Semidefinite Relaxation Method for Moving Object Localization Using a Stationary Transmitter at Unknown Position

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Background

- Multistatic localization becomes a popular research topic as it finds applications in target tracking, automatic driving, robotics, and many others.
- The information of time-delay (TD), differential-arrival-times, Doppler-frequency-shift (DFS) can be used.
- Transmitter position may not be known
  - The transmitter may be located somewhere in which the GPS is not accessible.
  - It could also happen in the passive coherent location (PCL) systems, where an existing facility is used as the transmitter and which facility the signal comes from for positioning is not known.
  - The position of the transmitter can even be intentionally left unknown for lower hardware and implementation costs.

Measurement Model

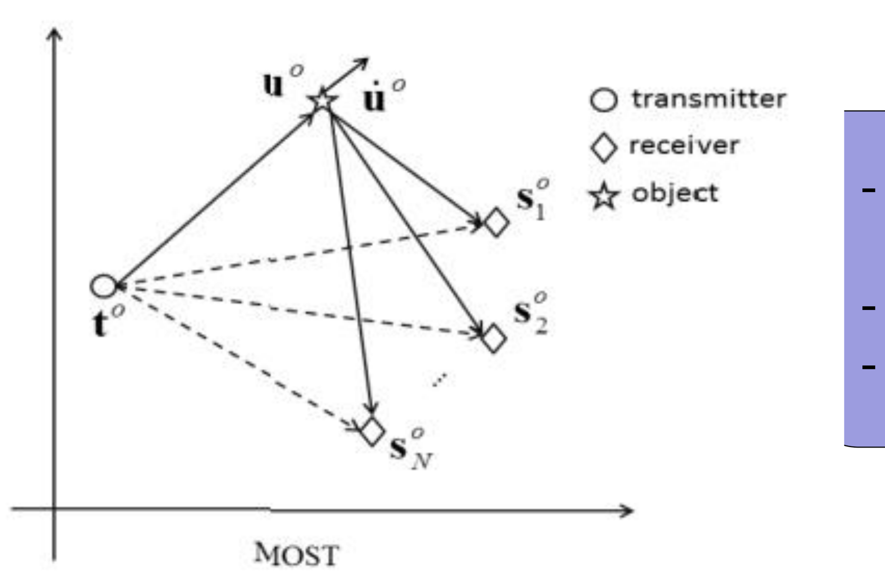

Fig. Illustration of the localization scenario of MOST, solid lines represent the indirect paths and dashed lines denote the direct paths.

- $\boldsymbol{u}^o$ and $\dot{\boldsymbol{u}}^o$: object position and velocity, respectively (unknown);
- $\boldsymbol{t}^o$: transmitter position (unknown);
- $\boldsymbol{s}_j^o$ j-th receiver position (known);

- The measurement range in the indirect path and direct path are

$$r_j = \left\|\boldsymbol{u}^o - \boldsymbol{s}_j^o\right\| + \left\|\boldsymbol{u}^o - \boldsymbol{t}^o\right\| + b_\tau^o + \varepsilon_{r,j}, \quad d_j = \left\|\boldsymbol{t}^o - \boldsymbol{s}_j^o\right\| + b_\tau^o + \varepsilon_{d,j},\ j = 1,\ldots,N.$$

- The measurement range rate in the indirect path and direct path are is

$$\dot{r}_j = \boldsymbol{\rho}_{\boldsymbol{u}-\boldsymbol{s}_j^o}^T \dot{\boldsymbol{u}}^o + \boldsymbol{\rho}_{\boldsymbol{u}-\boldsymbol{t}^o}^T \dot{\boldsymbol{u}}^o + b_f^o + \dot{\varepsilon}_{r,j}, \quad \dot{d}_j = d_j^o + b_f^o + \dot{\varepsilon}_{d,j},\ j = 1,\cdots,N.$$

We stack the noise into vectors $\boldsymbol{\varepsilon}_r = [\varepsilon_{r,1}, \varepsilon_{r,2}, \ldots, \varepsilon_{r,N}]^T$, $\dot{\boldsymbol{\varepsilon}}_{\dot{r}}$, $\boldsymbol{\varepsilon}_d$ and $\dot{\boldsymbol{\varepsilon}}_{\dot{d}}$ can be defined similarly. Composite vector $\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}_r^T, \dot{\boldsymbol{\varepsilon}}_r^T, \boldsymbol{\varepsilon}_d^T, \dot{\boldsymbol{\varepsilon}}_d^T]^T$. Assume that $\mathrm{E}(\boldsymbol{\varepsilon}) = 0$, $\mathrm{E}(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T) = \boldsymbol{Q}$.

