Semidefinite Relaxation Method for Moving Object Localization Using A Stationary Transmitter at Unknown Position

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Joint work with

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Introduction

- Background
  - Multistatic localization becomes a popular research topic as it finds applications in target tracking, automatic driving, robotics, and many others.
  - The information of time-delay (TD), differential-arrival-times, Doppler-frequency-shift (DFS) can be used.
- Unknown Transmitter Position
  - The transmitter may be located somewhere in which the GPS is not accessible.
  - It could happen in the passive coherent location (PCL) systems, where an existing facility is used as the transmitter.
  - The transmitter position can even be intentionally left unknown for lower hardware and implementation costs.
- Existing Methods
  - Methods based on TD measurements only:
    Two-step weighted least-squares (TSWLS) and Semidefinite relaxation (SDR).
  - Method based on TD and DFS measurements: TSWLS.
- Contribution
  - We propose an accurate SDR method for estimating the object position and velocity when the transmitter position is unknown.

Introduction

- Illustration of the localization scenario

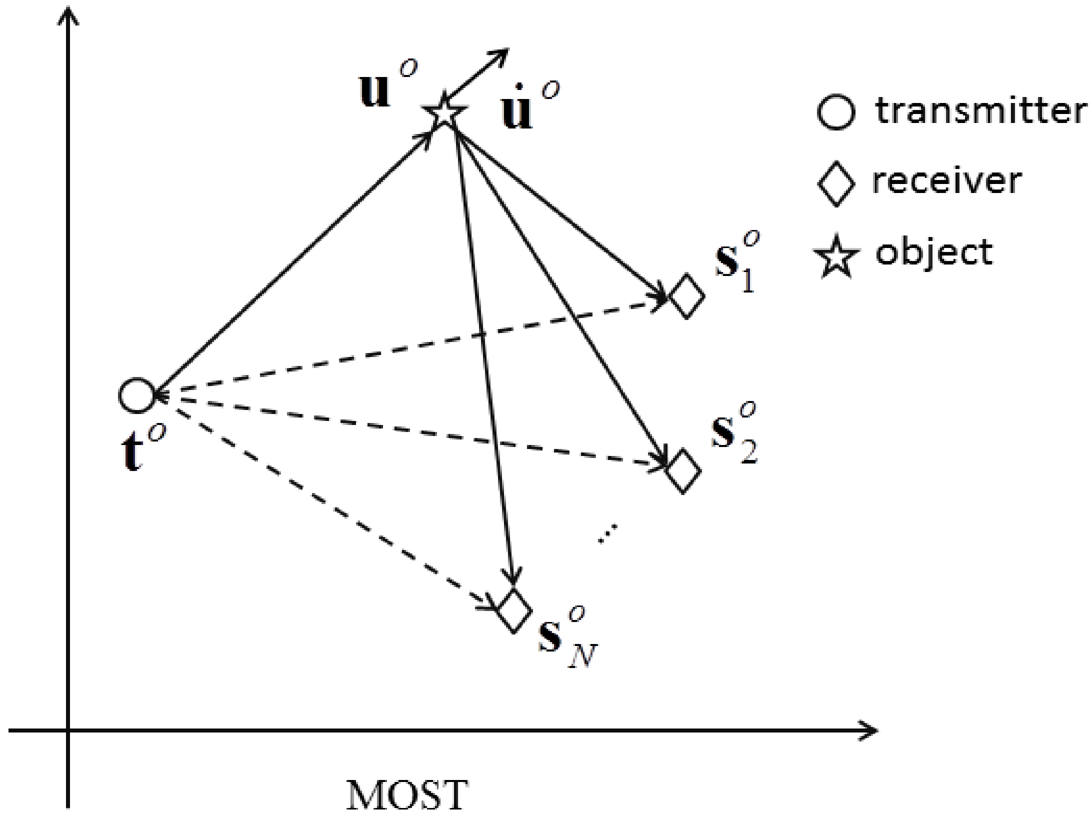

Figure 1: Illustration of the localization scenario of moving object and stationary transmitter (MOST): **Receivers are synchronized**.

