

# Waveform Design for Wireless Power Transfer with Power Amplifier and Energy Harvester Non- Linearities

Yumeng Zhang and Bruno Clerckx, *fellow, IEEE*

**Presenter: Yumeng Zhang**

Department of Electrical and Electronic Engineering  
Imperial College London

## Outline:

- **Motivation**
- **System Model**
- **Problem Formulation**
- **Optimization**
- **Simulation Results**



## Far-field wireless power transfer (WPT):

WPT generates and transmits suitable RF signals that propagate over the air before being captured and rectified into DC current via rectenna circuits at the receivers.

### Motivation:

- Free of periodic replacement or wires;
- Sustainable power sources for the internet of things (IoT) [1];
- Green technology and reduction in energy consumption [1].

### Feasibility:

- Lower power consumption on computing [2];
- Performance enhancement in WPT, i.e., advanced rectenna circuits, higher-efficient waveform design [3].

**Object: to boost the power harvesting performance in WPT by designing the waveform of the multi-carrier signals.**



Far-field wireless power transfer (WPT):



Fig 1. Block diagram of a generic WPT system.

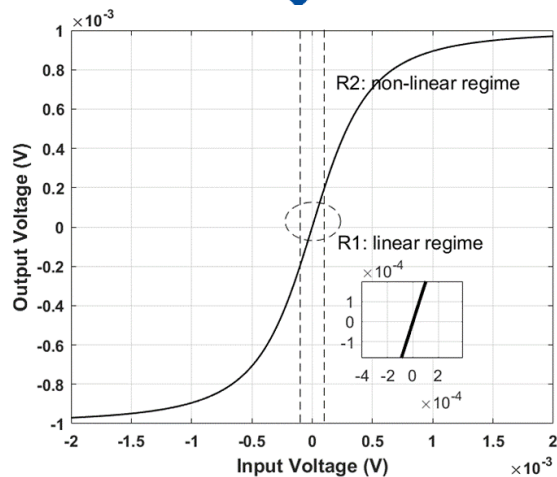


Fig 2. The transfer characteristics of the non-linear high power amplifier (HPA).

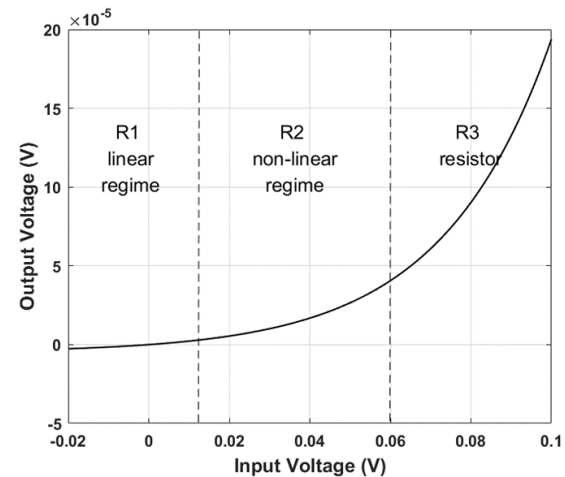


Fig 3. The transfer characteristics of the non-linear rectifier.