Semidefinite Relaxation Method

- Model Transformation：

- In the indirect path, let $\xi^o = \left\|\boldsymbol{u}^o - \boldsymbol{t}^o\right\|$. Moving ξ^o and b_τ^o to the left-hand side and squaring both sides yield

$$\frac{1}{2}(r_j^2 - \|\boldsymbol{s}_j^o\|^2) + \boldsymbol{s}_j^{oT}\boldsymbol{u}^o - r_j\xi^o - r_j b_\tau^o - \frac{1}{2}\varphi_1^o = \|\boldsymbol{u}^o - \boldsymbol{s}_j^o\|\varepsilon_{r,j},\ j = 1,\ldots,N, \quad (1)$$

where $\varphi_1^o = \|\boldsymbol{u}^o\|^2 - \xi^{o2} - 2\xi^o b_\tau^o - b_\tau^{o2}$.

- The equation for range rate is obtained based on time-derivative of (1) and incorporating the frequency offset,

$$r_j\dot{r}_j + \boldsymbol{s}_j^{oT}\dot{\boldsymbol{u}}^o - \dot{r}_j\xi^o - r_j\dot{\xi}^o - \dot{r}_j b_\tau^o - r_j b_f^o - \varphi_2^o = \boldsymbol{\rho}_{\boldsymbol{u}^o-\boldsymbol{s}_j^o}^T\dot{\boldsymbol{u}}^o\varepsilon_{r,j} + \|\boldsymbol{u}^o - \boldsymbol{s}_j^o\|\dot{\varepsilon}_{\dot{r}}, \quad (2)$$

where $\dot{\xi}^o = \boldsymbol{\rho}_{\boldsymbol{u}^o-\boldsymbol{t}^o}^T\dot{\boldsymbol{u}}^o$ and $\varphi_2^o = \boldsymbol{u}^{oT}\dot{\boldsymbol{u}}^o - \xi^o\dot{\xi}^o - \xi^o\dot{\xi}^o - \dot{\xi}^o b_\tau^o - \xi^o b_f^o - b_\tau^o b_f^o$.

- Similar processing for direct path gives

$$\frac{1}{2}(d_j^2 - \|\boldsymbol{s}_j^o\|^2) + \boldsymbol{s}_j^{oT}\boldsymbol{t}^o - d_j b_\tau^o - \frac{1}{2}\varphi_3^o = \|\boldsymbol{t}^o - \boldsymbol{s}_j^o\|\varepsilon_{d,j},\ j = 1,\ldots,N, \quad (3)$$

$$\dot{d}_j = b_f^o + \dot{\varepsilon}_{\dot{d},j},\ j = 1,\ldots,N. \quad (4)$$

- By defining the new vector,

$$\boldsymbol{y}^o = [\underbrace{\boldsymbol{u}^{oT}}_{1\times k}, \underbrace{\dot{\boldsymbol{u}}^{oT}}_{1\times k}, \underbrace{\boldsymbol{t}^{oT}}_{1\times k}, \underbrace{b_\tau^o}_{1}, \underbrace{b_f^o}_{1}, \underbrace{\xi^o}_{1}, \underbrace{\dot{\xi}^o}_{1}, \underbrace{\varphi_1^o}_{1}, \underbrace{\varphi_2^o}_{1}, \underbrace{\varphi_3^o}_{1}]^T.$$

- the pseudo-linear equations in (1), (2), (3), and (4) can be written in the matrix form:

$$\boldsymbol{b} - \boldsymbol{A}\boldsymbol{y}^o \simeq \boldsymbol{B}\boldsymbol{\varepsilon}, \quad (5)$$

$$\boldsymbol{B} = \mathrm{blkdiag}\left(\begin{bmatrix}\boldsymbol{B}_r & \boldsymbol{0}\\ \boldsymbol{B}_{\dot{r}} & \boldsymbol{B}_r\end{bmatrix}, \begin{bmatrix}\boldsymbol{B}_d & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}\right),$$

$$\boldsymbol{b} = [\boldsymbol{b}_r^T, \boldsymbol{b}_{\dot{r}}^T, \boldsymbol{b}_d^T, \boldsymbol{b}_{\dot{d}}^T]^T,\ \boldsymbol{A} = [\boldsymbol{A}_r^T, \boldsymbol{A}_{\dot{r}}^T, \boldsymbol{A}_d^T, \boldsymbol{A}_{\dot{d}}^T]^T.$$