Measurement Model

- The true ranges in the indirect and direct paths can be expressed as

$$r_j^o = \|\boldsymbol{u}^o - \boldsymbol{s}_j^o\| + \|\boldsymbol{u}^o - \boldsymbol{t}^o\| + b_\tau^o,\ j = 1, \ldots, N,$$
$$d_j^o = \|\boldsymbol{s}_j^o - \boldsymbol{t}^o\| + b_\tau^o,\ j = 1, \ldots, N. \quad (1)$$

where
  - $\boldsymbol{u}^o$ and $\dot{\boldsymbol{u}}^o$: object position and velocity, respectively (unknown);
  - $\boldsymbol{t}^o$: transmitter position (unknown);
  - $\boldsymbol{s}_j^o$: j-th receiver position (known);
  - b_τ^o time offset at the transmitter (unknown).

- The true range rates, which are transformed from the DFS, in the indirect and direct paths are:

$$\dot{r}_j^{\,o} = \boldsymbol{\rho}_{\boldsymbol{u}^o - \boldsymbol{s}_j^o}^T \dot{\boldsymbol{u}}^o + \boldsymbol{\rho}_{\boldsymbol{u}^o - \boldsymbol{t}^o}^T \dot{\boldsymbol{u}}^o + b_f^o,\ j = 1, \ldots, N,$$
$$\dot{d}_j^{\,o} = b_f^o,\ j = 1, \ldots, N, \quad (2)$$

where b_f^o is the unknown frequency offset.

Measurement Model

- In practice, the measurements observed by the receiver are contaminated by noise, i.e.,

$$
\begin{aligned}
r_j &= r_j^o + \varepsilon_{r,j},\ \ j = 1, \ldots, N, \\
d_j &= d_j^o + \varepsilon_{d,j},\ \ j = 1, \ldots, N, \\
\dot{r}_j &= \dot{r}_j^o + \dot{\varepsilon}_{\dot{r},j},\ \ j = 1, \ldots, N, \\
\dot{d}_j &= \dot{d}_j^o + \dot{\varepsilon}_{\dot{d},j},\ \ j = 1, \ldots, N, \qquad (3)
\end{aligned}
$$

where $\varepsilon_{r,j}$, $\dot{\varepsilon}_{\dot{r},j}$, $\varepsilon_{d,j}$, and $\dot{\varepsilon}_{\dot{d},j}$ are the additive noise.

Problem Formulation

- Dealing with the indirect path models.

  Transforming the indirect path range measurement model gives

$$\frac{1}{2}(r_j^2 - \|\boldsymbol{s}_j^o\|^2) + \boldsymbol{s}_j^{oT}\boldsymbol{u}^o - r_j\rho^o - r_j b_\tau^o - \frac{1}{2}\varphi_1^o$$
$$= \|\boldsymbol{u}^o - \boldsymbol{s}_j^o\|\varepsilon_{r,j},\ j = 1, \ldots, N, \tag{4}$$

where $\varphi_1^o = \|\boldsymbol{u}^o\|^2 - \rho^{o2} - 2\rho^o b_\tau^o - b_\tau^{o2}$, $\rho^o = \|\boldsymbol{u}^o - \boldsymbol{t}^o\|$, and the second-order noise terms $\varepsilon_{r,j}^2$ are neglected.

