldsymbol{y}^{\mathsf{H}} \boldsymbol{A}_{yd} \hat{\boldsymbol{d}} + \hat{\boldsymbol{d}}^{\mathsf{H}} \boldsymbol{A}_{yd}^{\mathsf{H}} \boldsymbol{y} + \boldsymbol{y}^{\mathsf{H}} \boldsymbol{A}_{yy} \boldsymbol{y}$$

$$\begin{pmatrix} \boldsymbol{A}_{dd} := \int_{\Omega} \boldsymbol{z}_d^*(\boldsymbol{r}) \boldsymbol{z}_d^{\mathsf{T}}(\boldsymbol{r})\, \mathrm{d}\boldsymbol{r} \\ \boldsymbol{A}_{yd} := \int_{\Omega} \boldsymbol{\zeta}_y^*(\boldsymbol{r}) \boldsymbol{z}_d^{\mathsf{T}}(\boldsymbol{r})\, \mathrm{d}\boldsymbol{r} \\ \boldsymbol{A}_{yy} := \int_{\Omega} \boldsymbol{\zeta}_y^*(\boldsymbol{r}) \boldsymbol{\zeta}_y^{\mathsf{T}}(\boldsymbol{r})\, \mathrm{d}\boldsymbol{r} \end{pmatrix}$$

- Update filter matrix based on NLMS algorithm

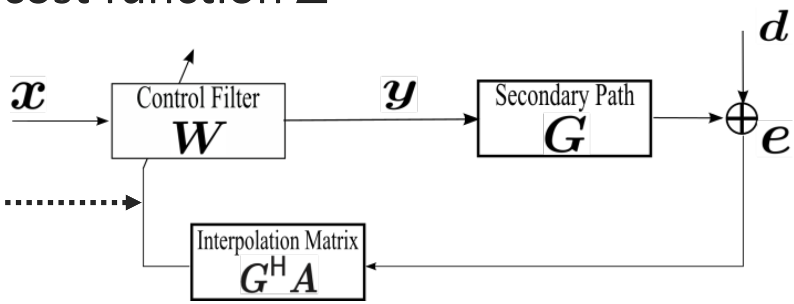
$$\boldsymbol{W}(n+1) = \boldsymbol{W}(n)$$
$$- \underbrace{\frac{\mu_0}{\left\| \boldsymbol{A}_{yy} \right\|_2 \left\| \boldsymbol{x}(n) \right\|_2^2 + \epsilon}}_{\text{Normalized step size}} \underbrace{\left[ \boldsymbol{A}_{yd} \boldsymbol{e}(n) + \left( \boldsymbol{A}_{yy} - \boldsymbol{A}_{yd} \boldsymbol{G} \right) \boldsymbol{y}(n) \right] \boldsymbol{x}(n)^{\mathsf{H}}}_{\text{Derivative of cost function w.r.t. filter matrix } \boldsymbol{W}}$$

# Relation to previous method

- Compare block diagrams and gradients of cost function ***Δ***

Total-KI-based method (Previous)

$$\boldsymbol{\Delta} = \boldsymbol{G}^{\mathsf{H}} \boldsymbol{A} \boldsymbol{e}(n) \boldsymbol{x}(n)^{\mathsf{H}}$$

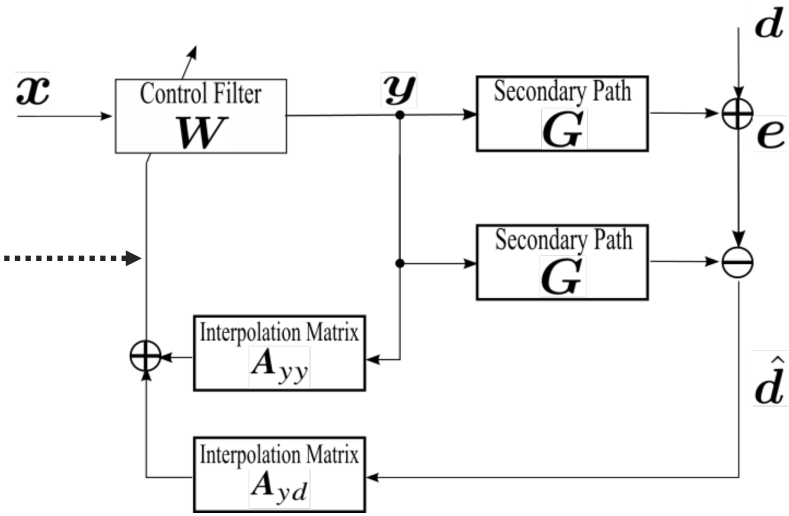



$\boldsymbol{A}_{yd} = \boldsymbol{G}^{\mathsf{H}} \boldsymbol{A}$, $\boldsymbol{A}_{yy} = \boldsymbol{A}_{yd} \boldsymbol{G}$