- The definitions of the vectors and matrices in (5) are

$$\boldsymbol{B}_r = \mathrm{diag}(\|\boldsymbol{u}^o - \boldsymbol{s}_1^o\|, \|\boldsymbol{u}^o - \boldsymbol{s}_2^o\|, \ldots, \|\boldsymbol{u}^o - \boldsymbol{s}_N^o\|),$$
$$\boldsymbol{B}_{\dot{r}} = \mathrm{diag}(\boldsymbol{\rho}_{\boldsymbol{u}^o-\boldsymbol{s}_1^o}^T\dot{\boldsymbol{u}}^o, \boldsymbol{\rho}_{\boldsymbol{u}^o-\boldsymbol{s}_2^o}^T\dot{\boldsymbol{u}}^o, \ldots, \boldsymbol{\rho}_{\boldsymbol{u}^o-\boldsymbol{s}_N^o}^T\dot{\boldsymbol{u}}^o),$$
$$\boldsymbol{B}_d = \mathrm{diag}(\|\boldsymbol{t}^o - \boldsymbol{s}_1^o\|, \|\boldsymbol{t}^o - \boldsymbol{s}_2^o\|, \ldots, \|\boldsymbol{t}^o - \boldsymbol{s}_N^o\|),$$
$$\boldsymbol{b}_r = \frac{1}{2}\left[(r_1^2 - \|\boldsymbol{s}_1^o\|^2), \ldots, (r_N^2 - \|\boldsymbol{s}_N^o\|^2)\right]^T,$$
$$\boldsymbol{b}_d = \frac{1}{2}\left[(d_1^2 - \|\boldsymbol{s}_1^o\|^2), \ldots, (d_N^2 - \|\boldsymbol{s}_N^o\|^2)\right]^T,$$
$$\boldsymbol{b}_{\dot{r}} = [r_1\dot{r}_1, r_2\dot{r}_2, \ldots, r_N\dot{r}_N]^T, \boldsymbol{b}_{\dot{d}} = [\dot{d}_1, \dot{d}_2, \ldots, \dot{d}_N]^T,$$

and

$$\boldsymbol{A}_r = [\boldsymbol{a}_{r_1}, \boldsymbol{a}_{r_2}, \ldots, \boldsymbol{a}_{r_N}]^T, \boldsymbol{A}_{\dot{r}} = [\boldsymbol{a}_{\dot{r}_1}, \boldsymbol{a}_{\dot{r}_2}, \ldots, \boldsymbol{a}_{\dot{r}_N}]^T,$$
$$\boldsymbol{A}_d = [\boldsymbol{a}_{d_1}, \boldsymbol{a}_{d_2}, \ldots, \boldsymbol{a}_{d_N}]^T, \boldsymbol{A}_{\dot{d}} = [\boldsymbol{a}_{\dot{d}_1}, \boldsymbol{a}_{\dot{d}_2}, \ldots, \boldsymbol{a}_{\dot{d}_N}]^T,$$

with

$$\boldsymbol{a}_{r_j} = [-\boldsymbol{s}_j^{oT}, \boldsymbol{0}_k^T, \boldsymbol{0}_k^T, r_j, 0, r_j, 0, \frac{1}{2}, \boldsymbol{0}_2^T]^T,$$
$$\boldsymbol{a}_{\dot{r}_j} = [\boldsymbol{0}_k^T, -\boldsymbol{s}_j^{oT}, \boldsymbol{0}_k^T, \dot{r}_j, r_j, \dot{r}_j, r_j, 0, 1, 0]^T,$$
$$\boldsymbol{a}_{d_j} = [\boldsymbol{0}_k^T, \boldsymbol{0}_k^T, -\boldsymbol{s}_j^{oT}, d_j, 0, \boldsymbol{0}_4^T, \frac{1}{2}]^T,$$
$$\boldsymbol{a}_{\dot{d}_j} = [\boldsymbol{0}_k^T, \boldsymbol{0}_k^T, \boldsymbol{0}_k^T, 0, 1, \boldsymbol{0}_5^T]^T.$$

