Efficiently and Globally Solving Joint Beamforming and Compression Problem in the Cooperative Cellular Network via Lagrangian Duality

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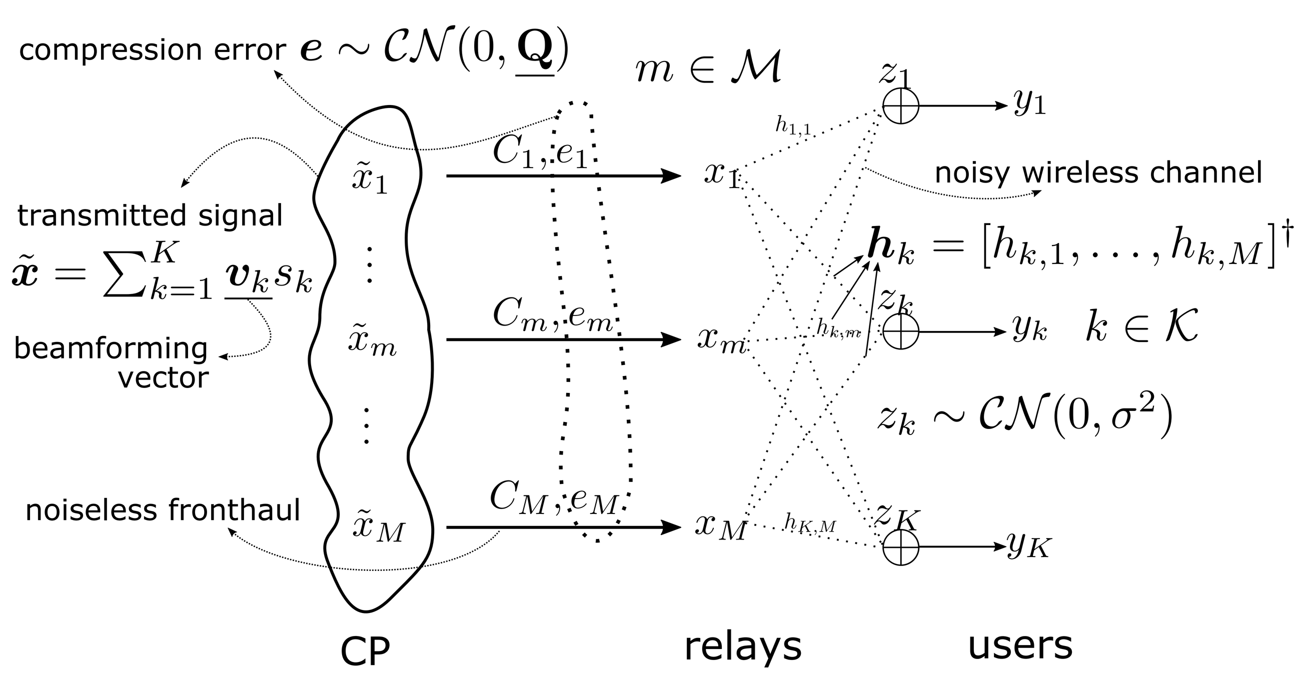
Main Contribution

- The downlink joint beamforming and compression problem is a challenging task in the cooperative cellular network.
- The rate-limited fronthaul between the central processor (CP) and base stations (BSs) poses constraints that are difficult to handle.
- The joint optimization of beamforming vectors and the covariance matrix of the compression error also makes the problem highly non-convex.
- We establish the tightness of the semidefinite relaxation (SDR) of the considered problem and thus the equivalence of the two problems.
- We propose a global and efficient algorithm for the considered problem via Lagrangian duality.

System Model



Problem Formulation

- The equivalent joint beamforming and compression problem [1, Proposition 4]:

$$\begin{aligned} \min_{\{v_k\},\mathbf{Q}} \quad & \sum_{k=1}^{K} \|v_k\|^2 + \mathbf{Q} \bullet \mathbf{I} \\ \text{s.t.} \quad & v_k^\dagger \mathbf{H}_k v_k - \gamma_k \left(\sum_{j \neq k} v_j^\dagger \mathbf{H}_k v_j + \mathbf{Q} \bullet \mathbf{H}_k + \sigma^2 \right) \geq 0, \ \forall\, k \in \mathcal{K}, \\ & \eta_m \tilde{\mathbf{Q}}_m - \mathbf{E}_m^\dagger \left(\sum_{k=1}^{K} v_k v_k^\dagger + \mathbf{Q} \right) \mathbf{E}_m \succeq \mathbf{0}, \ \forall\, m \in \mathcal{M}, \\ & \mathbf{Q} \succeq \mathbf{0}, \end{aligned} \tag{P}$$

where $\mathbf{H}_k = \boldsymbol{h}_k \boldsymbol{h}_k^\dagger$, γ_k is the rate target for user k, $\eta_m = 2^{C_m}$ and the right bottom of $\tilde{\mathbf{Q}}_m$ is filled with $\mathbf{Q}^{(m:M,m:M)}$ and the other parts are zero.

