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Trade-offs in Decentralized Multi-Antenna Architectures: The WAX decomposition.

JUAN VIDAL ALEGRÍA, FREDRIK RUSEK, OVE EDFORS.

Outline

- Introduction
- General Scenario
- Centralized VS decentralized
- System model
- WAX decomposition
- Recent results
- Future lines
- Wrap-up

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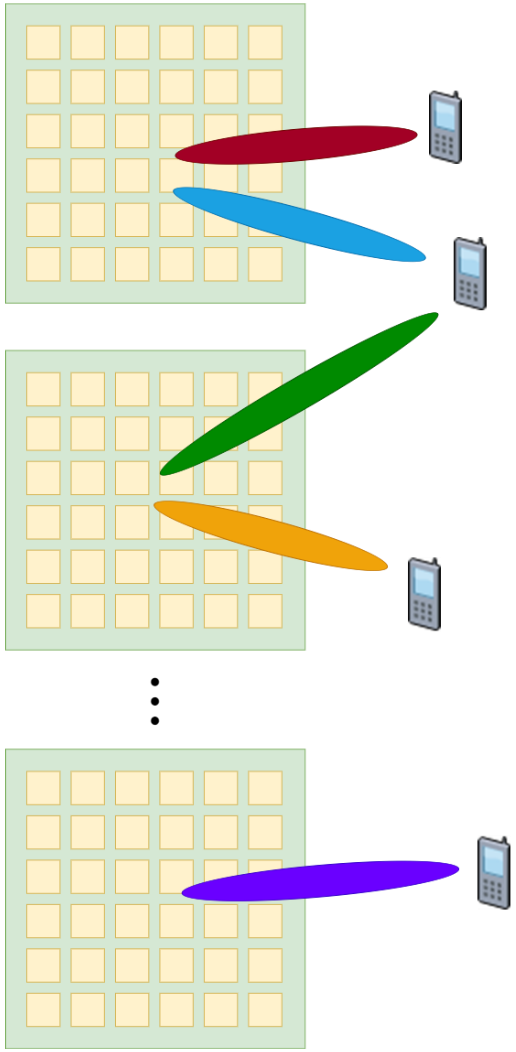
Introduction



- Mobile broadband communications (5G, 6G, etc)
- Massive MIMO, cell-free massive MIMO, LIS…
- Massive number of antennas.
- Centralized processing: Increased interconnection bandwidth to CPU.
- Trend towards decentralized. Increase complexity at the nodes to reduce information transfer to CPU.

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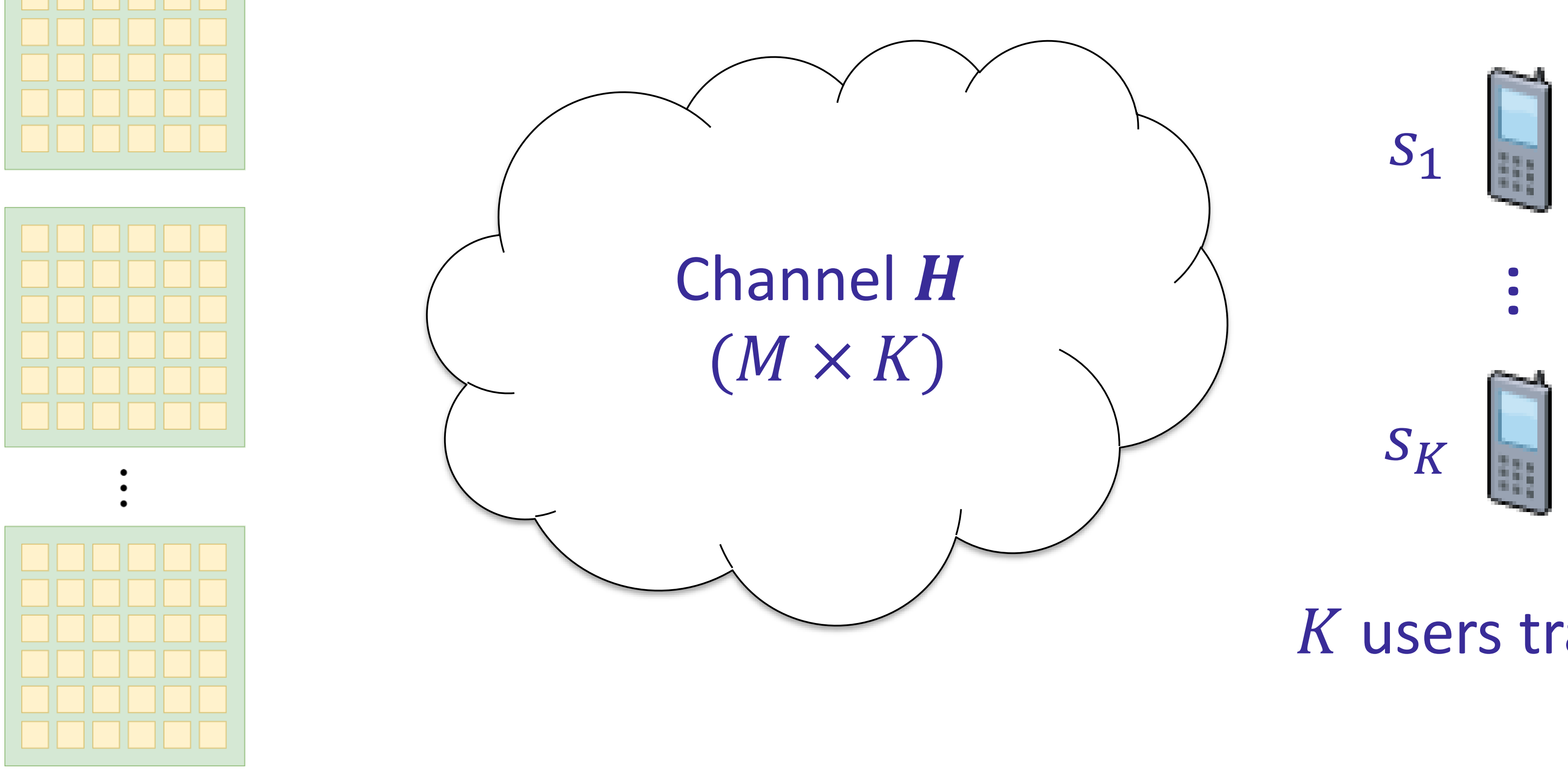
General scenario (uplink)