- Introducing $\boldsymbol{Y} = \boldsymbol{y}\boldsymbol{y}^T$ and dropping the nonconvex rank-1 constraint, we obtain the following SDP：

$$\begin{aligned}
\min_{\boldsymbol{Y},\boldsymbol{y}}\ & \mathrm{tr}\left\{\begin{bmatrix}\boldsymbol{Y} & \boldsymbol{y}\\ \boldsymbol{y}^T & 1\end{bmatrix}\boldsymbol{\Phi}\right\}\\
\text{s.t. } & Y_{(3k+3,3k+3)} = \mathrm{tr}\{\boldsymbol{Y}_{(1:k,1:k)}\}\\
& - 2\mathrm{tr}\{\boldsymbol{Y}_{(1:k,(2k+1):3k)}\}\\
& + \mathrm{tr}\{\boldsymbol{Y}_{((2k+1):3k,(2k+1):3k)}\},\\
& Y_{(3k+3,3k+4)} = \mathrm{tr}\{\boldsymbol{Y}_{(1:k,(k+1):2k)}\}\\
& - \mathrm{tr}\{\boldsymbol{Y}_{((2k+1):3k,(k+1):2k)}\},\\
& y_{(3k+5)} = \mathrm{tr}\{\boldsymbol{Y}_{(1:k,1:k)}\}\\
& - Y_{(3k+3,3k+3)} - 2Y_{(3k+1,3k+3)}\\
& - Y_{(3k+1,3k+1)},\\
& y_{(3k+6)} = \mathrm{tr}\{\boldsymbol{Y}_{(1:k,k+1:2k)}\} - Y_{(3k+3,3k+4)}\\
& - Y_{(3k+1,3k+4)} - Y_{(3k+3,3k+2)} - Y_{(3k+1,3k+2)},\\
& y_{(3k+7)} = \mathrm{tr}\{\boldsymbol{Y}_{(2k+1:3k,2k+1:3k)}\} - Y_{(3k+1,3k+1)},\\
& Y_{(3k+4,3k+4)} \le \mathrm{tr}\{\boldsymbol{Y}_{((k+1):2k,(k+1):2k)}\},\\
& \begin{bmatrix}\boldsymbol{Y} & \boldsymbol{y}\\ \boldsymbol{y}^T & 1\end{bmatrix} \succeq \boldsymbol{0},
\end{aligned}$$

where $\boldsymbol{\Phi} = \begin{bmatrix}\boldsymbol{A}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{A} & -\boldsymbol{A}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{b}\\ -\boldsymbol{b}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{A} & \boldsymbol{b}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{b}\end{bmatrix}$.

Simulation Results

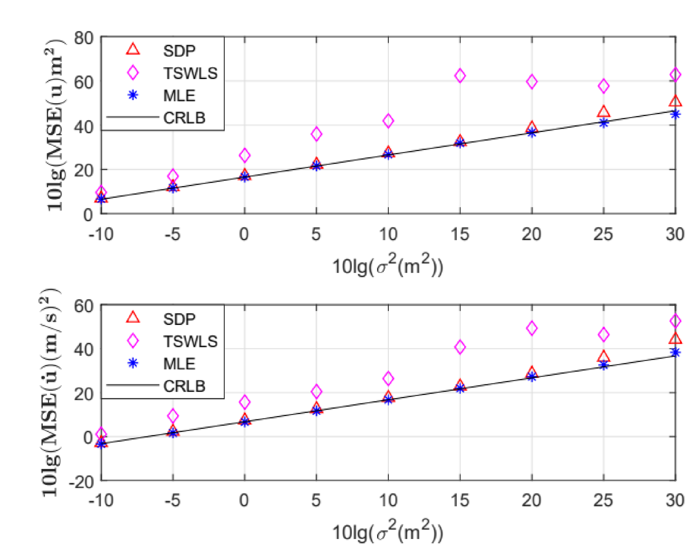

Fig. 2 MSE comparison for 3-D localization: ten randomly generated configurations each with one transmitter and five receivers.

