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Cell-free Massive MIMO: Exploiting the WAX Decomposition.

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Acknowledgments

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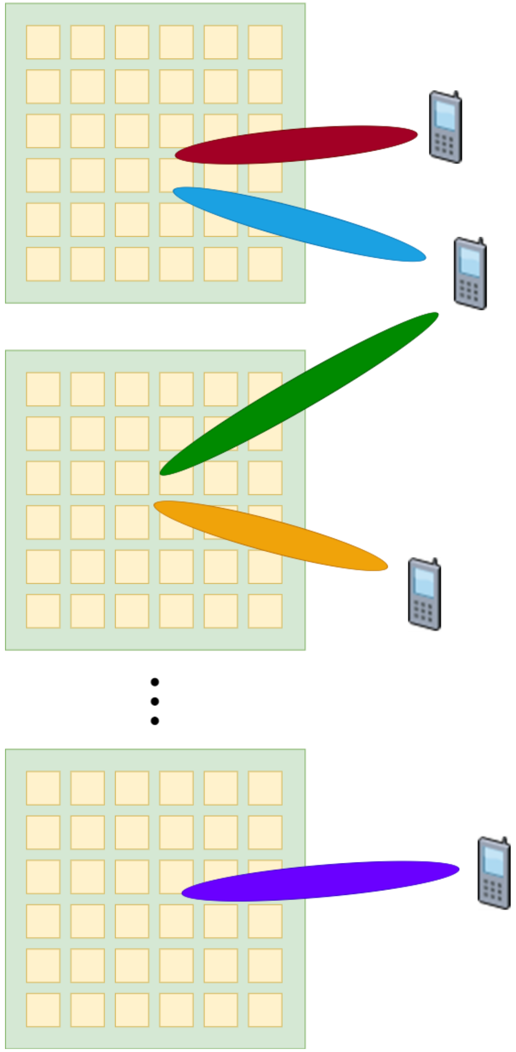
Outline

- Introduction
- General Scenario
- Background
- System model
- WAX decomposition
- WAX decomposition of sparse channels
- Numerical results
- Wrap-up