(Results will be extendable to downlink with some considerations)

(cplx BB vector Rx)

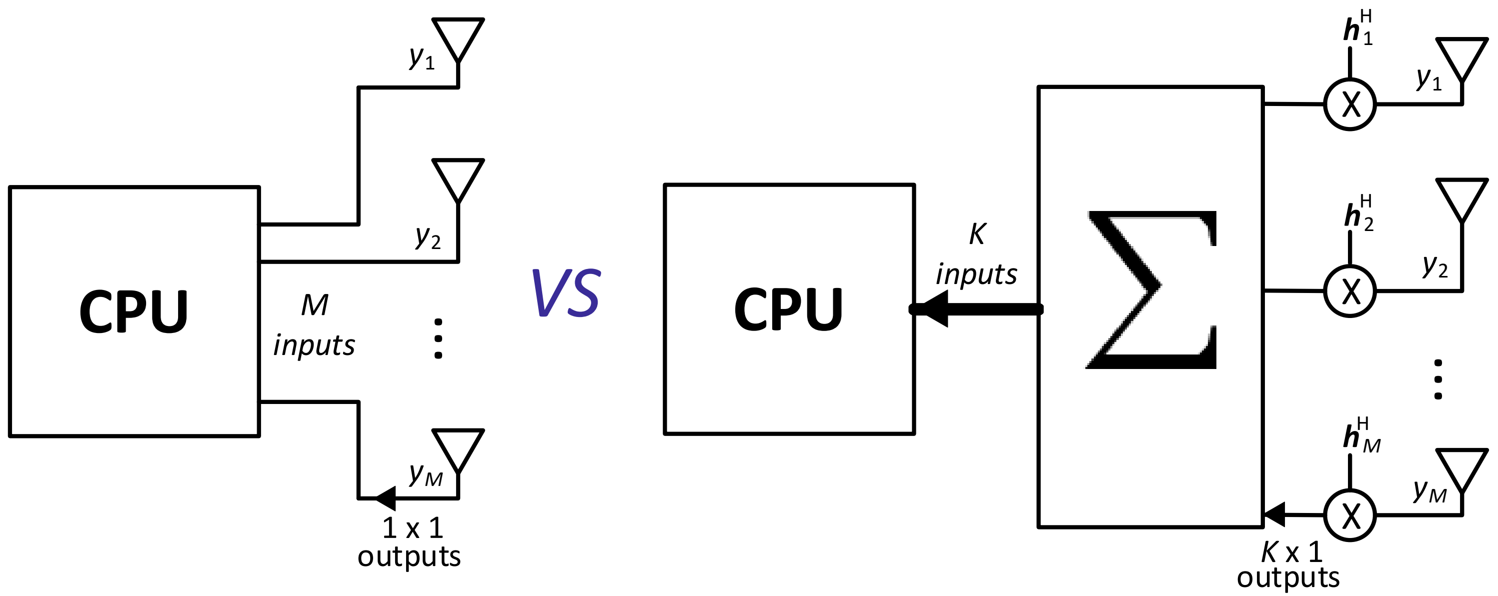
$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}$$