**Object: designing waveforms considering both HPA's and rectifier's non-linearity.**

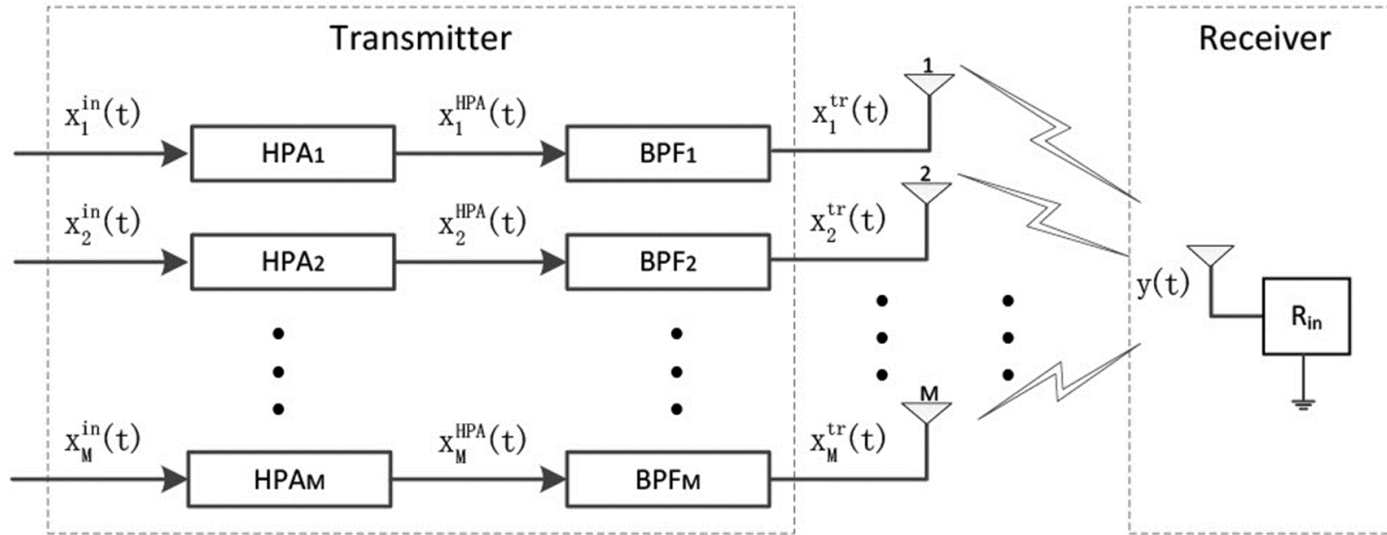


Fig 4. The WPT structure with transmitter's and rectenna's non-linearity. The transmitter is composed of a non-linear HPA and a band pass filter (BPF). The rectenna is composed of a non-linear rectifier and a low pass filter.

$$\tilde{x}_m^{\text{in}}(t) = \sum_{n=0}^{N-1} \tilde{w}_{n,m}^{\text{in}} e^{j2\pi f_n t}, \quad \tilde{x}_m^{\text{tr}}(t) = \sum_{n=0}^{N-1} \tilde{w}_{n,m}^{\text{tr}} e^{j2\pi f_n t}, \quad \tilde{y}(t) = \sum_{m=1}^M \sum_{n=0}^{N-1} \tilde{h}_{n,m}^{\text{tr}} \tilde{w}_{n,m}^{\text{tr}} e^{j2\pi f_n t},$$

After HPA
After BPF
After wireless propagation

$$\tilde{x}_m^{\text{HPA}}(t) = f_{\text{HPA}}(\tilde{x}_m^{\text{tr}}(t)) = G \tilde{x}_m^{\text{tr}}(t) / \left[ 1 + \left( \frac{G \tilde{x}_m^{\text{tr}}(t)}{A_s} \right)^{2\beta} \right]^{\frac{1}{2\beta}},$$

We use the solid state power amplifier (SSPA)'s model here.  $G$  is the small signal gain of SSPA,  $A_s$  is the saturation power of SSPA, and  $\beta$  is the smoothing parameter[4].

