Robust Recovery of Jointly-Sparse Signals Using Minimax Concave Loss Function

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Outline

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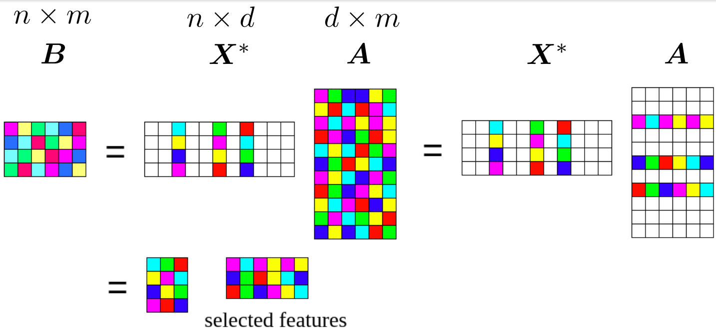
1. Background
2. Main Results
3. Numerical Examples
   - Experiment A: Robustness in Overdetermined Case
   - Experiment B: Support Recovery in Underdetermined Case
   - Experiment C: MECG Signal Recovery
4. Conclusion

Feature Selection Problem

Select an important subset of features
→ enhancing the performances (classification accuracy, training time, *etc*)



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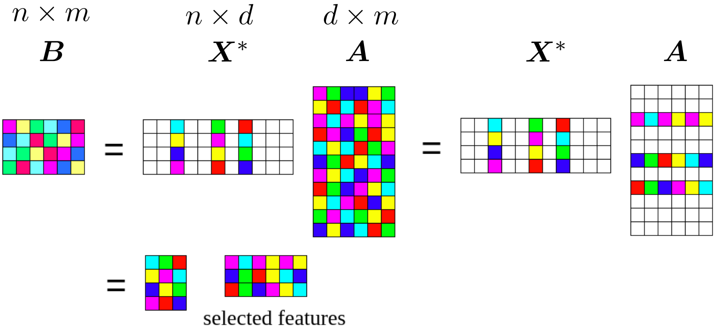

Without the condition $d > m$, this can be applied to multiple measurement vector (MMV) problem.

Applications: MECG, DNA microarrays, source localization, *etc*

Mathematical Model and Robust Feature Selection

$$\boldsymbol{B} = \boldsymbol{X}^* \boldsymbol{A} + \underset{\text{noise} \,\lrcorner\!\!\uparrow}{\boldsymbol{E}} + \underset{\uparrow\!\!\llcorner \, \text{outlier matrix (column sparse)}}{\boldsymbol{O}} \tag{1}$$

Mathematical Model and Robust Feature Selection

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Robust Feature Selection (RFS, Nie *et al.*, '10 [1]):

$$(\mathrm{P}_0) \qquad \min_{\boldsymbol{X} \in \mathcal{X}} \underbrace{\|\boldsymbol{B} - \boldsymbol{X}\boldsymbol{A}\|_{2,1}}_{\uparrow\ \boxed{\text{outlier robustness}}} + \underbrace{\lambda \|\boldsymbol{X}\|_{2,1}}_{\uparrow\ \boxed{\text{column sparsity}}}$$

$\|\boldsymbol{X}\|_{2,1} := \sum_{i=1}^{d} \|\boldsymbol{x}_i\|_2$ (sum of the column norms)

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[1] F. Nie, H. Huang, X. Cai, and C. H. Ding, "Efficient and robust feature selection via joint $\ell_{2,1}$-norms minimization," in Proc. Adv. Neural Inf. Process. Syst., 2010, pp. 1813–1821.