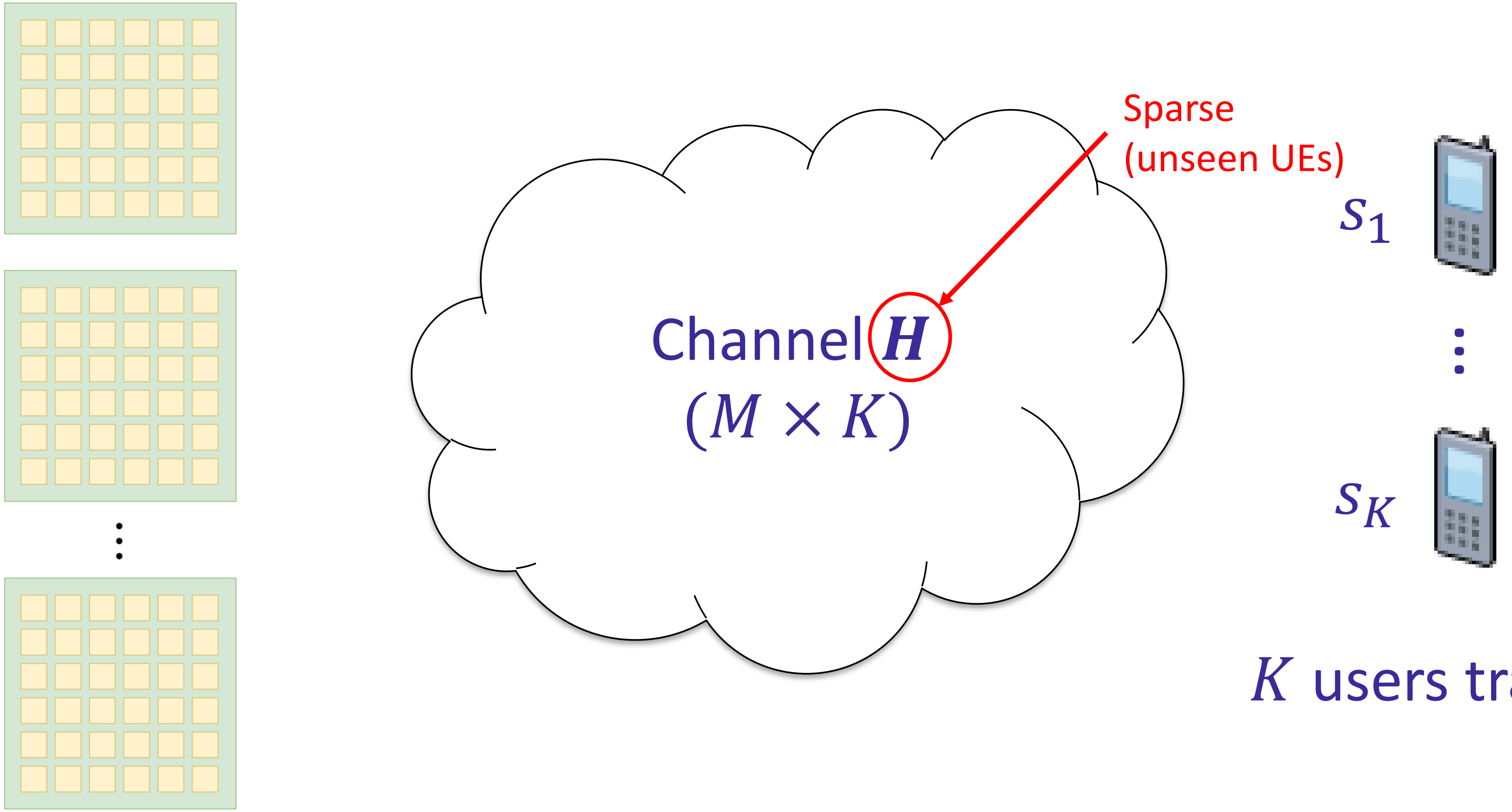
Introduction



- Mobile broadband communications (5G, 6G, etc)
- Cell-free massive MIMO
- Massive number of antennas/APs.
- Centralized processing: Increased interconnection bandwidth to CPU.
- Trend towards decentralized. Increase complexity at the nodes to reduce information transfer to CPU.

General scenario (uplink)

(cplx BB vector Rx)
$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}$

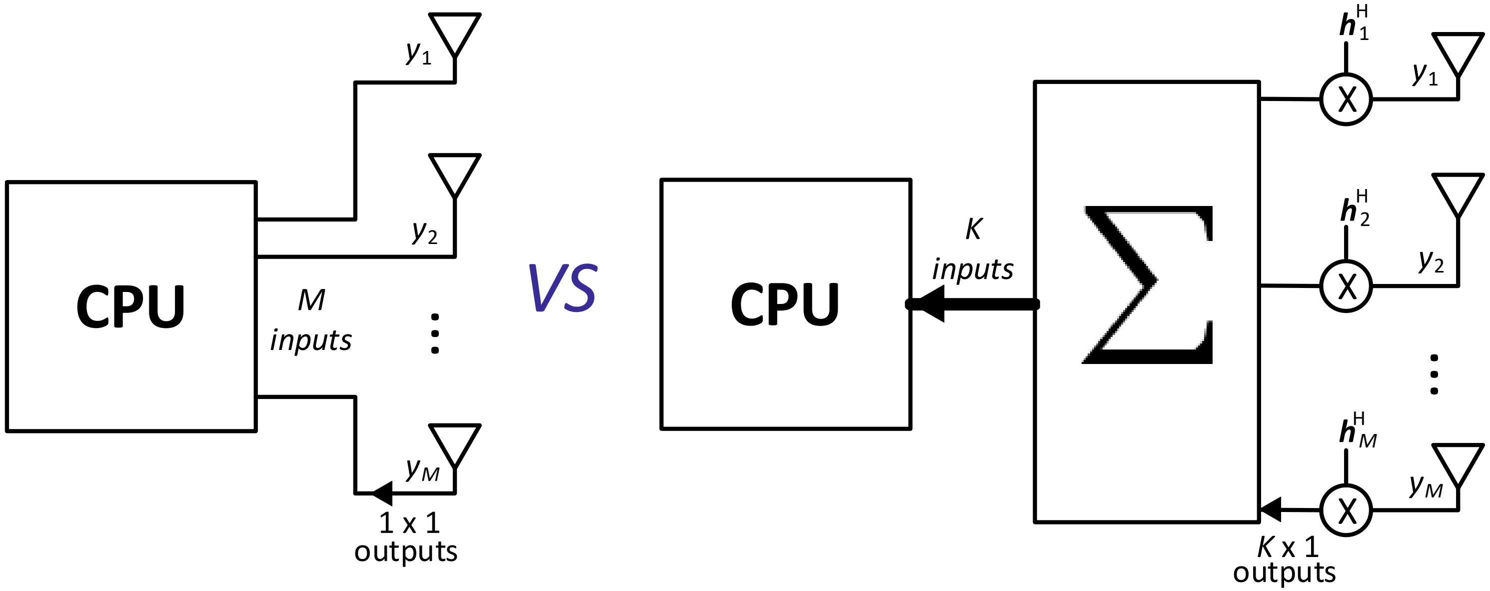


K users transmit $\boldsymbol{s}$

P cell-free mMIMO APs, N antennas each

$\mathrm{M} = PN$ total antennas

Centralized VS Decentralized (Background)



- Information lossless (both)
- M vs K inputs to CPU
- 0 vs K multiplications per antenna

[1] *J. Rodríguez Sánchez et al.*, "**Decentralized Massive MIMO: Is there Anything to be Discussed?**", *ISIT* 2019