The equation for range rate is obtained based on time-derivative of (4) and incorporating the frequency offset,

$$r_j\dot{r}_j + \boldsymbol{s}_j^{oT}\dot{\boldsymbol{u}}^o - \dot{r}_j\rho^o - r_j\dot{\rho}^o - \dot{r}_j b_\tau^o - r_j b_f^o - \varphi_2^o$$
$$= \boldsymbol{\rho}_{\boldsymbol{u}^o - \boldsymbol{s}_j^o}^T\dot{\boldsymbol{u}}^o\varepsilon_{r,j} + \|\boldsymbol{u}^o - \boldsymbol{s}_j^o\|\dot{\varepsilon}_{\dot{r},j},\ j = 1, \ldots, N, \tag{5}$$

where $\dot{\rho}^o = \boldsymbol{\rho}_{\boldsymbol{u}^o - \boldsymbol{t}^o}^T\dot{\boldsymbol{u}}^o$ and $\varphi_2^o = \boldsymbol{u}^{oT}\dot{\boldsymbol{u}}^o - \rho^o\dot{\rho}^o - \dot{\rho}^o b_\tau^o - \rho^o b_f^o - b_\tau^o b_f^o$.

- Dealing with the direct path models. The direct path range measurement model can be manipulated similarly and approximated by

$$\frac{1}{2}(d_j^2 - \|\boldsymbol{s}_j^o\|^2) + \boldsymbol{s}_j^{oT}\boldsymbol{t}^o - d_j b_\tau^o - \frac{1}{2}\varphi_3^o = \|\boldsymbol{t}^o - \boldsymbol{s}_j^o\|\varepsilon_{d,j},$$
$$j = 1, \ldots, N, \tag{6}$$

where $\varphi_3^o = \|\boldsymbol{t}^o\|^2 - b_\tau^{o2}$.
The frequency offset is present in the direct path DFS observations

$$\dot{d}_j = b_f^o + \dot{\varepsilon}_{\dot{d},j},\ j = 1, \ldots, N. \tag{7}$$

Problem Formulation

- Define the unknown vector $\boldsymbol{y}^o$:

$$\boldsymbol{y}^o = [\underbrace{\boldsymbol{u}^{oT}}_{1\times k}, \underbrace{\dot{\boldsymbol{u}}^{oT}}_{1\times k}, \underbrace{\boldsymbol{t}^{oT}}_{1\times k}, \underbrace{b_\tau^o}_{1}, \underbrace{b_f^o}_{1}, \underbrace{\rho^o}_{1}, \underbrace{\dot{\rho}^o}_{1}, \underbrace{\varphi_1^o}_{1}, \underbrace{\varphi_2^o}_{1}, \underbrace{\varphi_3^o}_{1}]^T.$$

Using $\boldsymbol{y}^o$, we can express the pseudo-linear equations in (4), (5), (6), and (7) as the following matrix forms:

$$\boldsymbol{b_r} - \boldsymbol{A_r}\boldsymbol{y}^o \simeq \boldsymbol{B_r}\boldsymbol{\varepsilon_r}, \tag{8a}$$

$$\boldsymbol{b_{\dot{r}}} - \boldsymbol{A_{\dot{r}}}\boldsymbol{y}^o \simeq \boldsymbol{B_{\dot{r}}}\boldsymbol{\varepsilon_r} + \boldsymbol{B_r}\dot{\boldsymbol{\varepsilon}}_{\dot{\boldsymbol{r}}}, \tag{8b}$$

$$\boldsymbol{b_d} - \boldsymbol{A_d}\boldsymbol{y}^o \simeq \boldsymbol{B_d}\boldsymbol{\varepsilon_d}, \tag{8c}$$

$$\boldsymbol{b_{\dot{d}}} - \boldsymbol{A_{\dot{d}}}\boldsymbol{y}^o = \dot{\boldsymbol{\varepsilon}}_{\dot{\boldsymbol{d}}}. \tag{8d}$$

Combining the approximate equations in (8a)-(8d) yields

$$\boldsymbol{b} - \boldsymbol{A}\boldsymbol{y}^o \simeq \boldsymbol{B}\boldsymbol{\varepsilon}. \tag{9}$$