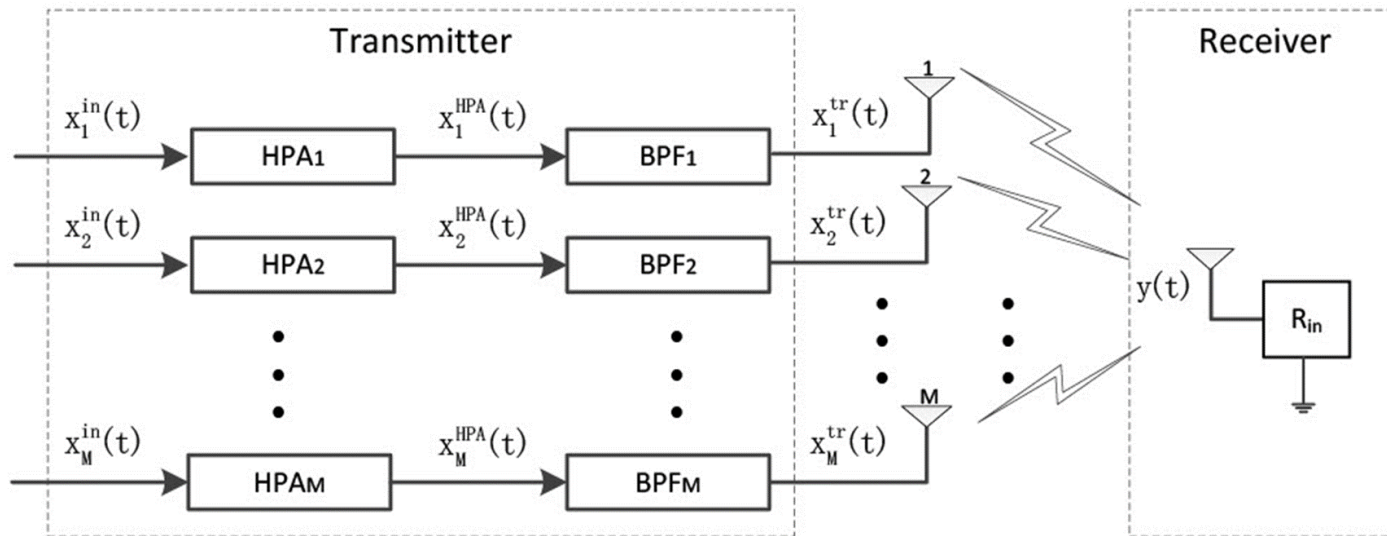


Fig 3. The WPT structure with transmitter's and rectenna's non-linearity. The transmitter is composed of a non-linear HPA and a band pass filter (BPF). The rectenna is composed of a non-linear rectifier and a low pass filter.

**The harvested power is proportional to the scaling term [3]:**

$$z_{DC} = k_2 R_{\text{ant}} \mathcal{E} \left\{ \Re \{ \tilde{y}(t) \}^2 \right\} + k_4 R_{\text{ant}} \mathcal{E} \left\{ \Re \{ \tilde{y}(t) \}^4 \right\},$$

where  $k_i = i_s / (i! (\eta_0 V_0)^i)$  with  $i_s$  being the reverse bias saturation current,  $\eta_0$  being the ideality factor,  $V_0$  being the thermal voltage of the diode and  $R_{\text{ant}}$  being the characteristic impedance of the receiving antenna.

$$\max_{\{\tilde{w}_{n,m}^{\text{in}}\}} z_{\text{DC}}(\{\tilde{w}_{n,m}^{\text{in}}\})$$

$$\text{s.t.} \quad \frac{1}{2} \sum_{m=1}^M \sum_{n=0}^{N-1} |\tilde{w}_{n,m}^{\text{in}}|^2 \leq P_{\text{in}}^{\text{max}}, \quad \leftarrow \text{The input power budget}$$

$$\frac{1}{2} \sum_{m=1}^M \sum_{n=0}^{N-1} |\tilde{w}_{n,m}^{\text{tr}} \{\tilde{w}_{n,m}^{\text{in}}\}|^2 \leq P_{\text{in}}^{\text{tr}}. \quad \leftarrow \text{For human safety}$$

**For tractability**

$$\max_{\{\tilde{w}_{n,m}^{\text{tr}}\}} z_{\text{DC}}(\{\tilde{w}_{n,m}^{\text{tr}}\}) \quad \leftarrow \text{A convex function with respect to } \{\tilde{w}_{n,m}^{\text{tr}}\}$$

$$\text{s.t.} \quad \frac{1}{2T} \sum_{m=1}^M \left\{ \frac{|\tilde{x}_m^{\text{in}}(t)|}{G} \left[ 1 - \left( \frac{\tilde{x}_m^{\text{in}}(t)}{A_s} \right)^{2\beta} \right]^{-\frac{1}{2\beta}} \right\}^2 \leq P_{\text{in}}^{\text{max}}, \quad \leftarrow \text{Convex with respect to } \{\tilde{w}_{n,m}^{\text{tr}}\}$$

$$\frac{1}{2} \sum_{m=1}^M \sum_{n=0}^{N-1} |\tilde{w}_{n,m}^{\text{tr}}|^2 \leq P_{\text{tr}}^{\text{max}}. \quad \leftarrow \text{Convex with respect to } \{\tilde{w}_{n,m}^{\text{tr}}\}$$

$$\begin{aligned} \max_{\{\tilde{w}_{n,m}^{\text{tr}}\}} & z_{\text{DC}}(\{\tilde{w}_{n,m}^{\text{tr}}\}) \\ \text{s.t.} & \frac{1}{2T} \sum_{m=1}^M \left\{ \frac{|\tilde{x}_m^{\text{in}}(t)|}{G} \left[ 1 - \left( \frac{\tilde{x}_m^{\text{in}}(t)}{A_s} \right)^{2\beta} \right]^{-\frac{1}{2\beta}} \right\}^2 \leq P_{\text{in}}^{\text{max}}, \\ & \frac{1}{2} \sum_{m=1}^M \sum_{n=0}^{N-1} |\tilde{w}_{n,m}^{\text{tr}}|^2 \leq P_{\text{tr}}^{\text{max}}. \end{aligned}$$