Minimax Concave Function

Huber function: $\phi_\gamma^{\mathrm{HB}}(x) := \begin{cases} \frac{1}{2\gamma}x^2 & \text{if } |x| \leq \gamma \\ |x| - \frac{1}{2}\gamma & \text{if } |x| > \gamma \end{cases} \quad (\gamma > 0)$

MC function (Zhang 2010 [2], Selesnick 2017 [3])

$$\phi_\gamma^{\mathrm{MC}}(x) \coloneqq |x| - \phi_\gamma^{\mathrm{HB}}(x) = \begin{cases} |x| - \frac{1}{2\gamma}x^2 & \text{if } |x| \leq \gamma \\ \frac{1}{2}\gamma & \text{if } |x| > \gamma \end{cases} \quad (\gamma > 0)$$

Constant over $[\gamma, +\infty)$ → remarkable robustness against outliers

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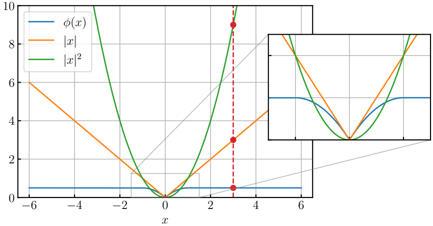

[2] C. H. Zhang, “Nearly unbiased variable selection under minimax concave penalty,” The Annals of Statistics, vol. 38, no. 2, pp. 894—942, 2010.
[3] I. Selesnick, “Sparse regularization via convex analysis,” IEEE Transactions on Signal Processing, vol. 65, no. 17, pp. 4481–4494, 2017.

Key Ideas

	robustness	convexity of loss	mathematical tractability (overall convexity)
Huber [4]	△	✓	✓
Tukey [5]	✓	×	×
Proposed	✓	(weakly convex)	✓

Key Ideas

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Ideas

1. MC loss
   → **Remarkable Outlier robustness** due to the nonconvexity
2. The squared Frobenius norm as an additional penalty
   → **Global optimality**
3. Split the cost function into convex terms
   → The reformulated problem can be solved by the efficient primal-dual splitting method
   → **Scalability**

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[4] P. J. Huber and E. Ronchetti, Robust Statistics, JohnWiley & Sons, 2009.
[5] R. A. Maronna, et al., Robust Statistics: Theory and Methods (with R), John Wiley & Sons, 2019.

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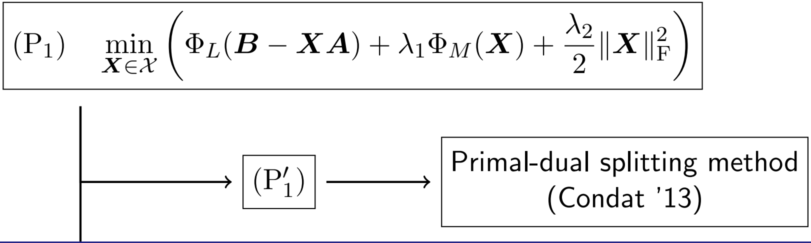
Proposed Formulation

$$(\mathrm{P}_1) \quad \min_{\boldsymbol{X}\in\mathcal{X}} \left(\Phi_L(\boldsymbol{B} - \boldsymbol{X}\boldsymbol{A}) + \lambda_1 \Phi_M(\boldsymbol{X}) + \frac{\lambda_2}{2}\|\boldsymbol{X}\|_{\mathrm{F}}^2 \right)$$

- $\Phi_L(\boldsymbol{Y}) := \|\boldsymbol{Y}\|_{2,1} - \min_{\boldsymbol{Z}\in\mathcal{Y}} \left(\|\boldsymbol{Z}\|_{2,1} + \frac{1}{2}\|\boldsymbol{Y} - \boldsymbol{Z}\|_L^2\right),\ \boldsymbol{Y} \in \mathcal{Y}$ $(\|\boldsymbol{Y}\|_L := \|L^{1/2}\boldsymbol{Y}\|_{\mathrm{F}})$
- $\Phi_M(\boldsymbol{X}) := \|\boldsymbol{X}\|_{2,1} - \min_{\boldsymbol{\Xi}\in\mathcal{X}} \left(\|\boldsymbol{\Xi}\|_{2,1} + \frac{1}{2}\|\boldsymbol{X} - \boldsymbol{\Xi}\|_M^2\right),\ \boldsymbol{X} \in \mathcal{X}$
- $L\boldsymbol{B} := \boldsymbol{B}\mathrm{diag}(l_1, \ldots, l_m),\ l_i > 0,\ \forall i = 1, \ldots, m$
- $M\boldsymbol{X} := \boldsymbol{X}\mathrm{diag}(\mu_1, \ldots, \mu_n),\ \mu_j > 0,\ \forall j = 1, \ldots, n$
- $\lambda_1 \geq 0$
- $\lambda_2 \geq 0$