Problem Formulation

- Base on (9), we can formulate a WLS problem to estimate $\boldsymbol{y}^o$:

$$\min_{\boldsymbol{y}} \ (\boldsymbol{b}-\boldsymbol{A}\boldsymbol{y})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{b}-\boldsymbol{A}\boldsymbol{y}), \tag{10a}$$

$$\text{s.t. } y_{(3k+3)}^2 = \|\boldsymbol{y}_{(1:k)} - \boldsymbol{y}_{((2k+1):3k)}\|^2, \tag{10b}$$

$$y_{(3k+4)} = \frac{(\boldsymbol{y}_{(1:k)} - \boldsymbol{y}_{((2k+1):3k)})^T}{\|\boldsymbol{y}_{(1:k)} - \boldsymbol{y}_{((2k+1):3k)}\|}\boldsymbol{y}_{((k+1):2k)}, \tag{10c}$$

$$y_{(3k+3)}y_{(3k+4)} = (\boldsymbol{y}_{(1:k)} - \boldsymbol{y}_{((2k+1):3k)})^T\boldsymbol{y}_{((k+1):2k)}, \tag{10d}$$

$$y_{(3k+5)} = \|\boldsymbol{y}_{(1:k)}\|^2 - (y_{(3k+3)} + y_{(3k+1)})^2, \tag{10e}$$

$$y_{(3k+6)} = -(y_{(3k+1)} + y_{(3k+3)})(y_{(3k+2)} + y_{(3k+4)}) + \boldsymbol{y}_{(1:k)}^T\boldsymbol{y}_{(k+1:2k)}, \tag{10f}$$

$$y_{(3k+7)} = \|\boldsymbol{y}_{((2k+1):3k)}\|^2 - y_{(3k+1)}^2, \tag{10g}$$

where $\boldsymbol{\Sigma} = \boldsymbol{B}\boldsymbol{Q}\boldsymbol{B}^T$.

- Let $\boldsymbol{Y} = \boldsymbol{y}\boldsymbol{y}^T$. Problem (10) can be relaxed into the following semidefinite program (SDP) by dropping the $\operatorname{rank}(\boldsymbol{Y}) = 1$ constraint:

$$\min_{\boldsymbol{y}} \ \operatorname{tr}\left\{\left[\begin{array}{cc} \boldsymbol{Y} & \boldsymbol{y} \\ \boldsymbol{y}^T & 1 \end{array}\right]\left[\begin{array}{cc} \boldsymbol{A}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{A} & -\boldsymbol{A}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{b} \\ -\boldsymbol{b}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{A} & \boldsymbol{b}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{b} \end{array}\right]\right\}, \tag{11a}$$

$$\text{s.t. } Y_{(3k+3,3k+3)} = \operatorname{tr}\{\boldsymbol{Y}_{(1:k,1:k)}\} - 2\operatorname{tr}\{\boldsymbol{Y}_{(1:k,(2k+1):3k)}\}$$

$$+ \operatorname{tr}\{\boldsymbol{Y}_{((2k+1):3k,(2k+1):3k)}\}, \tag{11b}$$

$$Y_{(3k+3,3k+4)} = \operatorname{tr}\{\boldsymbol{Y}_{(1:k,(k+1):2k)}\} - \operatorname{tr}\{\boldsymbol{Y}_{((2k+1):3k,(k+1):2k)}\}, \tag{11c}$$

$$y_{(3k+5)} = \operatorname{tr}\{\boldsymbol{Y}_{(1:k,1:k)}\} - Y_{(3k+3,3k+3)} - 2Y_{(3k+1,3k+3)}$$

$$- Y_{(3k+1,3k+1)}, \tag{11d}$$

$$y_{(3k+6)} = \operatorname{tr}\{\boldsymbol{Y}_{(1:k,k+1:2k)}\} - Y_{(3k+3,3k+4)}$$

$$- Y_{(3k+1,3k+4)} - Y_{(3k+3,3k+2)} - Y_{(3k+1,3k+2)}, \tag{11e}$$

$$y_{(3k+7)} = \operatorname{tr}\{\boldsymbol{Y}_{(2k+1:3k,2k+1:3k)}\} - Y_{(3k+1,3k+1)}, \tag{11f}$$

$$Y_{(3k+4,3k+4)} \leq \operatorname{tr}\{\boldsymbol{Y}_{((k+1):2k,(k+1):2k)}\}, \tag{11g}$$

$$\left[\begin{array}{cc} \boldsymbol{Y} & \boldsymbol{y} \\ \boldsymbol{y}^T & 1 \end{array}\right] \succeq \mathbf{0}. \tag{11h}$$

