Robust TDOA Source Localization Based on Lagrange Programming Neural Network

Paper Number: 9261

Wenxin Xiong*[⊙], Christian Schindelhauer[⊙], Dominik Jan Schott[¤], and Stefan Johann Rupitsch[¤]

Hing Cheung So[○]

[⊙]Department of Computer Science
[¤]Department of Microsystems Engineering
University of Freiburg, Freiburg 79110, Germany

*Presenter

[○]Department of Electrical Engineering
City University of Hong Kong,
Hong Kong, China

icassp 2022 Singapore

ABSTRACT

● We revisit here the problem of **time-difference-of-arrival** (**TDOA**) based localization under the mixed line-of-sight (LOS)/**non-line-of-sight** (**NLOS**) propagation conditions.

● Adopting the strategy of statistically robustifying the non-outlier-resistant l_2 loss, we formulate it as the minimization of a **possibly non-differentiable generalized robust cost function**, which is rooted in the analog **locally competitive algorithm** (**LCA**) for sparse approximation.

● We then present a **Lagrange programming neural network** (**LPNN**) to address the optimization formulation, with the non-differentiability issues being handled by grafting thereon the LCA concept of **internal state dynamics**.

● Compared with the existing algorithms, our approach is computationally less expensive, less reliant on the use of a priori error information, and observed to be capable of producing higher localization accuracy.

PROBLEM STATEMENT

● Our source localization (SL) scenario comprises $L \geq k$ synchronized sensors and a single source deployed in k-dimensional space.

● The known position of the ith sensor and unknown source location are denoted by $\boldsymbol{x}_i \in \mathbb{R}^k$ (for $i = 1, \dots, L$) and $\boldsymbol{x} \in \mathbb{R}^k$, respectively.

● The source-emitted radio or acoustic signal travels over the LOS or NLOS path, and is finally received by the ith sensor at time t_i (for $i = 1, \dots, L$).

● The nonredundant TDOA measurements are modeled as $t_{i,1} = t_i - t_1 = (\|\boldsymbol{x} - \boldsymbol{x}_i\|_2 - \|\boldsymbol{x} - \boldsymbol{x}_1\|_2 + n_{i,1} + b_{i,1})/c$ (for $i = 1, \dots, L$).

● c: Signal propagation velocity; $n_{i,1} = n_i - n_1$: Measurement noise in the TDOA-based range difference (RD) observation $r_{i,1} = ct_{i,1}$; n_i follows the uncorrelated zero-mean Gaussian distribution; $b_{i,1} = q_i - q_1$; q_i: Possible NLOS bias occurring in the ith path without any prior statistical knowledge.

● The task of TDOA-based SL under possible NLOS propagation conditions is to determine $\boldsymbol{x}$ given $\{r_{i,1}\}$ (possibly unreliable) and perfectly known $\{\boldsymbol{x}_i\}$.

FRAMEWORK OF LPNN

● As a locally stable Lagrange-type neurodynamic technique [i], the augmented LPNN is used to search for a critical point solution of the equality constrained optimization problem (ECOP) with differentiable objective:

$$\min_{\boldsymbol{y} \in \mathbb{R}^N} f(\boldsymbol{y}), \qquad \text{s.t.}\, \boldsymbol{h}(\boldsymbol{y}) = \boldsymbol{0}_M$$

with $\boldsymbol{h}(\boldsymbol{y}) = [h_1(\boldsymbol{y}), \dots, h_M(\boldsymbol{y})]^T$, by setting up its augmented Lagrangian as

$$\mathcal{L}_\rho(\boldsymbol{y}, \boldsymbol{\lambda}) = f(\boldsymbol{y}) + \boldsymbol{\lambda}^T \boldsymbol{h}(\boldsymbol{y}) + \frac{\rho}{2} \sum_{i=1}^{M} [h_i(\boldsymbol{y})]^2,$$

where $\boldsymbol{\lambda} = [\lambda_1, \dots, \lambda_M]^T$ is the Lagrange multiplier vector.

● Two types of neurons are then defined, known as the variable neurons and Lagrangian neurons, holding $\boldsymbol{y}$ to be optimized and Lagrange multipliers in $\boldsymbol{\lambda}$, respectively.