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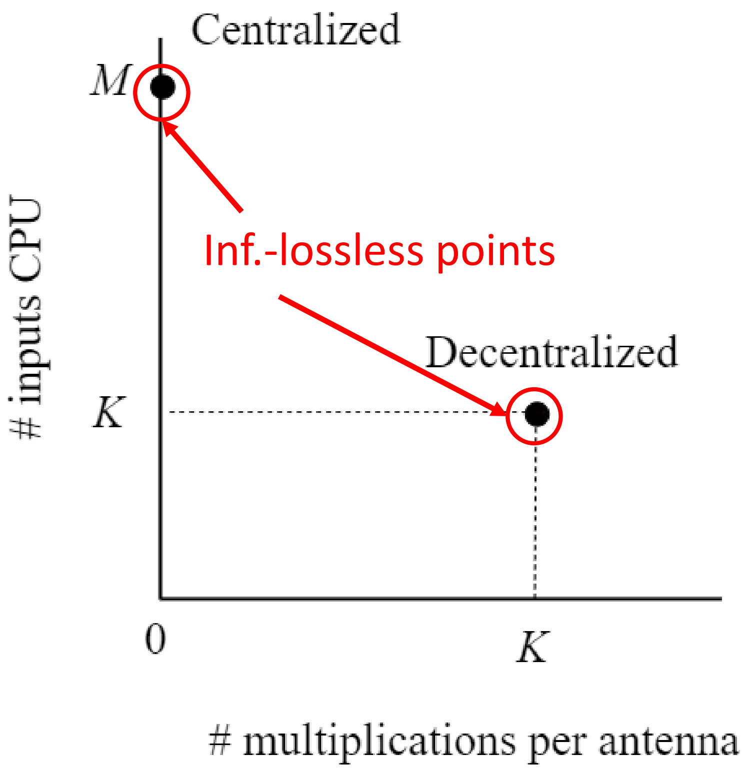
Trade-off (Background)

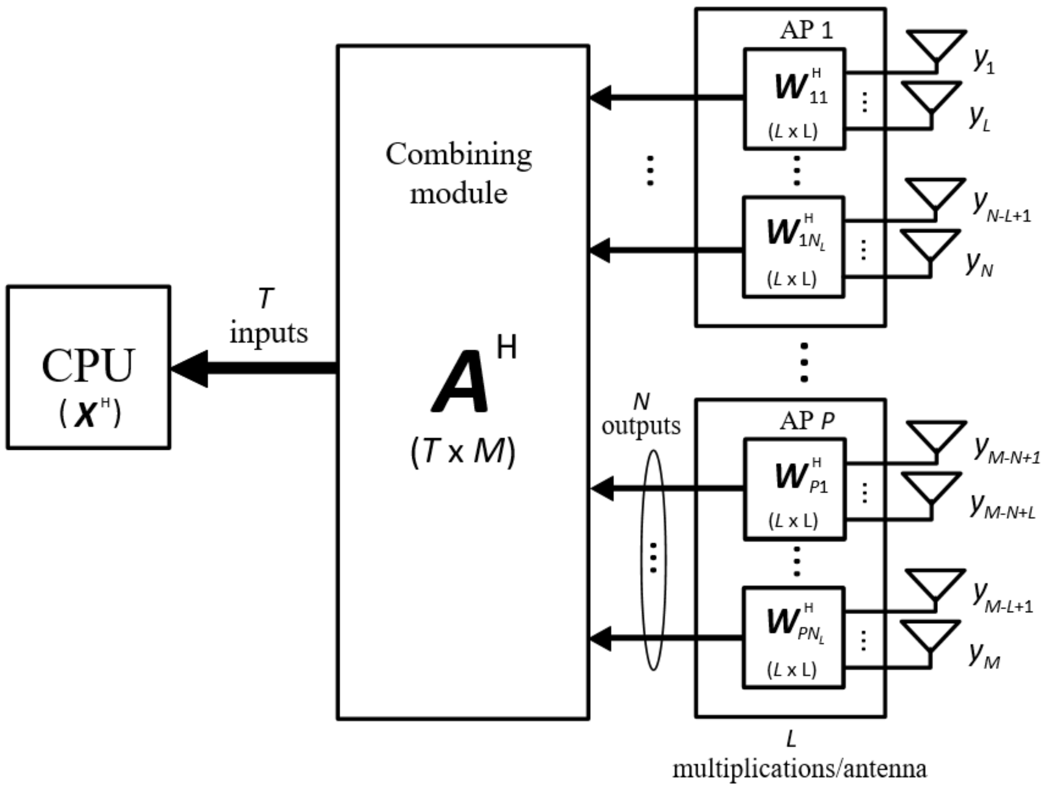


Level of decentralization (inputs CPU)

VS

Decentralized processing complexity (mult. per antenna)