K users transmit $\boldsymbol{s}$

BS with a total of M antennas (mMIMO, cell-free mMIMO, LIS, or whatever)

Centralized VS Decentralized



- Information lossless (both)
- M vs K inputs to CPU
- 0 vs K multiplications per antenna

J. Rodríguez Sánchez et al.,
"Decentralized Massive MIMO: Is there Anything to be Discussed?", *ISIT* 2019

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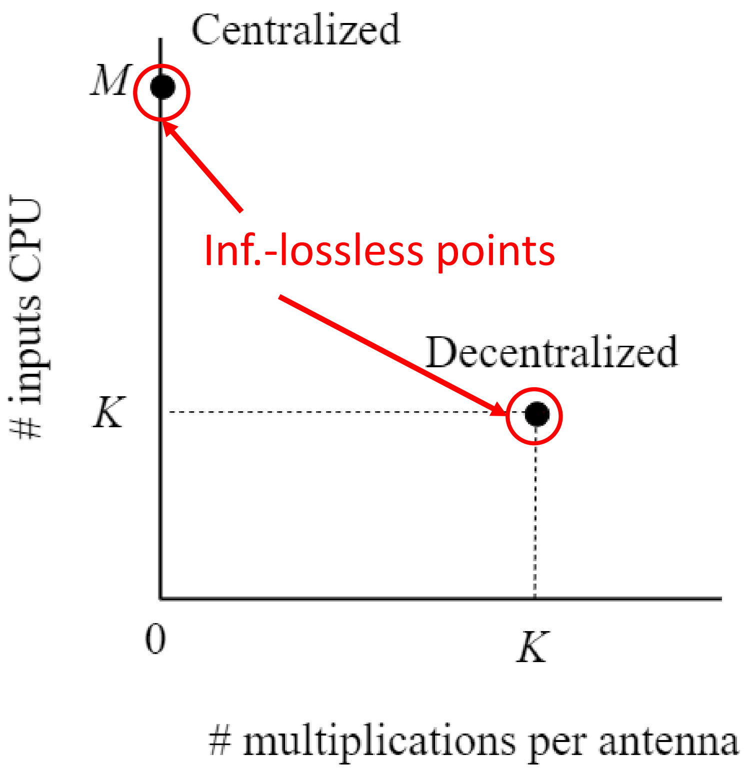
Trade-off



Level of decentralization (inputs CPU)

VS

Decentralized processing complexity (mult. per antenna)