● Their time-domain behaviors are defined by $\frac{d\boldsymbol{y}}{dt} = -\nabla_{\boldsymbol{y}} \mathcal{L}_\rho(\boldsymbol{y}, \boldsymbol{\lambda})$ and $\frac{d\boldsymbol{\lambda}}{dt} = \nabla_{\boldsymbol{\lambda}} \mathcal{L}_\rho(\boldsymbol{y}, \boldsymbol{\lambda})$.

FRAMEWORK OF LCA

● The LCA [ii] is a neural architecture aiming to solve the sparse approximation problem by descending an energy function:

$$\min_{\boldsymbol{z} \in \mathbb{R}^J} \mathrm{C}_\delta(\boldsymbol{z}) := \frac{1}{2} \|\boldsymbol{b} - \boldsymbol{\Phi} \boldsymbol{z}\|_2^2 + \delta \sum_{i=1}^{J} \psi_{(\kappa,\tau,\delta)}(z_i),$$

where $\boldsymbol{\Phi} \in \mathbb{R}^{H \times J}$ is the dictionary matrix with $H < J$, $\boldsymbol{b} \in \mathbb{R}^H$ is the observation vector, and $\psi_{(\kappa,\tau,\delta)}(z_i)$ is a sparsity-inducing penalty term whose specific form is determined by that of a smooth sigmoidal thresholding function:

$$\mathcal{T}_{(\kappa,\tau,\delta)}(u_i) = \mathrm{sgn}(u_i) \frac{|u_i| - \kappa\delta}{1 + \exp(-\tau(|u_i| - \delta))}.$$

● It consists of J neurons, holding the newly introduced internal state vector $\boldsymbol{u} = [u_1, \dots, u_J]^T$ instead of the sparse vector $\boldsymbol{z} = [z_1, \dots, z_J]^T$ to be estimated.

● The dynamical system is established according to $\frac{d\boldsymbol{z}}{dt} = -(\boldsymbol{u} - \boldsymbol{z}) + \boldsymbol{\Phi}^T(\boldsymbol{b} - \boldsymbol{\Phi}\boldsymbol{z})$ and the mapping from $\boldsymbol{u}$ to $\boldsymbol{z}$ via the thresholding function $z_i = \mathcal{T}_{(\kappa,\tau,\delta)}(u_i)$.

OUR FORMULATION

● A traditional l_1-norm based robust formulation is [iii]:

$$\min_{\boldsymbol{x}} \|\boldsymbol{e}\|_1,$$

where $\boldsymbol{e} = [e_{2,1}, \dots, e_{L,1}]^T$ is a dummy vector satisfying $e_{i,1} = r_{i,1} - \|\boldsymbol{x} - \boldsymbol{x}_i\|_2 + \|\boldsymbol{x} - \boldsymbol{x}_1\|_2$ (for $i = 2, \dots, L$).

● We propose to deal with an extension of it: $\min_{\boldsymbol{x}} \sum_{i=2}^{L} \psi(e_{i,1})$, where $\psi(\cdot)$ represents a generalized robust loss function, whose form is specified by the LCA-defined thresholding function.

● To avoid ill-posing in applying the gradient-type neurodynamic solver to the problem, we re-express the source-sensor constraints in a quadratic form:

$$\min_{\boldsymbol{x},\boldsymbol{d},\boldsymbol{w},\boldsymbol{e}} \sum_{i=2}^{L} \psi(e_{i,1}), \text{s.t.}\, \boldsymbol{r} - \boldsymbol{e} = \boldsymbol{D}\boldsymbol{d}, d_i^2 = \|\boldsymbol{x} - \boldsymbol{x}_i\|_2^2, d_i = w_i^2, i = 1, \dots, L,$$

where $\boldsymbol{D} = [-\boldsymbol{1}_{L-1}, \boldsymbol{I}_{(L-1)\times(L-1)}]$, $\boldsymbol{r} = [r_{2,1}, \dots, r_{L,1}]^T$, and $\boldsymbol{w} = [w_1, \dots, w_L]^T$.

LCA-INCORPORATED LPNN

● The LPNN is not straightforwardly applicable since we do not premise the robust loss on any differentiability assumption.

● It is straightforward to settle the inapplicability of LPNN to the problem, in a manner similar to the construction of internal state dynamics when solving the unconstrained sparse approximation formulation using LCA.