## Step 1: successive convex programming (SCP) for the objective function (Algorithm 1)

At the  $l$ -th iteration, the objective function is approximated as:

$$z_{\text{DC}}^{(l)} = \sum_{m=1}^M \sum_{n=0}^{N-1} \bar{a}_{n,m}^{(l)} \bar{w}_{n,m}^{\text{tr}} + \hat{a}_{n,m}^{(l)} \hat{w}_{n,m}^{\text{tr}}$$

## Step 2: interior-point (IP) method (Algorithm 2)

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### Algorithm 1: Successive convex programming (SCP)

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**Input:**  $(\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^{(0)}, \epsilon_0 > 0, l \leftarrow 1$ ;

**Output:**  $(\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^*$ ;

**Repeat:**

- 1: Compute  $(\{\bar{\alpha}\}, \{\hat{\alpha}\})^{(l)}$  at the operating point  $(\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^{(l-1)}$  using Taylor expansion;
  - 2: Compute  $(\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^{(l)}$  using Algorithm 2;
  - 3: Update  $(\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^* \leftarrow (\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^{(l)}$ ;
  - 4: Quit if  $|\{(\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^{(l)} - (\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^{(l-1)}\}| < \epsilon_0$ ;
  - 5:  $l \leftarrow l + 1$ ;
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### Algorithm 2: Interior-point

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**Input:**  $(\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^{(B_0)} \leftarrow (\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^{(l-1)}, t > 0, \mu_B > 0, \epsilon_B > 0$ ;

**Output:**  $(\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^{(l)}$ ;

**Repeat:**

- 1: Compute  $(\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})$  by minimizing problem (16) using Newton's Method with initialised point  $(\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^{(B_0)}$ ;
  - 2: Update  $(\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^{(l)} \leftarrow (\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})$ ;
  - 3: Quit if  $2/t < \epsilon_B$ ;
  - 4:  $t \leftarrow \mu_B t, (\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^{(B_0)} \leftarrow (\{\bar{w}_n^{\text{tr}}\}, \{\hat{w}_n^{\text{tr}}\})^{(l)}$ ;
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## ➤ Simulation results

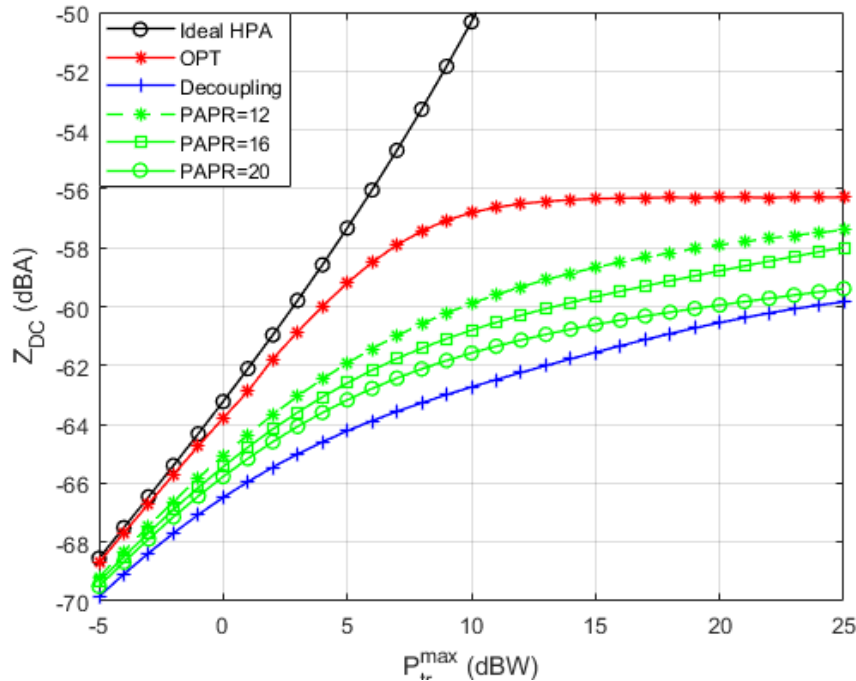


Fig.5 Power harvesting performance with  $G=1$ ,  $A_s=10$  dBV,  $P_{in}^{max}=25$  dBW,  $N=8$ ,  $M=1$ .