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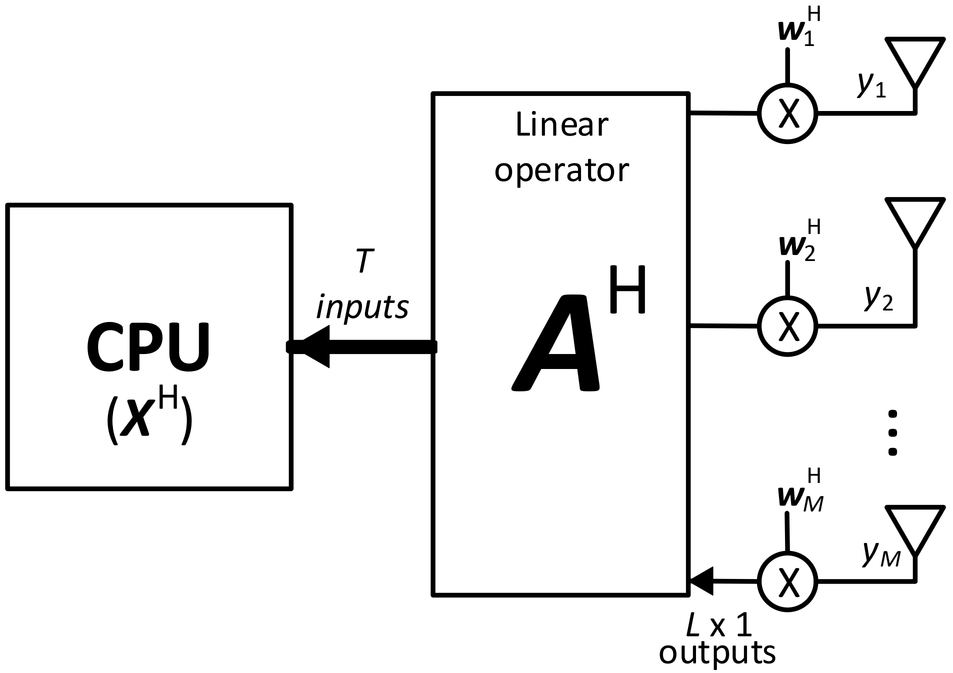
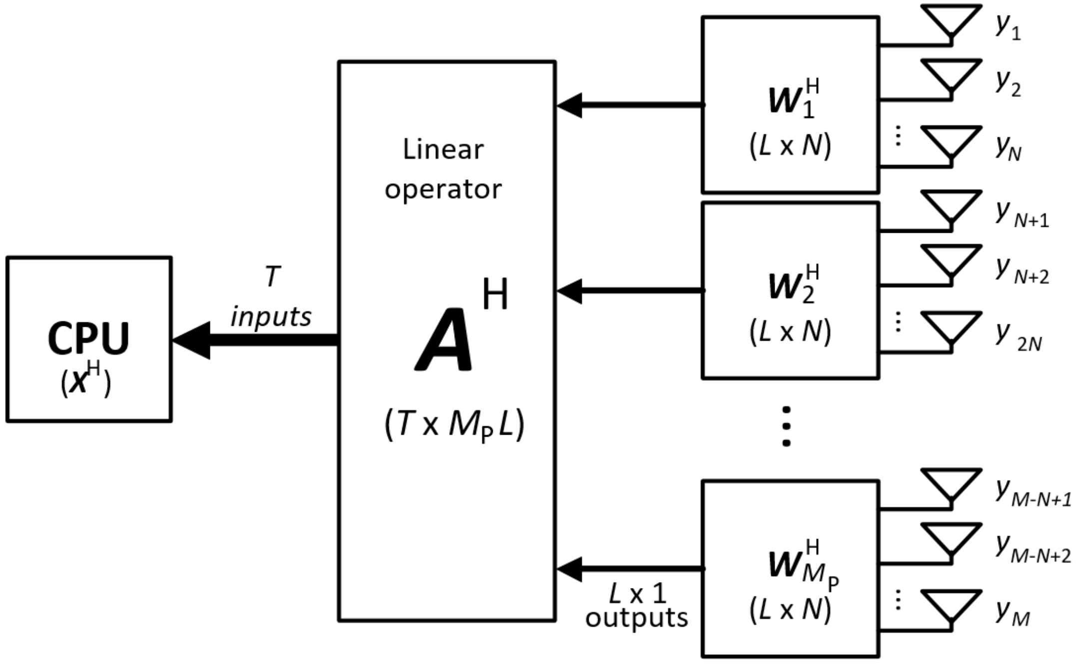
System model



- Trade-off parameters L, T
- Design variables $\boldsymbol{A}$ $(ML \times T)$
- Tunable variables $\boldsymbol{w}_m (1 \times L), \boldsymbol{X}$ $(T \times ?)$

- T ➡ # inputs to CPU
- L ➡ # mult. per ant.
- $\boldsymbol{A}$ ➡ Combining module (sparse with 1s/0s)
- $\boldsymbol{w}_m$ ➡ Decentralized filters
- $\boldsymbol{X}$ ➡ Processing at CPU

System model (general)



- Trade-off parameters L, T
- Design variables $\boldsymbol{A}$ $(M_{\mathrm{P}}L \times T)$
- Tunable variables $\boldsymbol{W}_m(N \times L), \boldsymbol{X}$ $(T \times ?)$

Resulting linear processing

$$\boldsymbol{z} = \boldsymbol{X}^{\mathrm{H}}\boldsymbol{A}^{\mathrm{H}}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{y}, \quad \boldsymbol{W} = \mathrm{diag}(\boldsymbol{W}_1, \dots, \boldsymbol{W}_{M_{\mathrm{P}}})$$

(Recall $\boldsymbol{y} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n}$)