● To be specific, letting $J = L - 1$, $\boldsymbol{z} = \boldsymbol{e}$, $\delta = 1$ and combining the use of both neural systems by substituting $\boldsymbol{e}$ held in the Lagrangian neurons with $\boldsymbol{u}$, we have finally:

$$\frac{d\boldsymbol{x}}{dt} = -2\sum_{i=1}^{L}\left[\lambda_{L-1+i} + \rho\left(d_i^2 - \|\boldsymbol{x} - \boldsymbol{x}_i\|_2^2\right)\right](\boldsymbol{x}_i - \boldsymbol{x}),$$

$$\frac{dd_i}{dt} = \left[\boldsymbol{D}^T \cdot [\lambda_1, \dots, \lambda_{L-1}]^T\right]_i - 2\lambda_{L-1+i}d_i + \rho\left\{\left[\boldsymbol{D}^T(\boldsymbol{r} - \boldsymbol{e} - \boldsymbol{D}\boldsymbol{d})\right]_i - 2\left(d_i^2 - \|\boldsymbol{x} - \boldsymbol{x}_i\|_2^2\right)d_i - \left(d_i - w_i^2\right)\right\} - \lambda_{2L-1+i},$$

$$\frac{dw_i}{dt} = 2\lambda_{2L-1+i}w_i + 2\rho\left(d_i - w_i^2\right)w_i, \quad i = 1, \dots, L,$$

$$\frac{d\boldsymbol{u}}{dt} = -\boldsymbol{u} + \boldsymbol{e} + [\lambda_1, \dots, \lambda_{L-1}]^T + \rho(\boldsymbol{r} - \boldsymbol{e} - \boldsymbol{D}\boldsymbol{d}),$$

$$e_{i-1} = \mathcal{T}_{(\kappa,\tau,\delta)}(u_{i-1}), \quad i = 2, \dots, L,$$

$$\frac{d\lambda_{i-1}}{dt} = r_{i,1} - e_{i,1} - d_i + d_1, \quad i = 2, \dots, L,$$

$$\frac{d\lambda_{L-1+i}}{dt} = d_i^2 - \|\boldsymbol{x} - \boldsymbol{x}_i\|_2^2, \quad i = 1, \dots, L,$$

$$\frac{d\lambda_{2L-1+i}}{dt} = d_i - w_i^2, \quad i = 1, \dots, L.$$

● Numerical complexity is $\mathcal{O}(N_{\mathrm{LPNN}}L)$, where N_{LPNN} is the number of iterations.

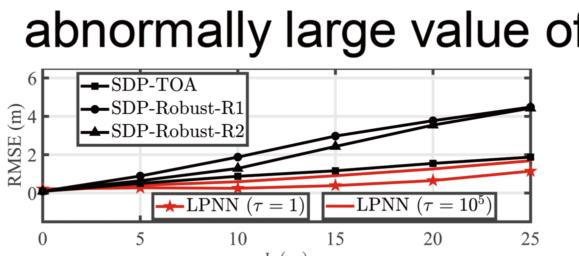
● Stability of LCA-incorporated LPNN remains an open issue for future research.

SIMULATION RESULTS

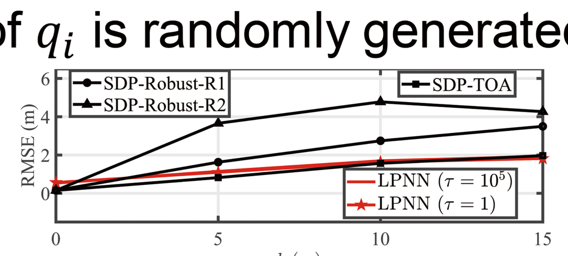
● The localization performance of the LPNN approach is evaluated using synthetic data. The robust loss is set as $\sum \psi_{(1,\tau,1)}(\cdot)$. State-of-the-art TDOA positioning methods with NLOS effects being countered, i.e., SDP-TOA [iv], SDP-Robust-R1 [v], and SDP-Robust-R2 [v] are implemented for comparison.

● The 1st configuration is deterministic, with $L = 8$ sensors evenly placed on the perimeter of a 20 m × 20 m square region and a single source fixed at $\boldsymbol{x} = [2,3]^T$ m, whereas the 2nd randomly generates positions of the source and $L = 10$ sensors from the same area in each of 500 Monte Carlo runs.