System model

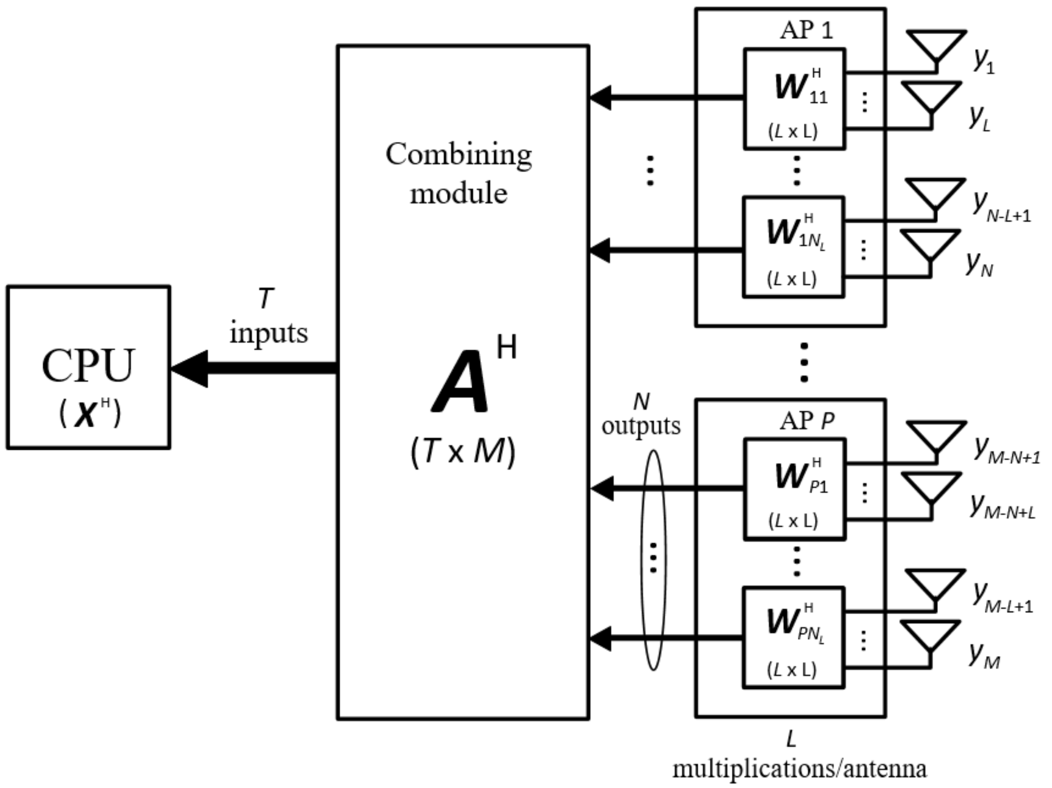


- Trade-off parameters L, T
- Design variables $\boldsymbol{A}$ $(M \times T)$
- Tunable variables $\boldsymbol{W}_{pn}(L \times L), \boldsymbol{X}$ $(T \times ?)$

- T ⟶ # inputs to CPU
- L ⟶ # mult. per ant.
- $\boldsymbol{A}$ ⟶ Combining module (HW?)
- $\boldsymbol{W}_{pn}$ ⟶ Decentralized filters
- $\boldsymbol{X}$ ⟶ Processing at CPU

[2] *J. Vidal Alegría et al.*, "**Trade-offs in Decentralized Multi-Antenna Architectures: The WAX Decomposition**", *TSP, 2020.* (General framework)

System model



- Trade-off parameters L, T
- Design variables $\boldsymbol{A}$ $(M \times T)$
- Tunable variables $\boldsymbol{W}_{pn}(L \times L), \boldsymbol{X}$ $(T \times ?)$

