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## Introduction

- Graph convolutional networks (GCNs) continue to suffer from oversmoothing - performance reduction with an increasing number of layers.
- We introduce a simple yet effective idea of feature gating over graph convolution layers to graph neural networks deeper and oversmoothing.
- The proposed feature gating is easy to implement without changing the underlying network architecture and is broadly applicable to GCN and almost any of its variants.
- We demonstrate the use of feature gating in assigning importance to node features and the nodes for the node classification task.

# Background

- The graph convolution layer is defined as:  $\mathbf{H}^{(l+1)} = \sigma(\widetilde{\mathbf{P}}\mathbf{H}^{(l)}\mathbf{W}^{(l)}), \text{ where } \widetilde{\mathbf{P}} = \widetilde{\mathbf{D}}^{-1/2}\widetilde{\mathbf{A}}\widetilde{\mathbf{D}}^{-1/2}$
- Such a fixed polynomial filter  $\widetilde{\mathbf{P}}^{K} \mathbf{x}$  converges to a distribution that is distant from the input feature x and hence, incurs vanishing gradients.
- Let  $d_{\mathcal{M}}(\mathbf{H}) := \inf\{ \|\mathbf{H} \mathbf{Y}\|_F | \mathbf{Y} \in \mathcal{M} \}$  denote the distance between **H** and  $\mathcal{M}$ .
- For any initial value  $\mathbf{H}^{(0)}$ , the output of  $l^{th}$  layer  $\mathbf{H}^{(l)}$ satisfies

 $d_{\mathcal{M}}(\mathbf{H}^{(l)}) \leq (s\lambda)^{l} d_{\mathcal{M}}(\mathbf{H}^{(0)})$ 

- Here, s and  $\lambda$  are the maximum singular and eigen values of the weight matrix W and is the normalised laplacian matrix  $\tilde{\mathbf{P}}$ , respectively.
- For,  $\lambda = 1$ , we can have  $s \ge 1$  and for  $\lambda \neq 1$ , the sweet spot falls in the range  $1 \leq s \leq \lambda^{-1}$ .

# **Exploring Deeper Graph Convolutions For Semi-Supervised Node** Classification

facilitate address

Combining initial residual and identity mapping:  $\mathbf{H}^{(l+1)} = \sigma \left( \left( \alpha_l \widetilde{\mathbf{P}} \mathbf{H}^{(l)} + (1 - \alpha_l) \mathbf{H}^{(0)} \right) \left( (1 - \beta_l) \mathbf{I} + \beta_l \mathbf{W}^{(l)} \right) \right)$ 

- Feature gating in GCN:  $\mathbf{H}^{(l+1)} = \widetilde{\mathbf{P}}(\phi(\mathbf{H}^{(l)}\mathbf{W}^{(l)}) \odot \sigma(\mathbf{H}^{(l)}\mathbf{G}^{(l)}))$
- GCN-IR-FG:  $\mathbf{H}^{(l+1)} = \beta_l \left( \widetilde{\mathbf{P}} \big( \phi(\mathbf{Z}^{(l)} \mathbf{W}^{(l)}) \odot \sigma(\mathbf{Z}^{(l)} \mathbf{G}^{(l)}) \big) + (1 - \beta_l) \big( \widetilde{\mathbf{P}} \mathbf{Z}^{(l)} \mathbf{I} \big) \right)$ such that,  $\mathbf{Z}^{(l)} = (1 - \alpha_l)\mathbf{H}^{(l)} + \alpha \mathbf{H}^{(0)}$