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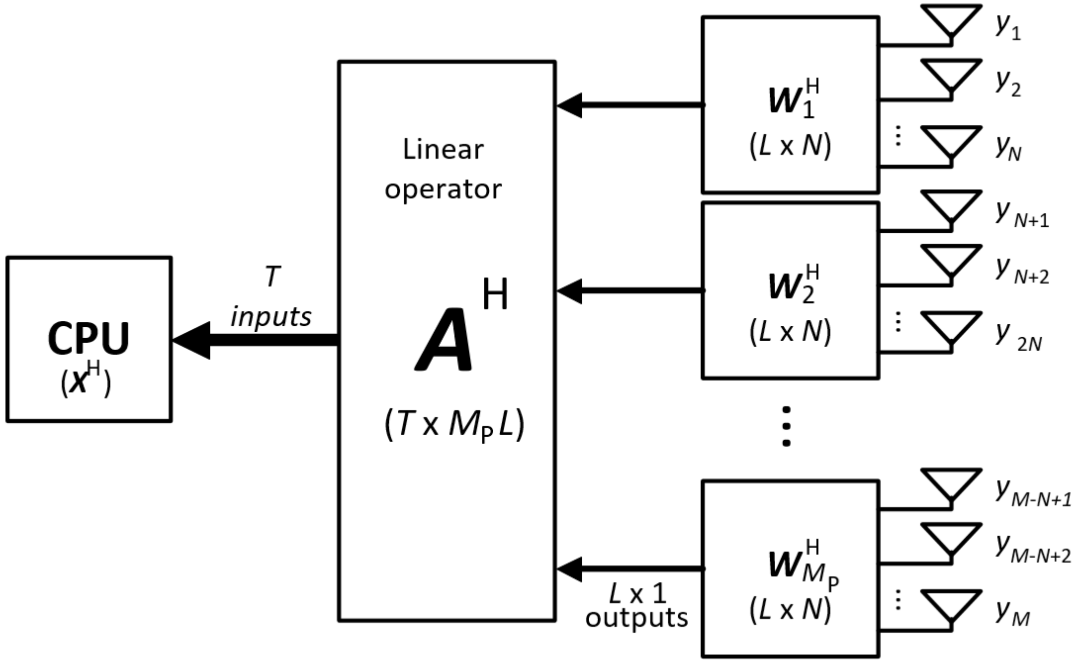
System model (general)



- Trade-off parameters L, T
- Design variables $\boldsymbol{A}$ $(M_P L \times T)$
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Resulting linear processing

$$\boldsymbol{z} = \boldsymbol{X}^H \boldsymbol{A}^H \boldsymbol{W}^H \boldsymbol{y}, \quad \boldsymbol{W} = \mathrm{diag}(\boldsymbol{W}_1, \ldots, \boldsymbol{W}_{M_P})$$

MF simple information-lossless transformation

$$\boldsymbol{z} = \boldsymbol{H}^H \boldsymbol{y}$$

Information lossless $\Leftrightarrow \mathrm{I}(\mathbf{z};\mathbf{s}) = \mathrm{I}(\mathbf{y};\mathbf{s}) \Leftrightarrow \boldsymbol{WAX} = \boldsymbol{H}$

Any inf.-lossless lin. processing can be applied by combining MF with something else

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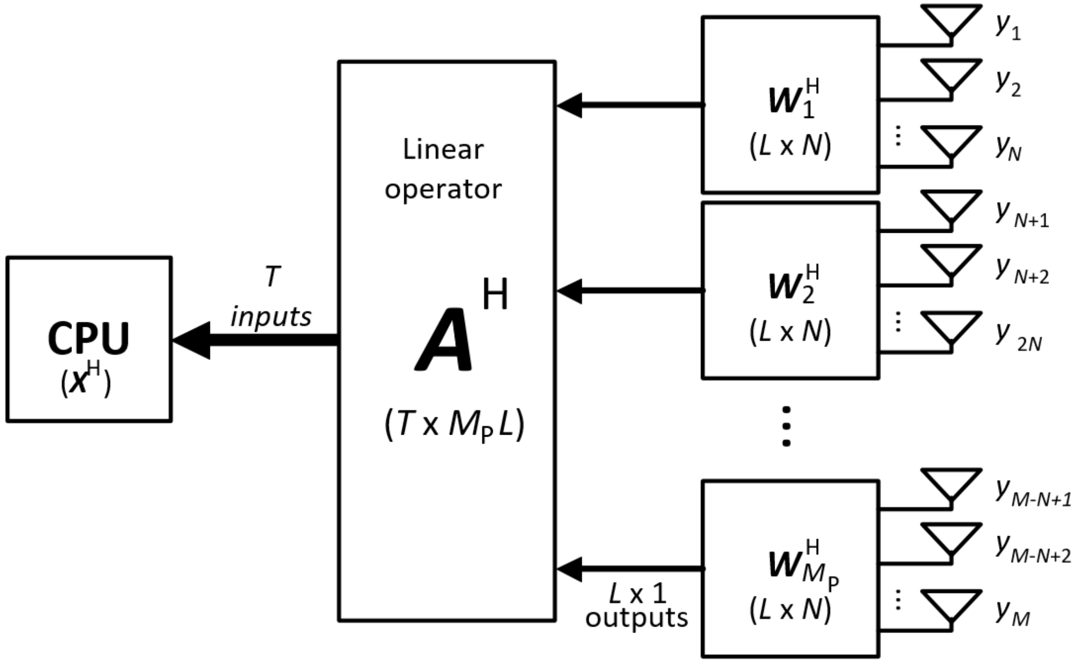
System model (general)