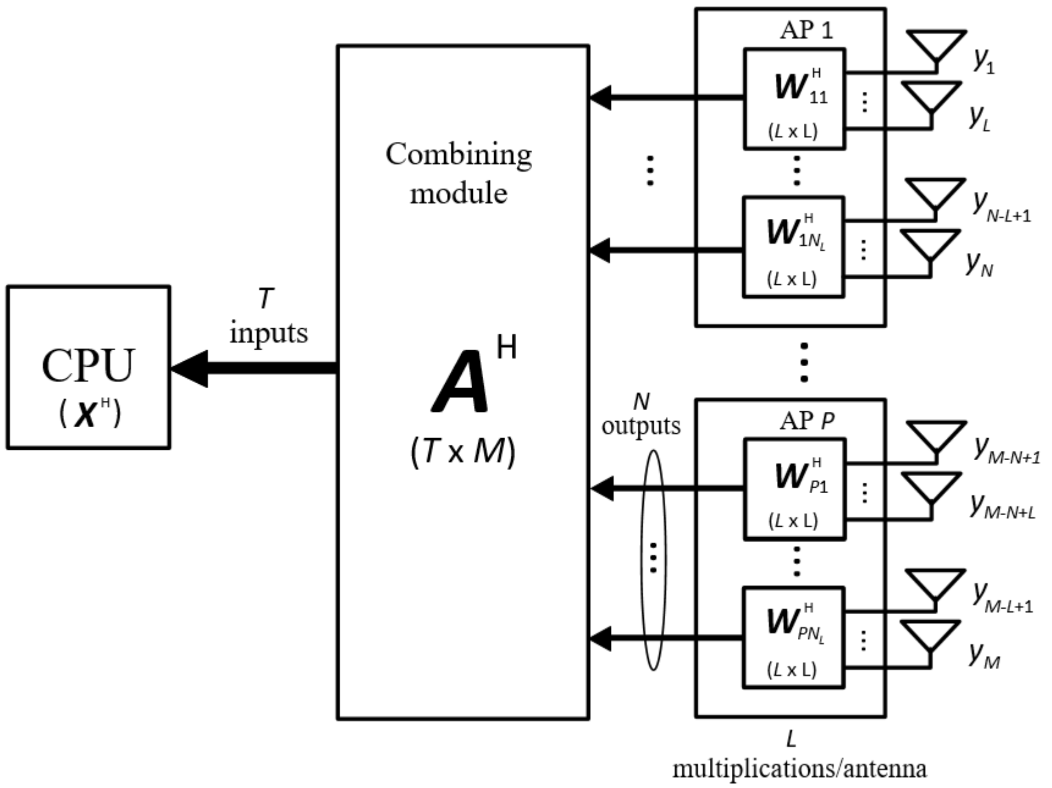
Resulting linear processing

$$\boldsymbol{z} = \boldsymbol{X}^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{y}, \quad \boldsymbol{W} = \mathrm{diag}(\boldsymbol{W}_{11}, \dots, \boldsymbol{W}_{PN_{\mathrm{L}}})$$

(Recall $\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}$)

[2] *J. Vidal Alegría et al.*, "**Trade-offs in Decentralized Multi-Antenna Architectures: The WAX Decomposition**", *TSP, 2020.* (General framework)

System model



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MF simple information-lossless transformation

$$\boldsymbol{z} = \boldsymbol{H}^{\mathrm{H}}\boldsymbol{y}$$

Information lossless $\Leftrightarrow \mathrm{I}(\mathbf{z};\mathbf{s}) = \mathrm{I}(\mathbf{y};\mathbf{s}) \Leftrightarrow \boldsymbol{WAX} = \boldsymbol{H}$, for some $\boldsymbol{W}, \boldsymbol{X}$

(See [2])

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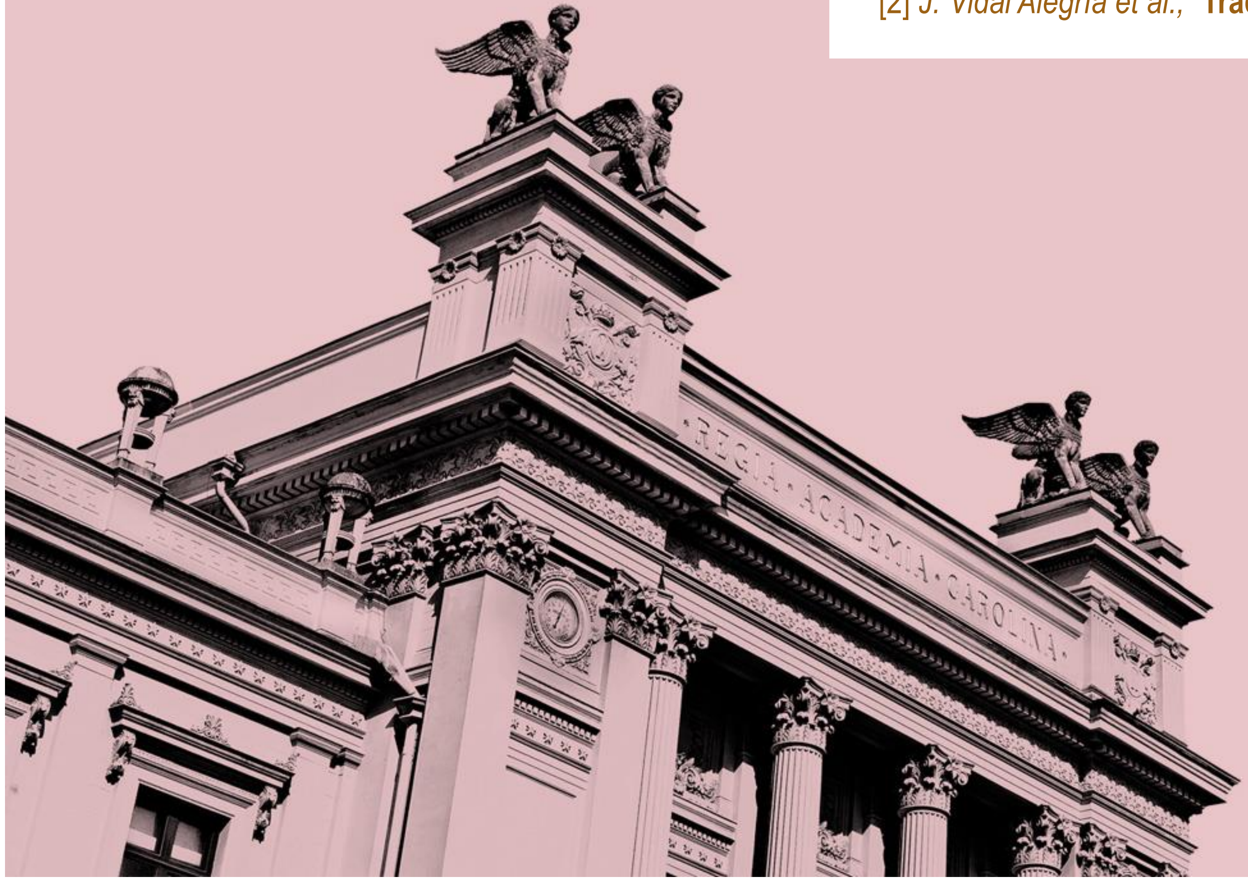
Previous results