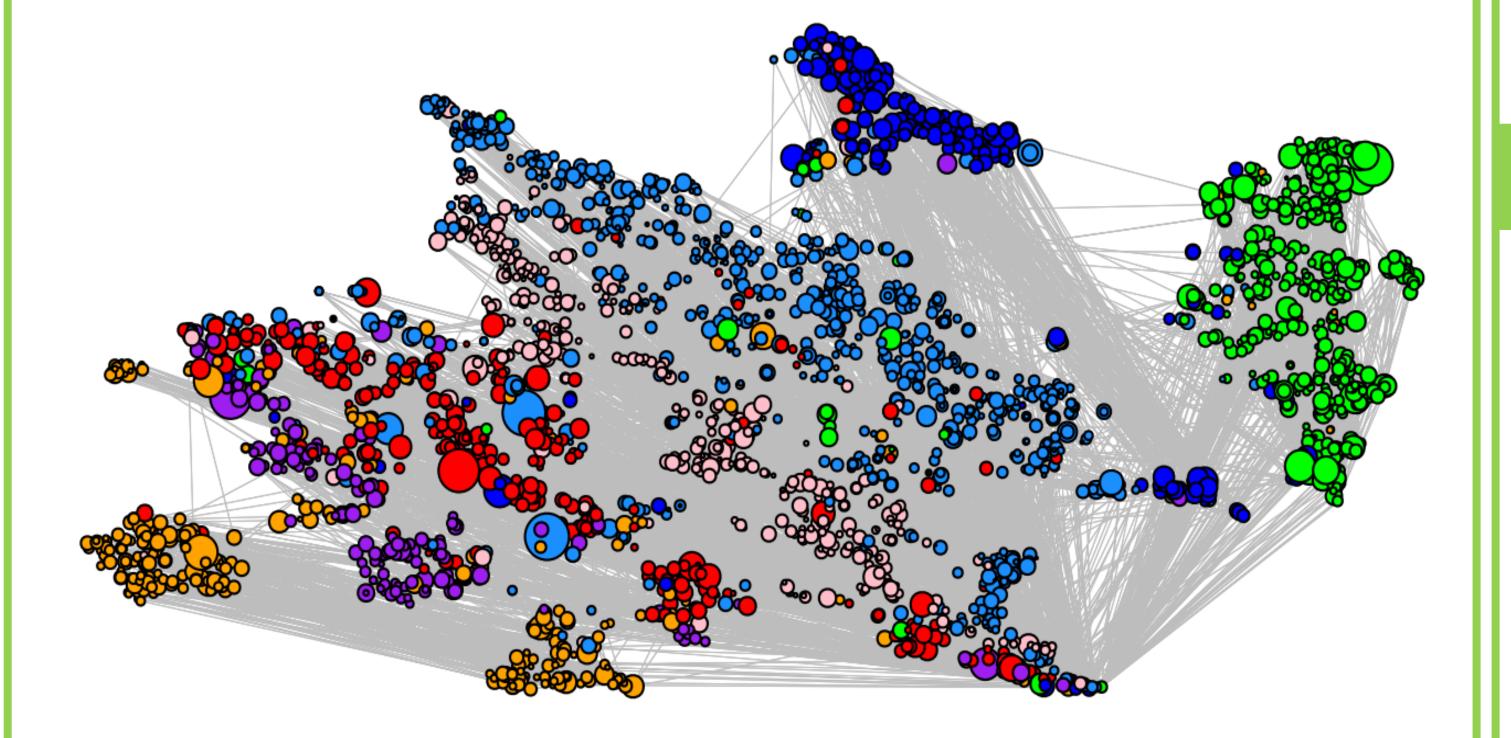
• For  $w_{ij}$  in  $\mathbf{W}^{(l)}$  and  $s_{ij}$  in  $\mathbf{S}^{(l)} = \sigma(\mathbf{H}^{(l)}\mathbf{G}^{(l)})$ , we have.

**1. Node feature importance**  $(\mathcal{I}_{n,m})$  : The importance of the  $m^{th}$  feature of the  $n^{th}$  node can be obtained as follows

 $\mathcal{I}_{n,m} = \sum_{i=1}^{n} w_{mj} S_{nj}$ 

**2. Node importance**  $(\mathcal{I}_n)$  : The importance of an  $n^{th}$ node can be obtained as follows

 $\mathcal{I}_n = \sum_{j=1}^{N} \|\boldsymbol{w}_j\|_2 s_{nj}$ Here,  $\boldsymbol{w}_j = [w_{1j}, w_{2j}, \dots, w_{dj}]$ 