Φ_L and Φ_M are slight extensions of the MC function to group-sparse matrices

Flow of the Derivation



$$\boxed{(\mathrm{P}_1) \quad \min_{\boldsymbol{X}\in\mathcal{X}} \left(\Phi_L(\boldsymbol{B} - \boldsymbol{X}\boldsymbol{A}) + \lambda_1 \Phi_M(\boldsymbol{X}) + \frac{\lambda_2}{2}\|\boldsymbol{X}\|_{\mathrm{F}}^2 \right)}$$

$$\longrightarrow \boxed{(\mathrm{P}_1')} \longrightarrow$$

Primal-dual splitting method
(Condat '13)

Moreau's decomposition

Let $f \in \Gamma_0(\mathcal{X})$. Then,

$$ {}^{1}f + {}^{1}f^{*} = \frac{1}{2}\|\cdot\|_{\mathrm{F}}^2, \tag{2}$$

where the Moreau envelope of f of index γ is

$$ {}^{\gamma}f : \boldsymbol{X} \mapsto \min_{\boldsymbol{Y}\in\mathcal{Y}} \left\{ f(\boldsymbol{Y}) + \frac{1}{2\gamma}\|\boldsymbol{X} - \boldsymbol{Y}\|_{\mathrm{F}}^2 \right\}, \tag{3}$$

and f^{*} is the convex conjugate of f.

Reformulation Based on Moreau's Decomposition

$$(\mathrm{P}_1) \quad \min_{\boldsymbol{X} \in \mathcal{X}} \left(\Phi_L(\boldsymbol{B} - \boldsymbol{X}\boldsymbol{A}) + \lambda_1 \Phi_M(\boldsymbol{X}) + \frac{\lambda_2}{2} \|\boldsymbol{X}\|_{\mathrm{F}}^2 \right)$$

Reformulation

$$(\mathrm{P}_1) \Leftrightarrow (\mathrm{P}_1') \;\; \min_{\boldsymbol{X} \in \mathcal{X}} [F(\boldsymbol{X}) + G(\boldsymbol{X}) + H(L_1\boldsymbol{X})] \tag{4}$$

- $F(\boldsymbol{X}) := \frac{\lambda_2}{2}\|\boldsymbol{X}\|_{\mathrm{F}}^2 - \frac{1}{2}\|L_1\boldsymbol{X}\|_L^2 - \frac{\lambda_1}{2}\|\boldsymbol{X}\|_M^2 + \langle L_1\boldsymbol{X}, \boldsymbol{B}\rangle_L$
  $+ {}^{1}(\iota_C \circ L^{1/2})(L^{1/2}(\boldsymbol{B} - L_1\boldsymbol{X})) + \lambda_1 {}^{1}(\iota_C \circ M^{1/2})(M^{1/2}\boldsymbol{X})$

  (a certain condition → convexity)
- $G(\boldsymbol{X}) := \lambda_1 \|\boldsymbol{X}\|_{2,1}$ (automatically convex)
- $H(\boldsymbol{Y}) := \|\boldsymbol{B} - \boldsymbol{Y}\|_{2,1}$ (automatically convex)
- $L_1\boldsymbol{X} := \boldsymbol{X}\boldsymbol{A}$