Problem Formulation (Cont.)

- The semidefinite relaxation problem (SDR) and its dual problem can be derived accordingly and have the following forms:

$$\begin{aligned} \min_{\mathbf{V}_k,\mathbf{Q}} \quad & P^{\text{dl}} \\ \text{s.t.} \quad & \beta_k : a_k \geq 0, \quad k \in \mathcal{K}, \\ & \boldsymbol{\Lambda}_m : \mathbf{B}_m \succeq \mathbf{0}, \quad m \in \mathcal{M}, \\ & \mathbf{V}_k, \ \mathbf{Q} \succeq \mathbf{0}, \quad k \in \mathcal{K}, \end{aligned} \qquad \begin{aligned} \max_{\beta_k,\boldsymbol{\Lambda}_m} \quad & P^{\text{ul}} \\ \text{s.t.} \quad & \mathbf{V}_k : \mathbf{C}_k - \beta_k \mathbf{H}_k \succeq \mathbf{0}, \quad \forall k \in \mathcal{K}, \\ & \mathbf{Q} : \mathbf{D} \succeq \mathbf{0}, \\ & \beta_k \geq 0, \quad \forall k \in \mathcal{K}, \\ & \boldsymbol{\Lambda}_m \succeq \mathbf{0}, \quad \forall m \in \mathcal{M}, \end{aligned}$$

where a_k, $\mathbf{B}_m$ and P^{dl} are functions of $\{\mathbf{V}_k\}$ and $\mathbf{Q}$; P^{ul} is a function of $\{\beta_k\}$, $\{\boldsymbol{\Lambda}_m\}$ and

$$\mathbf{C}_k(\{\beta_k\}, \{\boldsymbol{\Lambda}_m\}) = \mathbf{I} + \sum_{m=1}^{M} \mathbf{E}_m^\dagger \boldsymbol{\Lambda}_m \mathbf{E}_m + \sum_{j \neq k} \beta_j \gamma_j \mathbf{H}_j,$$

$$\mathbf{D}(\{\beta_k\}, \{\boldsymbol{\Lambda}_m\}) = \mathbf{I} - \sum_{m=1}^{M} \eta_m \tilde{\mathbf{Q}}_m + \sum_{k=1}^{K} \beta_k \gamma_k \mathbf{H}_k + \sum_{m=1}^{M} \mathbf{E}_m^\dagger \boldsymbol{\Lambda}_m \mathbf{E}_m.$$

The Proposed Algorithm

- Design an efficient algorithm for solving (P).
- We show that (SDR) is a tight relaxation of (P).
- Solve the KKT optimality conditions of (SDR) based on its special structure:

1. Write out the equivalent KKT conditions:

$$\begin{cases} \mathbf{D}(\{\beta_k\}, \{\boldsymbol{\Lambda}_m\}) = \mathbf{0}, & (1) \\ \left.\begin{array}{l} \operatorname{rank}(\boldsymbol{\Lambda}_m) = 1, \ \boldsymbol{\Lambda}_m \succeq \mathbf{0}, \ \forall\, m \in \mathcal{M}, \\ \boldsymbol{\Lambda}_m^{(1:m-1,1:m)} = \mathbf{0}, \ \boldsymbol{\Lambda}_m^{(m:M,1:m-1)} = \mathbf{0}, \ \forall\, m \in \mathcal{M}, \end{array}\right\} & (2) \\ \left.\begin{array}{l} \operatorname{rank}(\mathbf{C}_k(\{\beta_k\}, \{\boldsymbol{\Lambda}_m\}) - \beta_k \mathbf{H}_k) = M - 1, \ \forall\, m \in \mathcal{M}, \\ \mathbf{C}_k(\{\beta_k\}, \{\boldsymbol{\Lambda}_m\}) - \beta_k \mathbf{H}_k \succeq \mathbf{0}, \ \forall\, m \in \mathcal{M}, \end{array}\right\} & (3) \\ \beta_k \geq 0, \ \forall\, k \in \mathcal{K}, & (4) \\ \mathbf{V}_k \bullet (\mathbf{C}_k(\{\beta_k\}, \{\boldsymbol{\Lambda}_m\}) - \beta_k \mathbf{H}_k) = 0, \ \forall\, k \in \mathcal{K}, & (5) \\ \mathbf{V}_k \succeq \mathbf{0}, \ \operatorname{rank}(\mathbf{V}_k) = 1, \ \forall\, k \in \mathcal{K}, & (6) \\ a_k(\{\mathbf{V}_k\}, \mathbf{Q}) = 0, \ \forall\, k \in \mathcal{K}, & (7) \\ \mathbf{B}_m(\{\mathbf{V}_k\}, \mathbf{Q}) \succeq \mathbf{0}, \ \forall\, m \in \mathcal{M}, & (8) \\ \boldsymbol{\Lambda}_m \bullet \mathbf{B}_m(\{\mathbf{V}_k\}, \mathbf{Q}) = 0, \ \forall\, m \in \mathcal{M}, & (9) \\ \mathbf{Q} \succeq \mathbf{0}. & (10) \end{cases}$$