Simulations

- Parameter Setup
  - Transmitter and receiver positions: randomly selected in the 3-D space of size $(-4000, 4000) \times (-4000, 4000) \times (1000, 3000)$ m^3.
  - Object position and velocity: $\boldsymbol{u}^o = [-1000, 500, 1500]^T$ m, $\dot{\boldsymbol{u}}^o = [15, 15, 30]^T$ m/s.
  - Unknown time and frequency offsets: $b_\tau^o = 200$ m, $b_f^o = 5$ m/s.
  - Covariance matrix: $\boldsymbol{Q} = \mathrm{blkdiag}(\boldsymbol{Q}_{\boldsymbol{r}}, \boldsymbol{Q}_{\dot{\boldsymbol{r}}}, \boldsymbol{Q}_{\boldsymbol{d}}, \boldsymbol{Q}_{\dot{\boldsymbol{d}}})$, $\boldsymbol{Q}_{\dot{\boldsymbol{r}}} = 0.1\boldsymbol{Q}_{\boldsymbol{r}}$ and $\boldsymbol{Q}_{\dot{\boldsymbol{d}}} = 0.1\boldsymbol{Q}_{\boldsymbol{d}}$.
  - MSE: $\mathrm{MSE}(\hat{\boldsymbol{\zeta}}) = \frac{1}{KL}\sum_{j=1}^{K}\sum_{i=1}^{L}\|\hat{\boldsymbol{\zeta}}_{ji} - \boldsymbol{\zeta}_j^o\|^2$,
    where $\hat{\boldsymbol{\zeta}}_{ji}$ is the estimate of the true value $\boldsymbol{\zeta}_j^o$ in the i-th Monte Carlo (MC) run for the j-th configuration. K and L are the numbers of configurations and MC runs, respectively. Set $K = 10$ and $L = 1000$.