If same kernel function is used

Individual-KI-based method (Proposed)

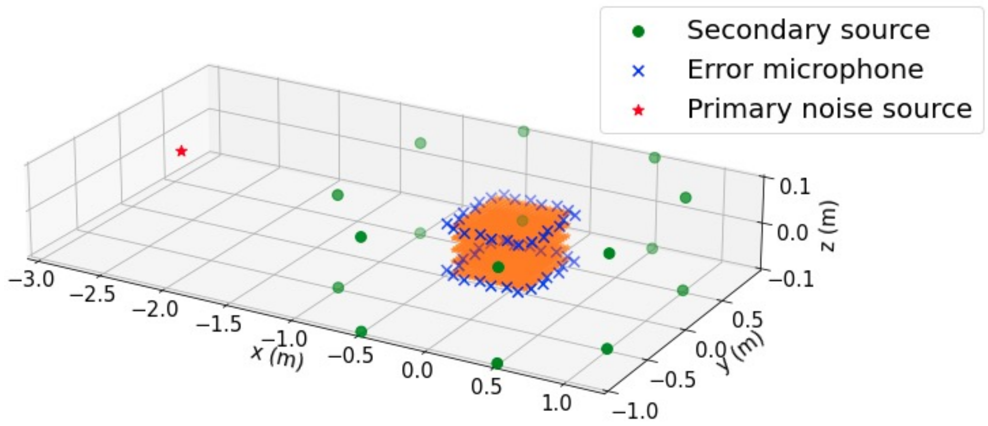
$$\boldsymbol{\Delta} = \boldsymbol{A}_{yd} \boldsymbol{e}(n) \boldsymbol{x}(n)^{\mathsf{H}} + (\boldsymbol{A}_{yy} - \boldsymbol{A}_{yd} \boldsymbol{G}) \boldsymbol{y}(n) \boldsymbol{x}(n)^{\mathsf{H}}$$




**Proposed method is generalization of previous method**

# Experimental setting

- 3D free field simulation




Target region ：
0.6 m × 0.6 m × 0.2 m
\# of eval points ： 1445

\# of error mics : 48
\# of loudspeakers : 16
\# of primary noise sources: 1

- Methods ：
  - Individual-KI-Based ANC (Proposed, $\beta = 10.0$)
  - Total-KI-Based ANC ($\beta = 0.0, 2.0$), MPC
- Performance measure:

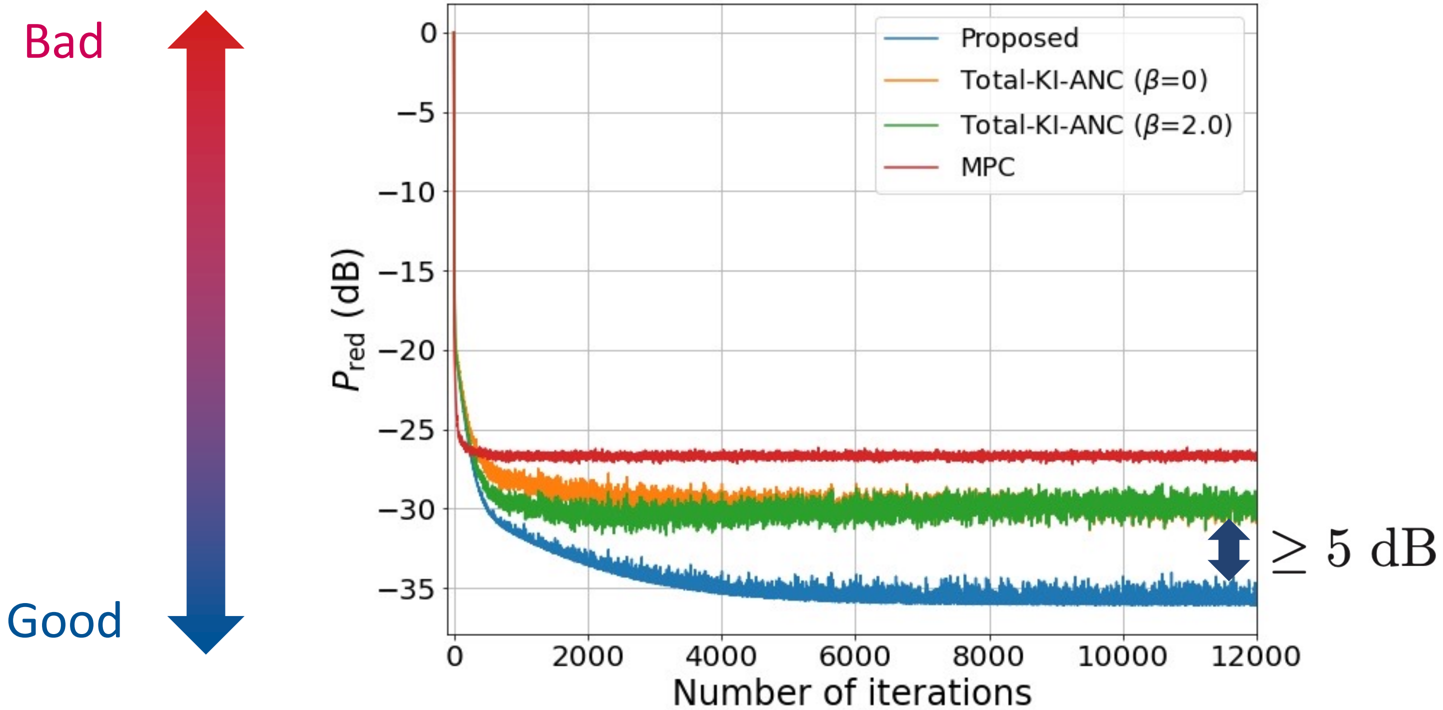
$$P_{\mathrm{red}}(n) = 10 \log_{10} \frac{\sum_j \underline{|u_{\mathrm{e}}^{(n)}(\boldsymbol{r}_j)|}^2}{\sum_j \underline{|u_{\mathrm{p}}^{(n)}(\boldsymbol{r}_j)|}^2}$$

Total pressure field at *j* th eval point

Primary noise field at *j* th eval point

# Noise reduction for each iteration at 200Hz

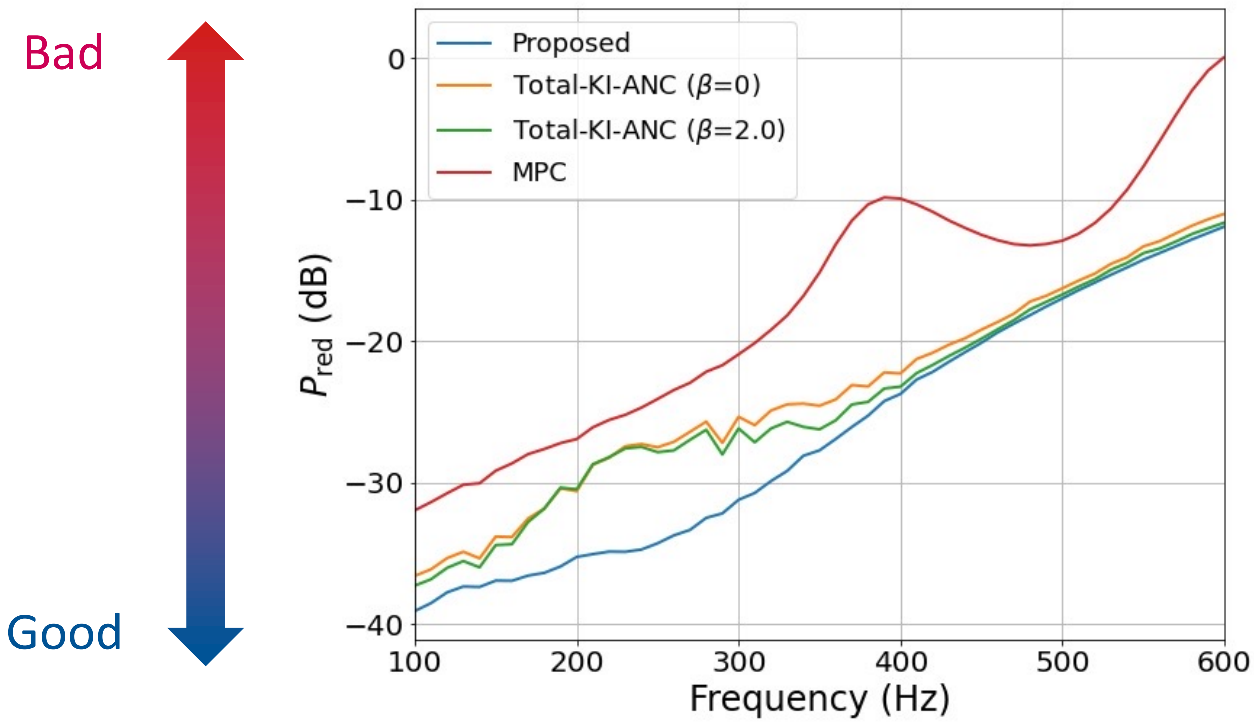
- Gaussian noise with SNR=40 dB is added to error mic signals