Convexity Results

$$(\mathrm{P}_1') \quad \min_{\boldsymbol{X}\in\mathcal{X}} [F(\boldsymbol{X}) + G(\boldsymbol{X}) + H(L_1\boldsymbol{X})] \tag{5}$$

$$\begin{aligned} F(\boldsymbol{X}) := {} & \frac{\lambda_2}{2}\|\boldsymbol{X}\|_{\mathrm{F}}^2 - \frac{1}{2}\|L_1\boldsymbol{X}\|_L^2 - \frac{\lambda_1}{2}\|\boldsymbol{X}\|_M^2 + \langle L_1\boldsymbol{X}, \boldsymbol{B}\rangle_L \\ & + {}^{1}(\imath_C \circ L^{1/2})(L^{1/2}(\boldsymbol{B} - L_1\boldsymbol{X})) + \lambda_1 {}^{1}(\imath_C \circ M^{1/2})(M^{1/2}\boldsymbol{X}) \end{aligned} \tag{6}$$

Convexity Results

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Proposition 1

1. The function F is convex ($\rightarrow$ **Global optimality**) if

$$\lambda_2 \geq \lambda_{\max}\{\boldsymbol{A}\mathrm{diag}(l_1, \ldots, l_m)\boldsymbol{A}^{\mathsf{T}} + \lambda_1 \mathrm{diag}(\mu_1, \ldots, \mu_n)\}. \tag{7}$$

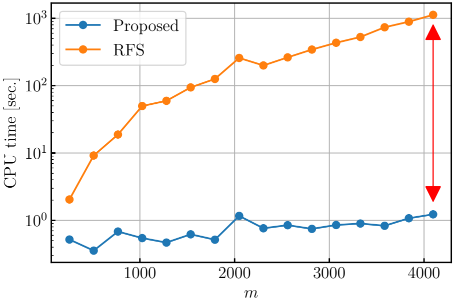
2. The condition (7) is also necessary when
$$K := \{\boldsymbol{X} \in \mathcal{X} \mid \|L(\boldsymbol{B} - \boldsymbol{X}\boldsymbol{A})\|_{2,\infty} \leq 1, \|M\boldsymbol{X}\|_{2,\infty} \leq 1\}$$
has a nonempty interior. ($\|\boldsymbol{X}\|_{2,\infty}$: maximal column norm)

Note: $K := \{\boldsymbol{X} \in \mathcal{X} \mid \left(\iota_C \circ L^{1/2} \boxdot q\right)(L^{1/2}(\boldsymbol{B} - \boldsymbol{X}\boldsymbol{A})) = 0\} \cap \{\boldsymbol{X} \in \mathcal{X} \mid \left(\iota_C \circ M^{1/2} \boxdot q\right)(M^{1/2}\boldsymbol{X}) = 0\}$ in the original paper.

Scalability of Proposed Method

Table: Computational complexity (q: the number of iterations)

Algorithm	Computational complexity
Proposed	$\mathcal{O}(\max\{qnmd, \min\{d^2, m^2\} \max\{d, m, p_\beta, p_\sigma\}\})$
RFS	$\mathcal{O}(qm(m+d) \max\{m+d, n\})$



18 min.

1 sec.

Scalability

Difficulties in Applying PDS or ADMM

$$(\mathrm{P}_1) \quad \min_{\boldsymbol{X}\in\mathcal{X}} \left(\Phi_L(\boldsymbol{B} - \boldsymbol{X}\boldsymbol{A}) + \lambda_1 \Phi_M(\boldsymbol{X}) + \frac{\lambda_2}{2}\|\boldsymbol{X}\|_{\mathrm{F}}^2 \right)$$

- Applying the primal-dual splitting method directly:
  - Each term should be convex
  - Proximity operator of the conjugate function of the first term is the zero operator
- Applying ADMM directly:
  - A certain proximity operator needs to be firmly nonexpansive, but... it is not even nonexpansive in the present case

Please refer to Section III-B of the following paper for detailed discussions:
K. Suzuki, and M. Yukawa, "Robust recovery of jointly-sparse signals using minimax concave loss function." IEEE Transactions on Signal Processing, pp. 669–681, 2021.