[2] *J. Vidal Alegría et al.*, "**Trade-offs in Decentralized Multi-Antenna Architectures: The WAX Decomposition**", *TSP, 2020.*



WAX decomposition (Previous work from [2])

- Decomposition of $\boldsymbol{H}$ $(M \times K)$ for fixed $\boldsymbol{A}$ $(M \times T)$, where $\boldsymbol{W} = \mathrm{diag}\left(\boldsymbol{W}_{11}, \dots, \boldsymbol{W}_{PN_{\mathrm{L}}}\right)$, $\boldsymbol{W}_{pn}$ $(L \times L)$.

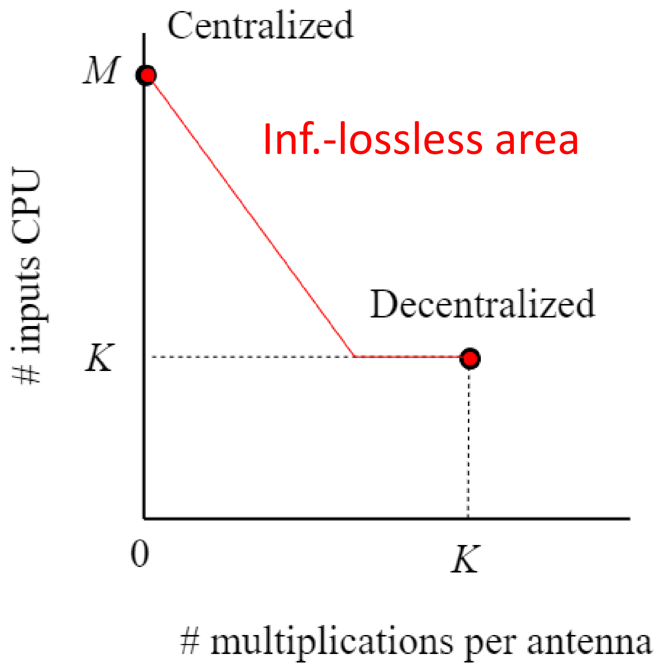
$$\boldsymbol{H} = \boldsymbol{W}\boldsymbol{A}\boldsymbol{X}$$

<u>Example:</u>

$$\underbrace{\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 \end{bmatrix}}_{\boldsymbol{W}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} -2 & -1 & -1 & 2 & 2 \\ 1 & -2 & -1 & 1 & 2 \\ 1 & -1 & -2 & -1 & -2 \\ 0 & 2 & 0 & -1 & 0 \\ 0 & -1 & 2 & 1 & -2 \end{bmatrix}}_{\boldsymbol{X}} = \underbrace{\begin{bmatrix} -3 & 1 & 0 & 1 & 0 \\ -2 & -4 & -3 & 5 & 6 \\ 1 & 1 & -2 & -2 & -2 \\ 0 & -2 & 0 & 1 & 0 \\ -2 & -2 & 5 & 4 & -4 \\ 2 & -2 & 3 & 0 & -4 \\ 1 & -2 & 2 & 3 & 4 \\ 4 & -2 & 2 & 0 & -8 \end{bmatrix}}_{\boldsymbol{H}}$$

WAX decomposition (Previous work from [2])

$$\boldsymbol{H} = \boldsymbol{W}\boldsymbol{A}\boldsymbol{X}$$



- Decomposition of $\boldsymbol{H}$ $(M \times K)$ for fixed $\boldsymbol{A}$ $(M \times T)$, where $\boldsymbol{W} = \mathrm{diag}\left(\boldsymbol{W}_{11}, \ldots, \boldsymbol{W}_{PN_\mathrm{L}}\right)$, $\boldsymbol{W}_{pn}(L \times L)$.
- Main result:
  - For a randomly chosen $\boldsymbol{A}$, iff

$$T > \min\left(M\frac{K-L}{K}, K-1\right)$$

a randomly chosen $\boldsymbol{H}$ (e.g., IID Rayleigh fading) <u>accepts WAX decomposition</u> with probability 1

Inf.-lossless processing within our system model for e.g. IID channel

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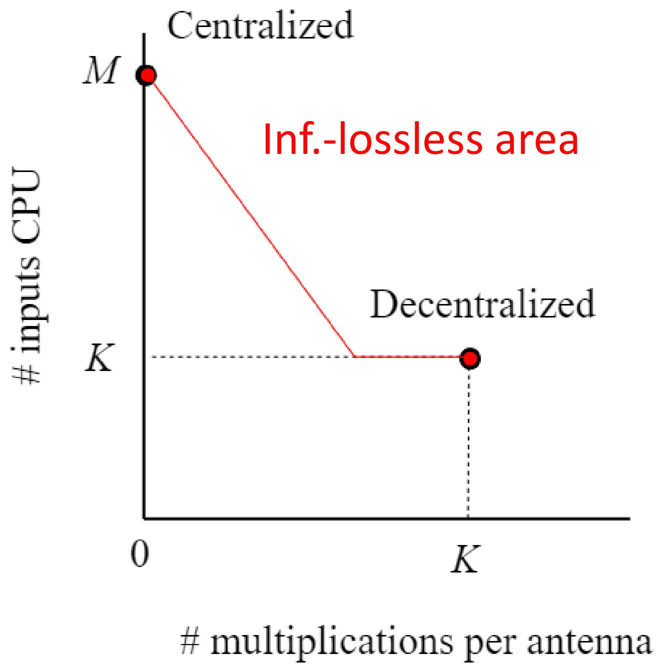
WAX decomposition (Previous work from [2])