Simulations

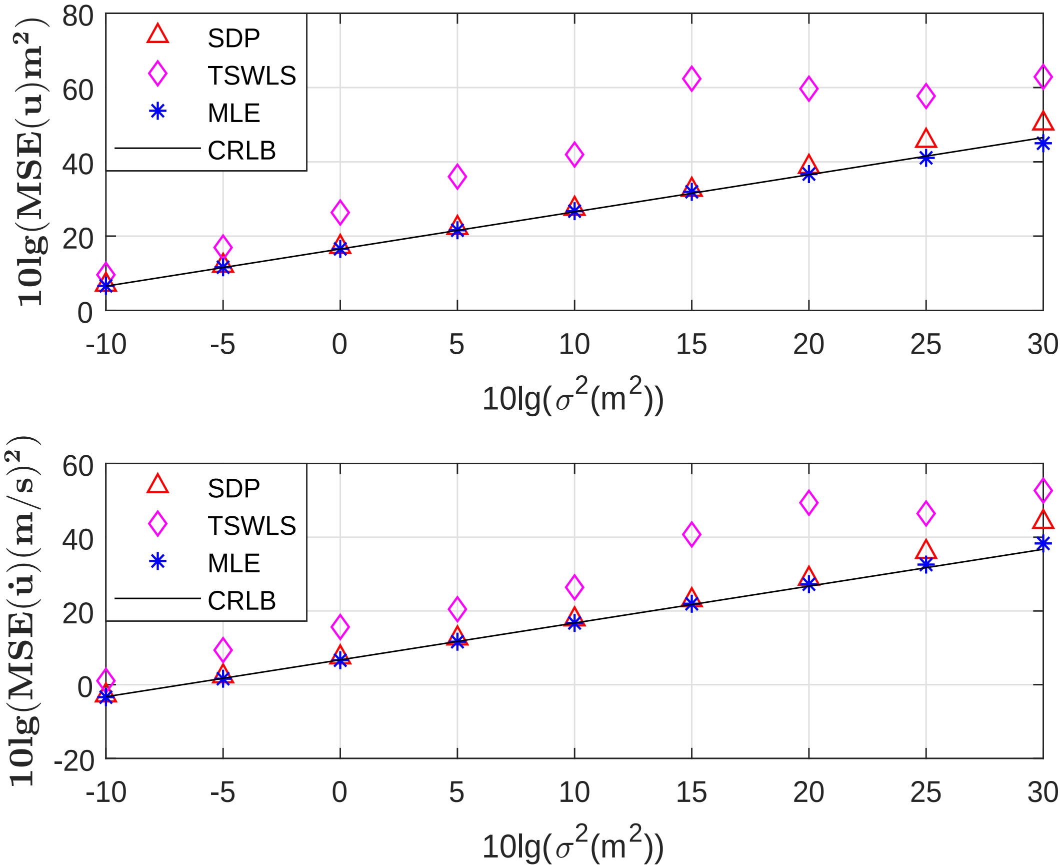

Figure 2: MSE comparison for 3-D localization: ten randomly generated configurations each with one transmitter and **five receivers**.