Proposed method achieves best performance among 4 methods

# Final noise reduction for each frequency

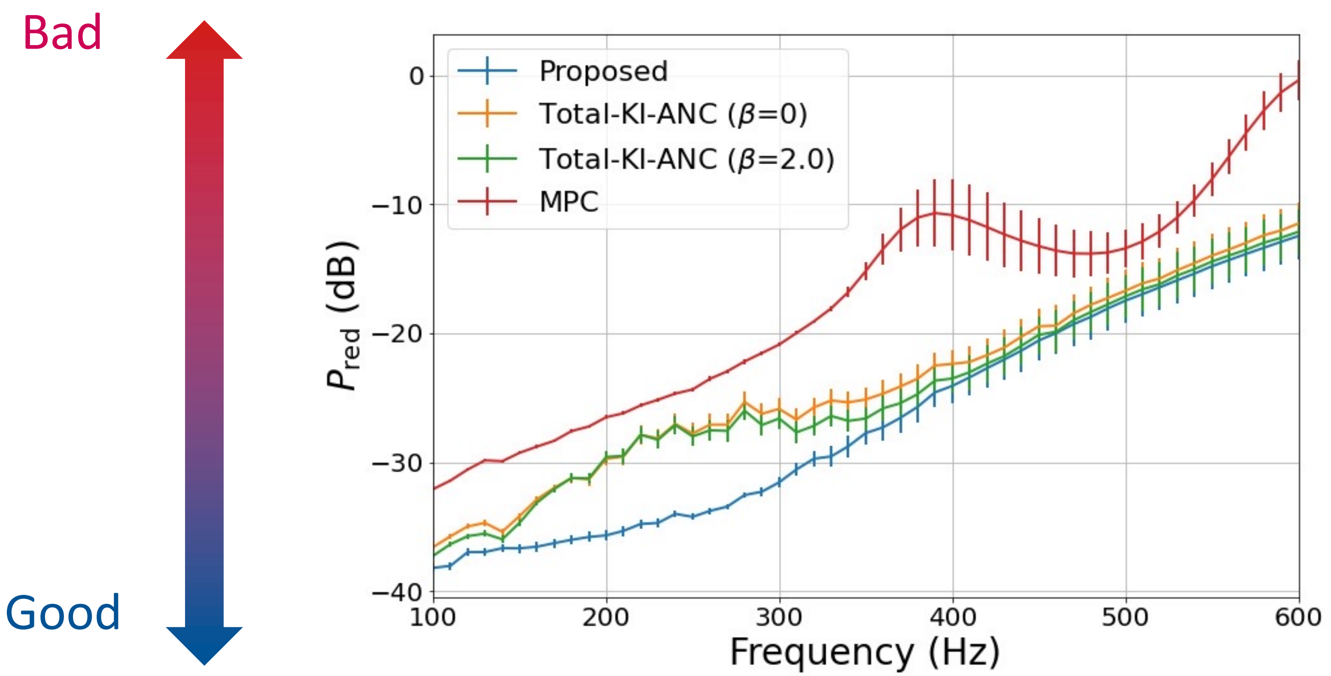
- $P_{\text{red}}$ after 12000 iterations w.r.t. noise frequency from 100 to 600 Hz




- Proposed method achieves best performance at all frequencies
- In particular, performance difference is large below 400 Hz

# Investigation of robustness against source perturbation

- Primary noise source position was perturbed randomly for 50 times
  - Perturbation was drawn from Gaussian distribution of standard deviations (0.05 m, 6°, 3°) in polar coordinates
  - Mean of final noise reductions is plotted with error bar




Robustness against primary noise perturbation is almost same as other methods

# Conclusion

- Spatial Active Noise Control based on kernel interpolation
  - Previous method estimates total sound field with single kernel function, which limits interpolation accuracy
  - By individually estimating primary and secondary sound fields, interpolation accuracy can be improved
  - NLMS algorithm based on individual kernel interpolation is derived
  - Proposed method can be seen as generalization of previous method
  - Achieves better performance at most frequencies
  - Robustness against primary noise source perturbation is almost same as previous method