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Experiment A: Robustness in Overdetermined Case

- Signal model: $\boldsymbol{B} = \boldsymbol{X}^* \boldsymbol{A} + \boldsymbol{E} + \boldsymbol{O}$
  - $\boldsymbol{X}^* \in \mathbb{R}^{n \times d}$, $\boldsymbol{A} \in \mathbb{R}^{d \times m}$: i.i.d. $\mathcal{N}(0, 1)$
    ($\boldsymbol{X}^*$ is dense so that the pure effects of robustification can be seen.)
  - $\boldsymbol{E}$: SNR $10, 30$ dB
  - $\boldsymbol{O}$: column sparse, i.i.d. $\mathcal{N}(0, 100)$

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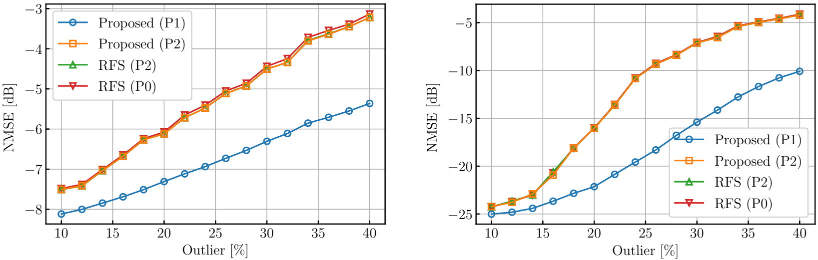
- Approaches to be tested:
  1. (P_1): $\min_{\boldsymbol{X} \in \mathbb{R}^{n \times d}} \Phi_L(\boldsymbol{B} - \boldsymbol{X}\boldsymbol{A}) + \frac{\lambda_2}{2} \|\boldsymbol{X}\|_{\mathrm{F}}^2$ by the proposed approach
     $(\lambda_1 = 0)$
  2. (P_2): $\min_{\boldsymbol{X} \in \mathbb{R}^{n \times d}} \|\boldsymbol{B} - \boldsymbol{X}\boldsymbol{A}\|_{2,1} + \frac{\lambda_2}{2} \|\boldsymbol{X}\|_{\mathrm{F}}^2$ by the proposed approach
     $(\lambda_1 = 0,\ L = O)$
  3. (P_2): $\min_{\boldsymbol{X} \in \mathbb{R}^{n \times d}} \|\boldsymbol{B} - \boldsymbol{X}\boldsymbol{A}\|_{2,1} + \frac{\lambda}{2} \|\boldsymbol{X}\|_{\mathrm{F}}^2$ by RFS (Nie *et al.*, '10)
  4. (P_0): $\min_{\boldsymbol{X} \in \mathbb{R}^{n \times d}} \|\boldsymbol{B} - \boldsymbol{X}\boldsymbol{A}\|_{2,1} + \lambda \|\boldsymbol{X}\|_{2,1}$ by RFS

Experiment A: Results

- Evaluation metric: normalized mean squared errors (NMSE)

$$\mathrm{NMSE} := \frac{\|\boldsymbol{X}^* - \hat{\boldsymbol{X}}\|_{\mathrm{F}}^2}{\|\boldsymbol{X}^*\|_{\mathrm{F}}^2} \tag{8}$$



(a) SNR 10 dB

(b) SNR 30 dB

Figure: NMSE for different column-sparsity of outlier matrix under $d = 128$, $m = 256$, and $n = 128$.

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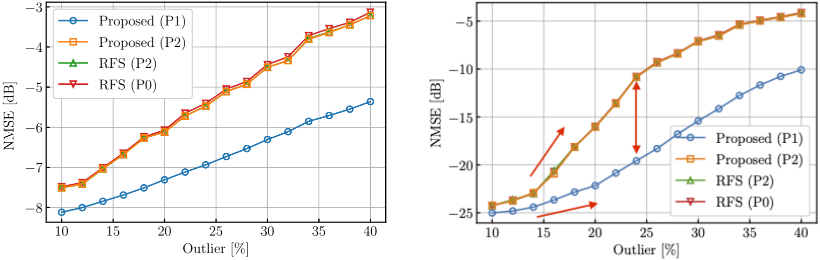