Simulations

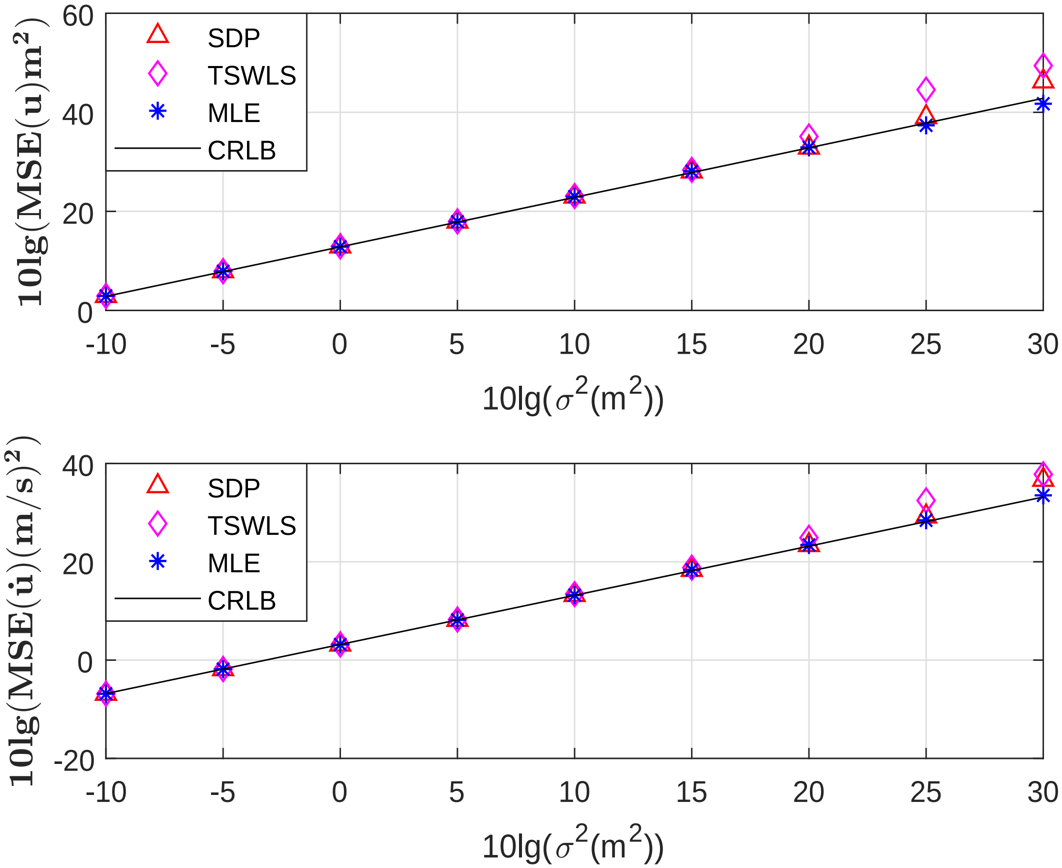

Figure 3: MSE comparison for 3-D localization: ten randomly generated configurations each with one transmitter and **six receivers**.

Simulations

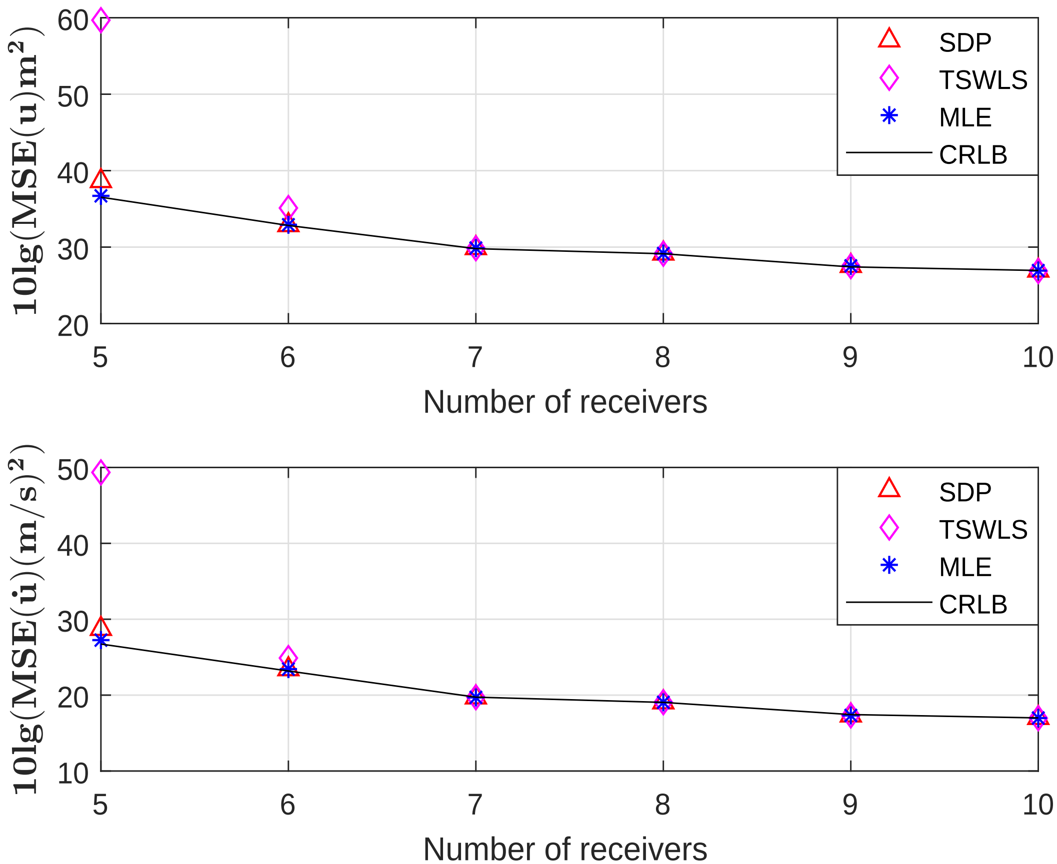

Figure 4: MSE comparison for 3-D localization at $\sigma = 10$ m as the number of receivers varies: ten randomly generated configurations.

Thank You