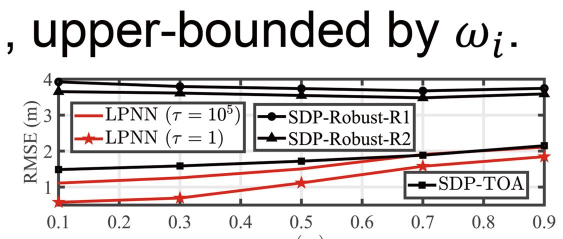
● n_i is assumed to be of constant variance σ^2 for all is, and the possibly abnormally large value of q_i is randomly generated, upper-bounded by ω_i.



Deterministic deployment.
$\sigma = 0.1$ m, $\omega_2 = \omega_3 = b$, $\omega_1 = \omega_4 = \cdots = \omega_8 = 0$



Random deployment.
$\sigma = 0.1$ m, $\omega_2 = \omega_3 = b$, $\omega_1 = \omega_4 = \cdots = \omega_{10} = 0$



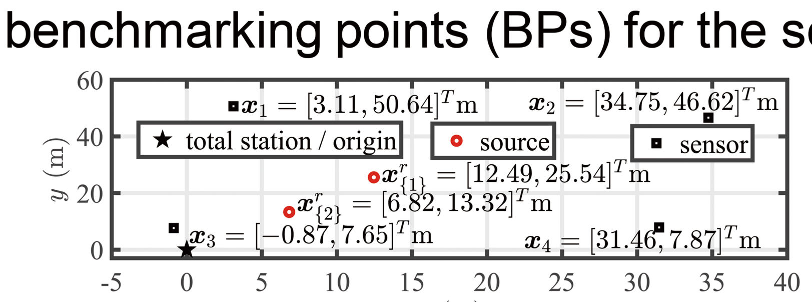
Deterministic deployment.
$\omega_2 = \omega_3 = 20$ m, $\omega_1 = \omega_4 = \cdots = \omega_8 = 0$

EXPERIMENTAL RESULTS

● We also conduct tests using the real experimental data collected in a 45 m × 60 m area outdoors by a ranging system comprising five equal-height deployed Decawave DWM1000 modules, each of which is an IEEE 802.15.4-2011 UWB implementation, based on the Decawave DW1000 UWB transceiver integrated circuit.

● While four of the modules are utilized as sensors, the one left acts as the source to be located. 50 Monte Carlo trials are performed.

● The localization geometry is illustrated below. The true positions are measured by a total station set up at the origin, and $\boldsymbol{x}_{\{1\}}^r$ and $\boldsymbol{x}_{\{2\}}^r$ are two benchmarking points (BPs) for the source.



Algorithm	RMSE (m)		Average run-time (s)	
—	BP 1	BP 2	BP 1	BP 2
LPNN ($\tau = 10^5$)	0.09	0.06	0.054	0.068
LPNN ($\tau = 1$)	0.07	0.10	0.140	0.115
SDP-TOA	0.10	0.07	0.788	0.777
SDP-Robust-R1	0.17	0.21	0.717	0.720
SDP-Robust-R2	0.51	0.61	1.020	0.945

[i] S. Zhang and A. G. Constantinides, ``Lagrange programming neural networks,'' *IEEE Trans. Circuits Syst. II: Anal. Digit. Signal Process.*, vol. 39, no. 7, pp. 441-452, Jul. 1992.
[ii] C. J. Rozell, D. H. Johnson, R. G. Baraniuk, and B. A. Olshausen, ``Sparse coding via thresholding and local competition in neural circuits,'' *Neural Comput.*, vol. 20, no. 10, pp. 2526-2563, Oct. 2008.
[iii] M. R. Gholami, S. Gezici, and E. G. Strom, ``A concave-convex procedure for TDOA based positioning,'' *IEEE Commun. Lett.*, vol. 17, no. 4, pp. 765-768, Apr. 2013.
[iv] Z. Su, G. Shao, and H. Liu, ``Semidefinite programming for NLOS error mitigation in TDOA localization,'' *IEEE Commun. Lett.*, vol. 22, no. 7, pp. 1430-1433, Jul. 2018.
[v] G. Wang, W. Zhu and N. Ansari, ``Robust TDOA-based localization for IoT via joint source position and NLOS error estimation,'' *IEEE Internet Things J.*, vol. 6, no. 5, pp. 8529-8541, Oct. 2019.

DOI of this paper: 10.1109/LSP.2021.3082035

Contact the corresponding author at w.x.xiong@outlook.com