Name (color)	Definition
'Ideal HPA' (black)	Assume an ideal linear HPA (benchmark)
'OPT' (red)	The proposed waveform
'Decoupling' (blue)	Waveform in [3] only considering rectenna's non-linearity
'PAPR' (green)	Add PAPR constraints compared with 'Decoupling'

**Setup:** a Wi-Fi-like scenario with  $f_0 = 5.18$  GHz.

**HPA:**  $\beta = 1$ ,  $G = 1$ .

**Rectenna:**  $i_s = 5\mu\text{A}$ ,  $\eta=1.05$ ,  $v_0 = 25.86\text{mV}$ ,

$$R_{\text{ant}} = 50\Omega$$

**Path loss:** 58 dBi; **Antenna gain:** 2 dBi.

- HPA's non-linearity **degrades** the power harvesting performance.
- HPA's saturation power **limits** the power harvesting performance.
- The proposed waveform (red) **outperforms** the waveform considering rectenna's non-linearity only (blue).
- Low-PAPR signals **suffer less** HPA's degradation.

## ➤ Simulation results

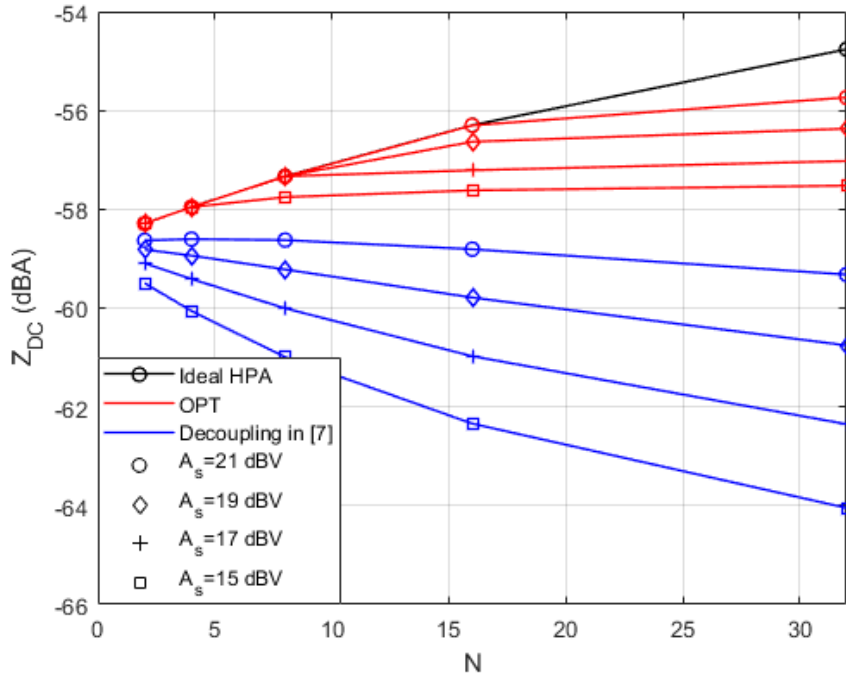


Fig.6 Power harvesting performance as a function of  $N$  with different  $A_s$ ,  $G=1$ ,  $P_{in}^{max}=25$  dBW,  $P_{tr}^{max}=25$  dBW,  $M=1$ .

- Larger HPA's saturation power, larger harvested power.
- For the saturation power **large enough**, the proposed waveform achieves the largest harvested power as with an ideal HPA.
- A larger  $N$  **does not** necessarily benefit the power harvesting performance.



(Larger  $N$  gives larger PAPR)

- The harvested power **saturates** with increasing  $N$ .



(In contrast with [3], where the harvested power is proportionally to  $N$ .)

**Thank you**

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