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$\mathrm{I}\left(\boldsymbol{A}^{\mathrm{H}}\boldsymbol{W}^{\mathrm{H}}\boldsymbol{y}; \mathbf{s}\right) = \mathrm{I}(\mathbf{y}; \mathbf{s})$

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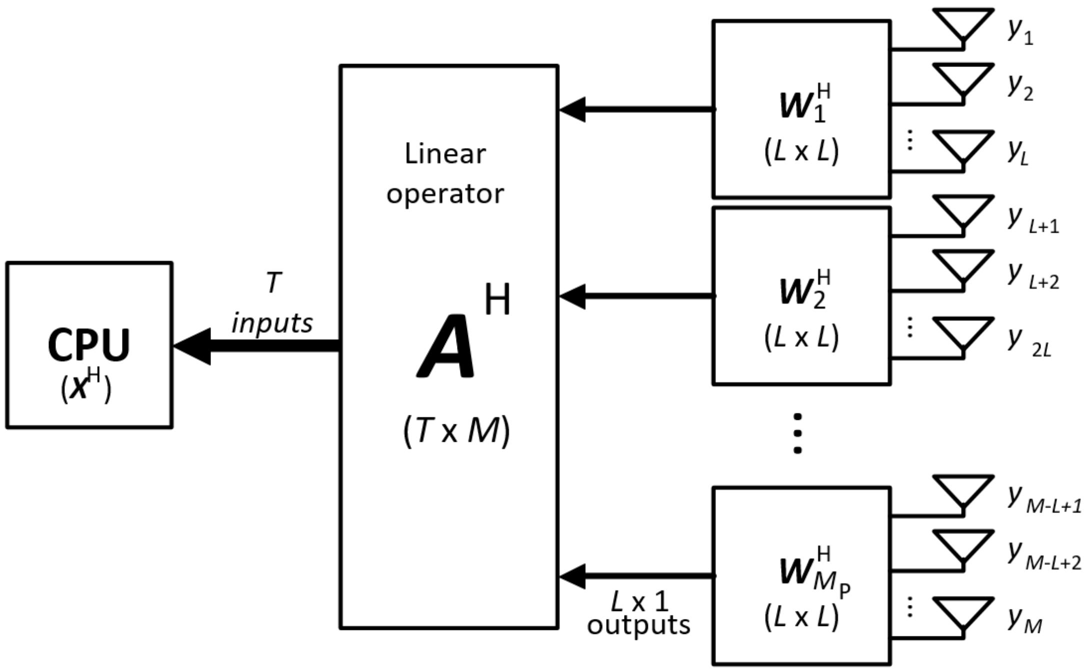
System model (general enough)



- Trade-off parameters L, T
- Design variables $\boldsymbol{A}$ $(M \times T)$
- Tunable variables $\boldsymbol{W}_m (L \times L), \boldsymbol{X}$ $(T \times K)$

Any N dividing L can be studied in this framework (sparse transformation on $\boldsymbol{A}$)

$N > L$ not interesting due to inf.-loss for $L \leq K$

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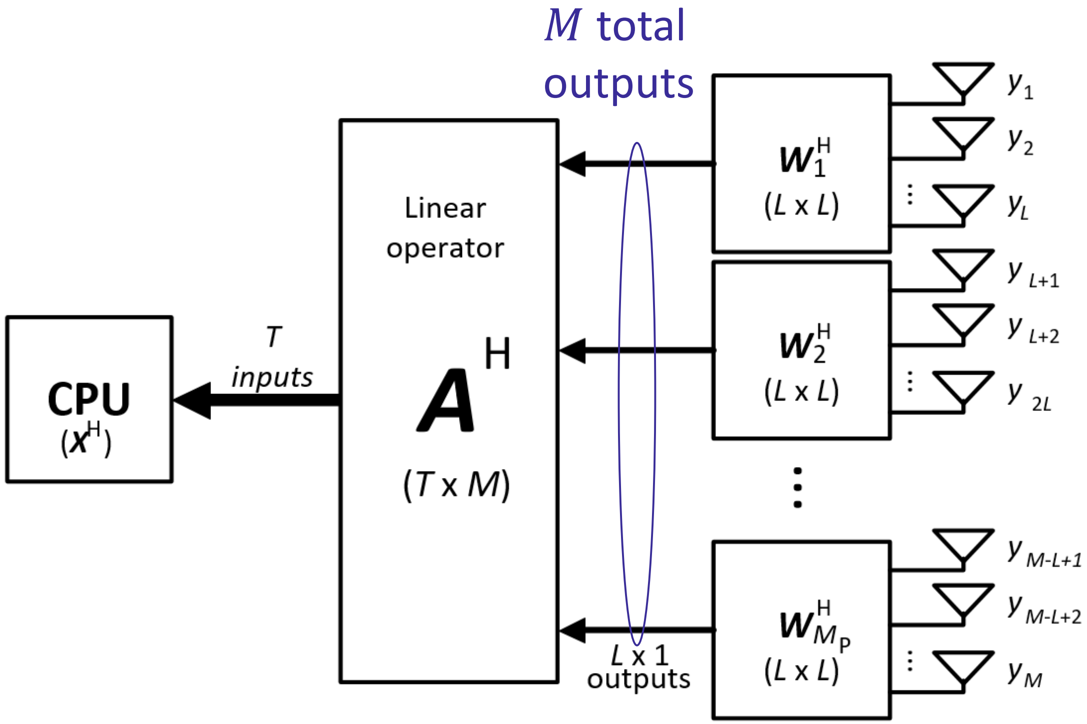
System model (general enough)