2. Separate the equations into two sets (Eqs. (1)–(4) and Eqs. (5)–(10)) and solve the equations involving the dual variables first and then the equations involving the primal variables.
3. Show that each set of equations can be solved elegantly via fixed-point iteration.

The Proposed Algorithm (Cont.)

- First, consider solving Eqs. (1)–(4).
- Given $\{\beta_k\}$, one equivalent form of Eq. (1) is

$$\sum_{m=1}^{M} \eta_m \tilde{\boldsymbol{\Lambda}}_m - \sum_{m=1}^{M} \mathbf{E}_m^\dagger \boldsymbol{\Lambda}_m \mathbf{E}_m = \mathbf{I} + \sum_{k=1}^{K} \beta_k \gamma_k \mathbf{H}_k \triangleq \boldsymbol{\Gamma}.$$

- Only $\boldsymbol{\Lambda}_1$ affects the first row and column of matrix $\boldsymbol{\Gamma}$. $\Rightarrow$ The entries in the first row of $\boldsymbol{\Lambda}_1$ should be $\left[\frac{1}{\eta_1 - 1}\boldsymbol{\Gamma}^{(1,1)}, \frac{1}{\eta_1}\boldsymbol{\Gamma}^{(1,2:M)}\right]$.
- $\boldsymbol{\Lambda}_1$ is of rank one. (Eq. (2)) $\Rightarrow$ Further obtain all entries of $\boldsymbol{\Lambda}_1$.
- Subtract all terms related to $\boldsymbol{\Lambda}_1$ from both sides:

$$\sum_{m=2}^{M} \eta_m \tilde{\boldsymbol{\Lambda}}_m - \sum_{m=2}^{M} \mathbf{E}_m^\dagger \boldsymbol{\Lambda}_m \mathbf{E}_m = \boldsymbol{\Gamma} - \eta_m \boldsymbol{\Lambda}_1 + \mathbf{E}_1 \boldsymbol{\Lambda}_1 \mathbf{E}_1 \triangleq \boldsymbol{\Gamma}'.$$

- Repeat the above procedure to find all $\boldsymbol{\Lambda}_m$. Denote the solution to Eqs. (1)–(2) as $\{\boldsymbol{\Lambda}_m(\{\beta_k\})\}$.
- Given $\{\boldsymbol{\Lambda}_m\}$, by Eq. (3) and since $\mathbf{H}_k \succeq \mathbf{0}$ is of rank one, we have

$$\beta_k\left(\{\boldsymbol{\Lambda}_m\}, \{\beta_j\}_{j \neq k}\right) = \left(\boldsymbol{h}_k^\dagger \mathbf{C}_k^{-1} \mathbf{h}_k\right)^{-1} > 0.$$

- If one find $(\{\beta_k\}, \{\boldsymbol{\Lambda}_m(\{\beta_k\})\})$ that satisfy

$$\beta_k = I_k(\{\beta_k\}) \triangleq \beta_k\left(\{\boldsymbol{\Lambda}_m(\{\beta_k\})\}, \{\beta_j\}_{j \neq k}\right), \quad \forall\, k \in \mathcal{K}, \tag{11}$$

then all Eqs. (1)–(4) holds.
- Define $\boldsymbol{\beta} = [\beta_1, \ldots, \beta_K]^\mathsf{T}$ and $I(\boldsymbol{\beta}) = [I_1(\{\beta_k\}), \ldots, I_K(\{\beta_k\})]^\mathsf{T}$, then (11) is to find the fixed-point of function $I(\cdot)$.
- We can show that $I(\cdot)$ is a standard interference function.
- The fixed-point iteration $\boldsymbol{\beta}^{(i+1)} = I(\boldsymbol{\beta}^{(i)})$ will converge to the unique fixed point of $I(\cdot)$. (Lemma and [3, Theorem 2])
- Solving Eqs. (5)–(10) can also be reduced to find a fixed-point of some standard interference function $J(\cdot)$. Hence, it can also be solved by performing a fixed-point iteration.