- Transmitter and receiver positions: randomly selected in the 3-D space of size (-4000,4000) x (-4000,4000) x (-4000,4000) m^3;
- $\boldsymbol{u}^o = [-1000, 500, 1500]^T$ m and $\dot{\boldsymbol{u}}^o = [15, 15, 30]^T$ m/s;
- $b_\tau^o = 200$ m and $b_f^o = 5$ m/s;
- Fig. 2 shows the simulation results. The proposed SDR method performs much better than TSWLS and achieves the CRLB accuracy at low noise levels.

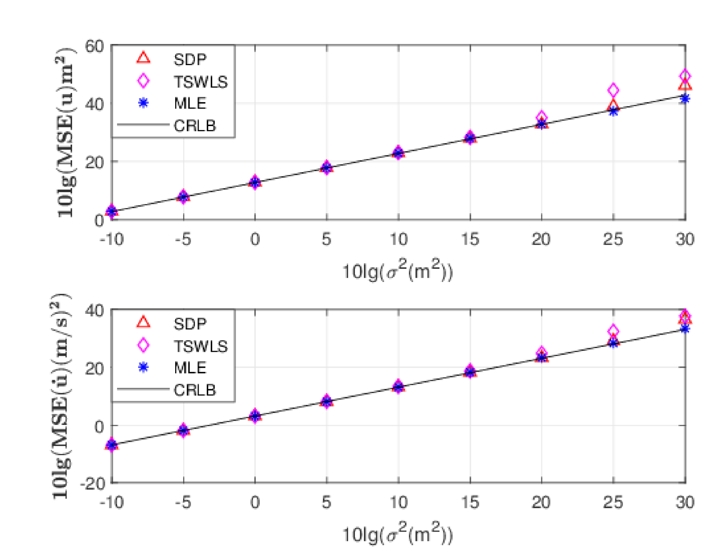

Fig. 3 MSE comparison for 3-D localization: ten randomly generated configurations each with one transmitter and six receivers.

- The proposed SDR method still outperforms the TSWLS method at large noise levels when using six receivers.
- We list in Table 1 the proportion of solutions of the proposed method in this simulation, indicating that rank-1 solutions can be achieved.

Table 1. Proportion of Rank-1 Solutions (10000 MC Runs in Total for Each Value of σ^2)

$10\lg(\sigma^2)$	-10	-5	0	5	10	15	20	25	30
Prop.(%)	100	100	100	99.2	96.7	93.4	87.8	77.8	67.7

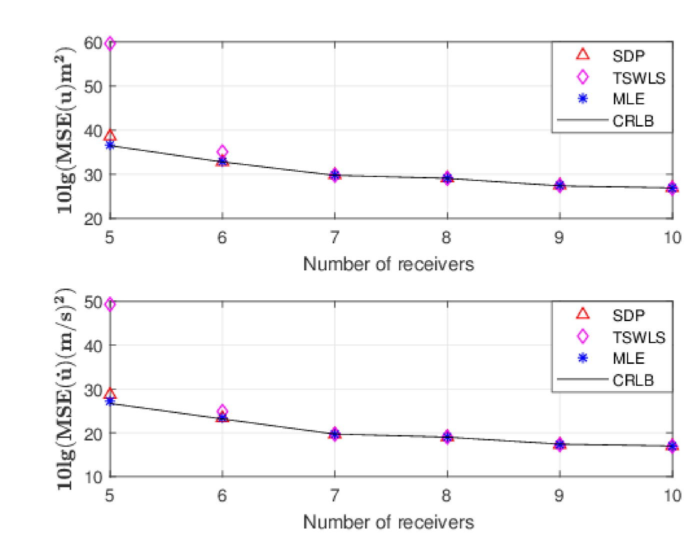

Fig. 4 MSE comparison for 3-D localization at $\sigma = 10$ m, as the number of receivers varies: ten randomly generated configurations.

- The estimation accuracy improves as the number of receivers increases.
- The proposed method outperforms the TSWLS method when using fewer receivers.

Conclusions

- We have formulated a WLS problem and solved it approximately using SDP by employing the SDR technique.
- The proposed SDR method is able to achieve the CRLB accuracy for the estimation of object position and velocity under mild Gaussian noise.