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Any N dividing L can be studied in this framework (sparse transformation on $\boldsymbol{A}$)

$N > L$ not interesting due to inf.-loss for $L \leq K$

WAX decomposition

$$\boldsymbol{H} = \boldsymbol{W}\boldsymbol{A}\boldsymbol{X}$$

- Decomposition of $\boldsymbol{H}$ $(M \times K)$ for fixed $\boldsymbol{A}$ $(M \times T)$, where $\boldsymbol{W} = \mathrm{diag}(\boldsymbol{W}_1, \dots, \boldsymbol{W}_{M_\mathrm{P}})$, $\boldsymbol{W}_m (L \times L)$.
- Iff it exists for any channel realization we can have inf.-lossless processing in our system model

WAX decomposition

- Decomposition of $\boldsymbol{H}$ $(M \times K)$ for fixed $\boldsymbol{A}$ $(M \times T)$, where $\boldsymbol{W} = \mathrm{diag}\left(\boldsymbol{W}_1, \dots, \boldsymbol{W}_{M_{\mathrm{P}}}\right)$, $\boldsymbol{W}_m (L \times L)$.

$$\boldsymbol{H} = \boldsymbol{W}\boldsymbol{A}\boldsymbol{X}$$

Example:

$$\underbrace{\begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 \end{bmatrix}}_{\boldsymbol{W}} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} -2 & -1 & -1 & 2 & 2 \\ 1 & -2 & -1 & 1 & 2 \\ 1 & -1 & -2 & -1 & -2 \\ 0 & 2 & 0 & -1 & 0 \\ 0 & -1 & 2 & 1 & -2 \end{bmatrix}}_{\boldsymbol{X}} = \underbrace{\begin{bmatrix} -3 & 1 & 0 & 1 & 0 \\ -2 & -4 & -3 & 5 & 6 \\ 1 & 1 & -2 & -2 & -2 \\ 0 & -2 & 0 & 1 & 0 \\ -2 & -2 & 5 & 4 & -4 \\ 2 & -2 & 3 & 0 & -4 \\ 1 & -2 & 2 & 3 & 4 \\ 4 & -2 & 2 & 0 & -8 \end{bmatrix}}_{\boldsymbol{H}}$$

WAX decomposition

$$\boldsymbol{H} = \boldsymbol{WAX}$$

- Decomposition of $\boldsymbol{H}$ $(M \times K)$ for fixed $\boldsymbol{A}$ $(M \times T)$, where $\boldsymbol{W} = \mathrm{diag}\left(\boldsymbol{W}_{11}, \dots, \boldsymbol{W}_{PN_\mathrm{L}}\right)$, $\boldsymbol{W}_{pn}(L \times L)$.

- Main result:

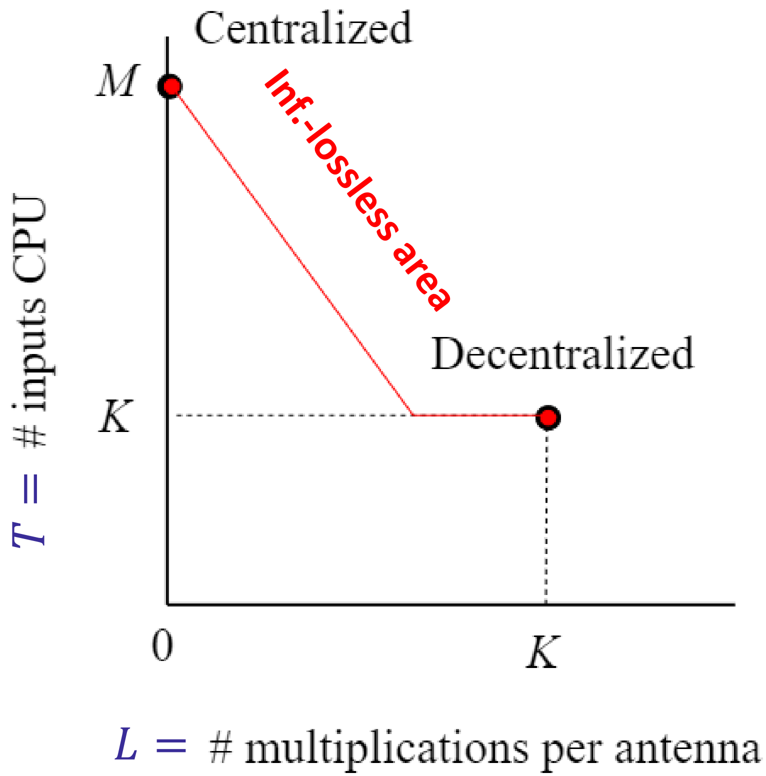
  For a randomly chosen $\boldsymbol{A}$, iff