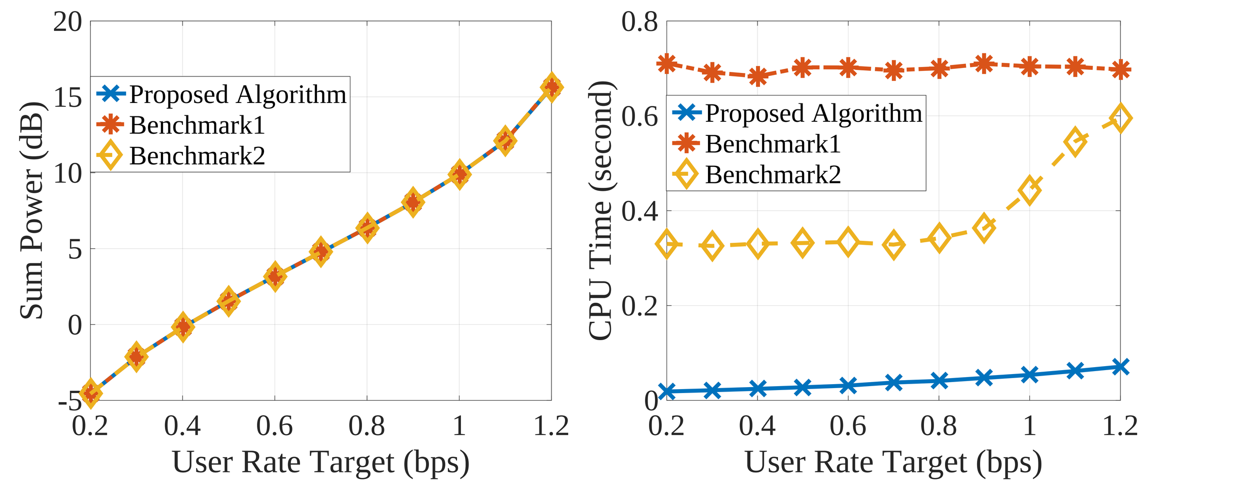
Algorithm 1 Proposed Algorithm for Solving Problem (P)

1: Find $\{\beta_k\}$ and $\{\boldsymbol{\Lambda}_m\}$ that satisfy Eqs. (1)–(4) by performing a fixed-point iteration via $I(\cdot)$ on $\{\beta_k\}$ until the desired error bound is met.
2: Find $\{\mathbf{V}_k\}$ and $\mathbf{Q}$ that satisfy Eqs. (5)–(10) by performing a fixed-point iteration via $J(\cdot)$ on $\{p_k\}$ until the desired error bound is met.
3: Find $\boldsymbol{v}_k$ such that $\mathbf{V}_k = \boldsymbol{v}_k \boldsymbol{v}_k^\dagger$, $\forall\, k \in \mathcal{K}$.
4: **Output:** $\{\boldsymbol{v}_k\}$ and $\mathbf{Q}$.

- Global optimality guarantee: If (SDR) is feasible, then proposed Algorithm 1 returns the optimal solution of problem (P).
- Computationally efficient: The evaluation of fixed-point iteration functions, namely $I(\cdot)$ and $J(\cdot)$, are cheap.

Simulation Results

- Consider a network with
  - $M = 8$ relays and $K = 10$ users,
  - the wireless channels between relays and users are generated based on the i.i.d. Rayleigh fading model following $\mathcal{CN}(0, 1)$,
  - and the fronthaul capacities between all relays and the CP are set to be 3 bits per symbol (bps).
- Moreover, the noise powers at the users are set to be $\sigma^2 = 1$.
- The rate targets for all the users are assumed to be identical.
- All results are obtained by averaging over 200 Monte-Carlo runs.



Left: Average sum power versus the user rate target;
Right: Average CPU time versus the user rate target.

- Benchmark1: directly call CVX to solve (SDR) $\Rightarrow$ verify the tightness
- Benchmark2: the proposed algorithm in [1] $\Rightarrow$ compare the efficiency
- The left figure verifies the tightness of the SDR and the global optimality of the solution returned by the proposed algorithm.
- The right figure shows the high efficiency of our proposed algorithm.

Conclusion

- Propose an efficient and global algorithm for solving the downlink beamforming and compression problem.
- Solve the KKT conditions by judiciously exploiting the problem structure.
- Achieve the global optimality as the state-of-the-art algorithm proposed in [1] but with a significant less CPU time.

References

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[3] R. D. Yates, "A framework for uplink power control in cellular radio systems," *IEEE J. Sel. Areas Commun.*, vol. 13, no. 7, pp. 1341-1347, Sept. 1995.