initializations.												
Method	Cora	Citeseer	PubMed	Coauthor	Coauthor	Amazon	Amazon					
				CS	Physics	Photos	Computers					
SGC	$80.7 \pm 0.8$	$72.8\pm0.6$	$77.3 \pm 1.1$	$78.72\pm0.7$	$76.28 \pm 1.2$	$75.14\pm0.5$	$79.85\pm0.7$					
GCN	$81.1 \pm 0.7$ (2)	$70.9 \pm 0.9$ (2)	$78.1 \pm 0.5$ (2)	$81.87 \pm 0.8$ (2)	$83.99 \pm 0.7$ (2)	$81.63 \pm 0.7$ (2)	$84.67 \pm 0.8$ (2)					
GCN-DE	$82.3 \pm 0.6$ (2)	$71.1 \pm 0.5$ (2)	$78.4 \pm 0.3$ (2)	$82.28 \pm 0.6$ (2)	$85.08 \pm 0.6$ (2)	$82.07 \pm 0.8$ (2)	$84.91 \pm 0.6$ (2)					
GCN-FG	$82.8 \pm 0.6$ (2)	$73.1 \pm 0.6$ (2)	$79.1 \pm 0.5$ (2)	$83.05 \pm 0.6$ (2)	$85.56 \pm 0.5$ (2)	$83.86 \pm 0.5$ (2)	$85.29 \pm 0.5$ (2)					
GAT	$81.9 \pm 0.5$ (2)	$70.6 \pm 0.6$ (2)	$77.4 \pm 0.9$ (2)	$82.01 \pm 0.5$ (2)	$84.57 \pm 1.1$ (2)	$82.53 \pm 0.9$ (2)	$85.09 \pm 0.7$ (2)					
GAT-DE	$82.1 \pm 0.6$ (2)	$70.8 \pm 0.4$ (2)	$77.5 \pm 0.8$ (2)	$82.84 \pm 0.7$ (2)	$85.22 \pm 0.6$ (2)	$83.09 \pm 0.7$ (2)	$85.57 \pm 0.5$ (2)					
GAT-FG	$82.0 \pm 0.5$ (2)	$70.9 \pm 0.7$ (2)	$77.8 \pm 0.9$ (2)	$82.56 \pm 0.4$ (2)	$84.98 \pm 0.5$ (2)	$82.84 \pm 0.6$ (2)	84.11 ± 0.6 (2)					
JKNet	$81.1 \pm 0.8$ (4)	$69.8 \pm 0.6$ (16)	$78.1 \pm 0.5$ (16)	$84.54 \pm 0.7$ (8)	$85.52 \pm 0.5$ (16)	$81.67 \pm 0.6$ (16)	$82.14 \pm 0.6$ (16)					
APPNP	$83.3 \pm 0.7$	$71.8\pm0.8$	$79.8\pm0.7$	$85.37\pm0.5$	$85.98\pm0.6$	$82.62\pm0.4$	$83.81\pm0.8$					
GRAND*	$84.3 \pm 0.6$ (8)	$72.8 \pm 0.5$ (8)	$78.5 \pm 0.6$ (4)	$88.31 \pm 0.5$ (8)	$87.89 \pm 0.6$ (8)	85.31 ± 0.5 (8)	$86.66 \pm 0.7$ (4)					
GCNII	$84.8 \pm 0.5$ (32)	$72.9 \pm 0.3$ (64)	$79.8 \pm 0.3$ (16)	88.83 ± 0.8 (16)	$88.51 \pm 0.7$ (32)	$88.27 \pm 0.6$ (32)	$87.25 \pm 0.6$ (32)					
GFGN*	$84.9 \pm 0.6$ (4)	$73.4 \pm 0.4$ (4)	$80.4 \pm 0.4$ (8)	$89.03 \pm 0.6$ (4)	$89.45 \pm 0.3$ (4)	89.13 ± 0.8 (8)	$87.92 \pm 0.5$ (4)					
GCN-IR	$85.3 \pm 0.7$ (32)	$73.2 \pm 0.6$ (32)	<b>80.1</b> $\pm$ <b>0.5</b> (32)	$89.98 \pm 0.7$ (32)	89.75 ± 0.6 (32)	$89.16 \pm 0.7$ (64)	<b>88.91</b> $\pm$ <b>0.6</b> (32)					
GCN-IR-FG	<b>85.7</b> $\pm$ <b>0.5</b> (32)	$\textbf{73.6} \pm \textbf{0.4}~(32)$	$80.0 \pm 0.6$ (64)	<b>90.79</b> $\pm$ <b>0.6</b> (64)	$90.21 \pm 0.4$ (32)	$89.94 \pm 0.4$ (64)	$88.49 \pm 0.5 \ (32)$					

### Performance analysis with increasing number of layers

Detect	Mathad	Layers						
Dataset	Method	2	4	8	16	32	64	
	GCN	81.1	80.4	69.5	64.9	60.3	28.7	
	GCN-DE	82.3	82.0	75.8	75.7	62.5	49.5	
	GCN-FG	82.8	78.6	69.9	72.7	59.3	47.4	
	GAT	81.9	79.8	69.5	47.8	45.1	25.3	
Cora	GAT-DE	82.1	80.8	72.9	66.3	51.3	43.4	
	GAT-FG	82.0	68.4	51.3	59.7	45.1	29.2	
	GRAND	72.8	79.6	84.3	80.1	77.2	73.4	
	GCNII	82.1	82.4	83.6	83.9	84.8	84.6	
	GFGN	83.7	84.9	82.3	79.6	65.2	61.9	
	GCN-IR	82.4	83.9	84.6	84.8	85.3	85.2	
	GCN-IR-FG	83.2	82.9	85.1	85.4	85.7	85.5	
	GCN	78.1	75.5	62.2	41.7	21.4	34.2	
	GCN-DE	78.4	77.9	78.1	78.2	77.0	61.5	
	GCN-FG	79.1	74.7	65.1	54.5	41.7	35.1	
	GAT	77.4	69.4	48.3	39.5	41.3	40.7	
PubMed	GAT-DE	77.5	76.8	76.4	67.1	54.7	68.2	
	GAT-FG	77.8	68.7	51.8	48.2	49.1	41.6	
	GRAND	72.4	74.7	78.5	76.2	71.9	68.6	
	GCNII	74.9	76.2	77.5	<b>79.8</b>	78.1	74.2	
	GFGN	80.4	79.7	77.6	72.4	67.6	62.8	
	GCN-IR	78.1	78.3	79.1	79.6	80.1	79.5	
	GCN-IR-FG	77.8	78.1	79.7	79.9	80.0	80.0	

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### Results

### Test accuracy and standard deviation over 50 random

### **Key References**

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