  $$T > \min\left[M\frac{K-L}{K}, K-1\right]$$

  a randomly chosen $\boldsymbol{H}$ (e.g., IID Rayleigh fading) accepts WAX decomposition with probability 1

Inf.-lossless processing within our system model for e.g. IID channel

Trade-off (revisited)



- ❑ Proved for randomly chosen ***A*** and ***H***

WAX decomposition (Previous work from [2])

$$\boldsymbol{H} = \boldsymbol{W}\boldsymbol{A}\boldsymbol{X}$$

- Decomposition of $\boldsymbol{H}$ $(M \times K)$ for fixed $\boldsymbol{A}$ $(M \times T)$, where $\boldsymbol{W} = \text{diag}(\boldsymbol{W}_{11}, \dots, \boldsymbol{W}_{PN_\text{L}})$, $\boldsymbol{W}_{pn}(L \times L)$.
- Main result:

  For a randomly chosen $\boldsymbol{A}$, iff

  $$T > \min\left[M\frac{K-L}{K}, K-1\right]$$

  a randomly chosen $\boldsymbol{H}$ (e.g., IID Rayleigh fading) accepts WAX decomposition with probability 1

Inf.-lossless processing within our system model for e.g. IID channel

Proof: Solving the equivalent linear equation (vectorizing) $\boldsymbol{A}\boldsymbol{X} = \boldsymbol{W}^{-1}\boldsymbol{H}$ with full-rank $\boldsymbol{W}^{-1}$ constraint

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WAX decomposition (Previous work from [2])

$$\boldsymbol{H} = \boldsymbol{W}\boldsymbol{A}\boldsymbol{X}$$

- Decomposition of $\boldsymbol{H}$ $(M \times K)$ for fixed $\boldsymbol{A}$ $(M \times T)$, where $\boldsymbol{W} = \mathrm{diag}(\boldsymbol{W}_{11}, \dots, \boldsymbol{W}_{PN_{\mathrm{L}}})$, $\boldsymbol{W}_{pn}(L \times L)$.
- Main result: What about sparse $\boldsymbol{A}$?

  For a randomly chosen $\boldsymbol{A}$, iff

$$T > \min\left[M\frac{K-L}{K}, K-1\right]$$

a randomly chosen $\boldsymbol{H}$ (e.g., IID Rayleigh fading) accepts WAX decomposition with probability 1

Inf.-lossless processing within our system model for e.g. IID channel

Proof: Solving the equivalent linear equation (vectorizing) $\boldsymbol{A}\boldsymbol{X} = \boldsymbol{W}^{-1}\boldsymbol{H}$ with full-rank $\boldsymbol{W}^{-1}$ constraint

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Valid $\boldsymbol{A}$ structures

- $\boldsymbol{A}$ iff a randomly chosen $\boldsymbol{H}$ accepts WAX decomposition for that $\boldsymbol{A}$ with probability 1.

- Important result for testing $\boldsymbol{A}$ structures:

  Iff we can perform WAX decomposition of **ONE** randomly chosen $\boldsymbol{H}$ with a given $\boldsymbol{A}$, said $\boldsymbol{A}$ is valid with probability 1
  (either $\boldsymbol{A}$ works for random $\boldsymbol{H}$ or it doesn't at all)

- Necessary conditions on valid $\boldsymbol{A}$
  (Follow-up on valid $\boldsymbol{A}$ structures coming for ICC 2022)

Wrap-up

- Centralized VS Decentralized. Connections to CPU VS Mult./antenna
- Generalized architecture. WAX decomposition for inf.-lossless proc.
- Inf.-lossless trade-off
- Follow-up work. $\boldsymbol{A}$ sparse (ICC2022), $\boldsymbol{H}$ sparse (ICASSP2022)

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