$$\boldsymbol{H} = \boldsymbol{W}\boldsymbol{A}\boldsymbol{X}$$



- Decomposition of $\boldsymbol{H}$ $(M \times K)$ for fixed $\boldsymbol{A}$ $(M \times T)$, where $\boldsymbol{W} = \mathrm{diag}\left(\boldsymbol{W}_{11}, \dots, \boldsymbol{W}_{PN_L}\right)$, $\boldsymbol{W}_{pn}(L \times L)$.
- Main result:
  - For a randomly chosen $\boldsymbol{A}$, iff

$$T > \min\left[M\frac{K-L}{K}, K-1\right]$$

What about sparse $\boldsymbol{H}$?

a randomly chosen $\boldsymbol{H}$ (e.g., IID Rayleigh fading) accepts WAX decomposition with probability 1

Inf.-lossless processing within our system model for e.g. IID channel

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Follow up work

Application of WAX decomposition to sparse $\boldsymbol{H}$ matrices

WAX decomposition for sparse $\boldsymbol{H}$ (2 APs)

$$\boldsymbol{H} = \begin{pmatrix} \overbrace{\boldsymbol{H}_{11}}^{N \times K_1} & \boldsymbol{0}_{N\times K_2} & \overbrace{\boldsymbol{H}_{13}}^{N \times K_3} \\ \boldsymbol{0}_{N\times K_1} & \underbrace{\boldsymbol{H}_{22}}_{N \times K_2} & \underbrace{\boldsymbol{H}_{23}}_{N \times K_3} \end{pmatrix}$$

- General channel for the 2 APs scenario

K_1 ➡ # UEs seen by AP1

K_2 ➡ # UEs seen by AP2

K_3 ➡ # UEs seen by AP1 and AP2

WAX decomposition for sparse $\boldsymbol{H}$ (2 APs)

$$\boldsymbol{H} = \begin{pmatrix} \overbrace{\boldsymbol{H}_{11}}^{N\times K_1} & \mathbf{0}_{N\times K_2} & \overbrace{\boldsymbol{H}_{13}}^{N\times K_3} \\ \mathbf{0}_{N\times K_1} & \underbrace{\boldsymbol{H}_{22}}_{N\times K_2} & \underbrace{\boldsymbol{H}_{23}}_{N\times K_3} \end{pmatrix}$$

- General channel for the 2 APs scenario

K_1 ➡ # UEs seen by AP1

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K_3 ➡ # UEs seen by AP1 and AP2

Still block-diagonal ($N \times N$ submatrices of $\boldsymbol{W}$)

$$\boldsymbol{H} = \begin{pmatrix} \boldsymbol{W}_1 & \mathbf{0}_{N\times N} \\ \mathbf{0}_{N\times N} & \boldsymbol{W}_2 \end{pmatrix} \begin{pmatrix} \boldsymbol{A}_1 \\ \boldsymbol{A}_2 \end{pmatrix} (\boldsymbol{X}_1\ \boldsymbol{X}_2\ \boldsymbol{X}_3)$$

2 WAX decomposition sub-problems

$$\Rightarrow \begin{cases} \boldsymbol{W}_1\widetilde{\boldsymbol{A}}_1\widetilde{\boldsymbol{X}}_1 = \boldsymbol{H}_{11} & \widetilde{\boldsymbol{A}}_1 \text{ is } N \times (T-N) \\ \boldsymbol{W}_2\widetilde{\boldsymbol{A}}_2\widetilde{\boldsymbol{X}}_2 = \boldsymbol{H}_{22} & \widetilde{\boldsymbol{A}}_2 \text{ is } N \times (T-N) \end{cases}$$

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WAX decomposition for sparse $\boldsymbol{H}$ (2 APs)

$$\boldsymbol{H} = \begin{pmatrix} \overbrace{\boldsymbol{H}_{11}}^{N \times K_1} & \boldsymbol{0}_{N\times K_2} & \overbrace{\boldsymbol{H}_{13}}^{N \times K_3} \\ \boldsymbol{0}_{N\times K_1} & \underbrace{\boldsymbol{H}_{22}}_{N \times K_2} & \underbrace{\boldsymbol{H}_{23}}_{N \times K_3} \end{pmatrix}$$

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K_1 ➡ # UEs seen by AP1

K_2 ➡ # UEs seen by AP2

K_3 ➡ # UEs seen by AP1 and AP2

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⬅

New conditions:

$$\begin{cases} (T-N) > \min\left(N\dfrac{K_1 - L}{K_1}, K_1 - 1\right) \\ (T-N) > \min\left(N\dfrac{K_2 - L}{K_2}, K_2 - 1\right) \end{cases}$$

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WAX decomposition for sparse $\boldsymbol{H}$ (general)

$$\boldsymbol{H} = \begin{pmatrix} b_{11}\overbrace{\boldsymbol{H}_{11}}^{N \times K_1} & \cdots & b_{1C}\overbrace{\boldsymbol{H}_{1C}}^{N \times K_C} \\ \vdots & \ddots & \vdots \\ b_{P1}\boldsymbol{H}_{P1} & \cdots & b_{PC}\boldsymbol{H}_{PC} \end{pmatrix}$$

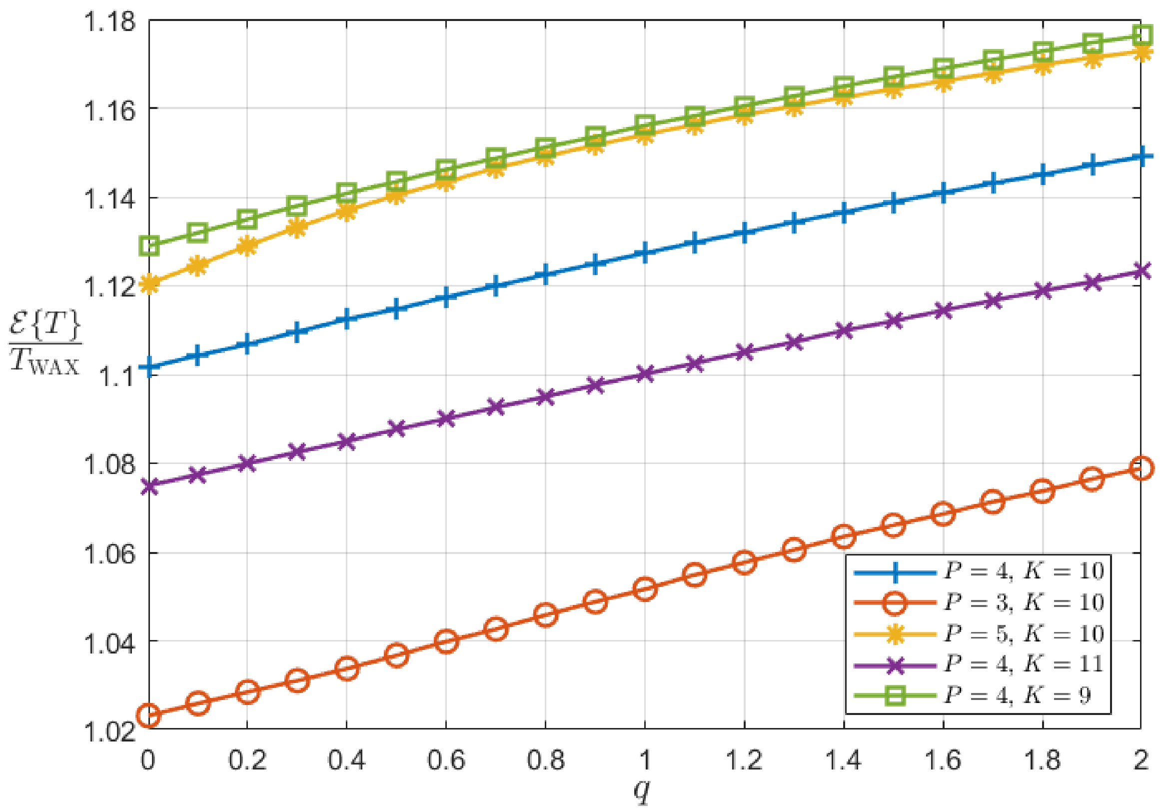
- General channel for any # of APs

$$C = 2^P - 1$$

$\boldsymbol{b}_j = (b_{j1} \quad \cdots \quad b_{jC})_2$ ➡ binary of j

<u>New conditions:</u> $T - \left(P - \|\boldsymbol{b}_j\|_1\right) N > \min\left(\|\boldsymbol{b}_j\|_1 N^{\frac{K_j - L}{K_j}}, K_1 - 1\right),\ j = 1, \ldots, C$

Numerical results



- ❑ $L = 2, N = 16$
- ❑ T_{WAX} ➡ opt. T for non-sparse $\boldsymbol{H}$
- ❑ # panels seen by each user is random, $p(n) = a/n^q$

Degradation over original WAX trade-off v.s. Sparsity of $\boldsymbol{H}$

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Wrap-up

- Centralized VS Decentralized. Connections to CPU VS Mult./antenna
- WAX decomposition for inf.-lossless processing
- Sparsity of $\boldsymbol{H}$ restricts trade-off
- More sparsity $\Longrightarrow$ greater degradation

Bibliography

[1] *J. Rodríguez Sánchez et al.*, "**Decentralized Massive MIMO: Is there Anything to be Discussed?**", *ISIT* 2019

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