Remarkable Outlier–Robustness

(a) SNR 10 dB

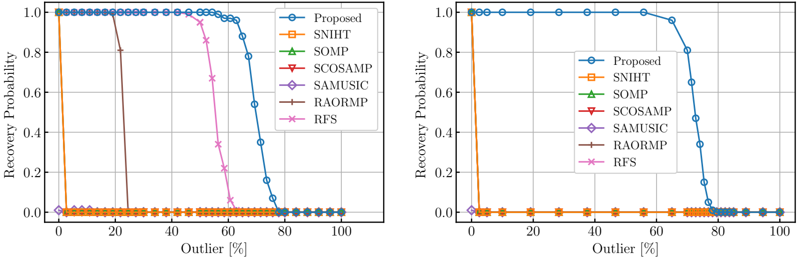
(b) SNR 30 dB

Figure: NMSE for different column-sparsity of outlier matrix under $d = 128$, $m = 256$, and $n = 128$.

Experiment B: Support Recovery in Underdetermined Case

- Signal model: $\boldsymbol{B} = \boldsymbol{X}^* \boldsymbol{A} + \boldsymbol{E} + \boldsymbol{O}$
  - $\boldsymbol{X}^* \in \mathbb{R}^{n \times d}$: column sparse ($k$ non-zero vectors), i.i.d. $\mathcal{N}(0, 1)$
  - $\boldsymbol{A} \in \mathbb{R}^{d \times m}$: i.i.d. $\mathcal{N}(0, 1)$
  - $\boldsymbol{E}$: SNR 30 dB
  - $\boldsymbol{O}$: column sparse (k' non-zero vectors) with $\mathrm{SNR_O} := \frac{\|\boldsymbol{X}^* \boldsymbol{A}\|_{\mathrm{F}}^2 / m}{\|\boldsymbol{O}\|_{\mathrm{F}}^2 / k'}$
- Algorithms to be tested:
  - Proposed approach
  - Subspace augmented MUSIC (SAMUSIC) (Tropp *et al.*, '06)
  - RFS (Nie *et al.*, '10)
  - Simultaneous OMP (SOMP) (Kim *et al.*, '12)
  - Rank aware order recursive matching pursuit (RAORMP) (Davies and Eldar, '12)
  - Simultaneous normalized IHT (SNIHT) (Blanchard *et al.*, '14)
  - Simultaneous COSAMP (SCOSAMP) (Blanchard *et al.*, '14)
- Evaluation metric: success probability of support recovery

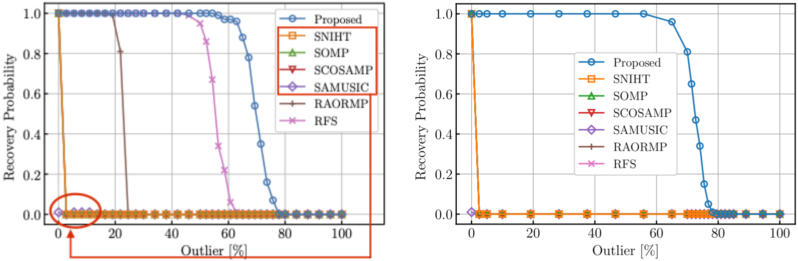
Experiment B: Results (1/2)



(a) $\mathrm{SNR_O}$ -30 dB (b) $\mathrm{SNR_O}$ -3000 dB

Figure: Recovery probability for different column sparsity of outlier matrix under $d = 256$, $m = 128$, $n = 32$, and $k = 16$.

Experiment B: Results (1/2)



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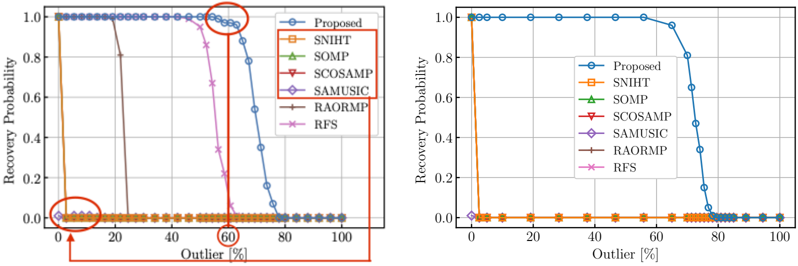

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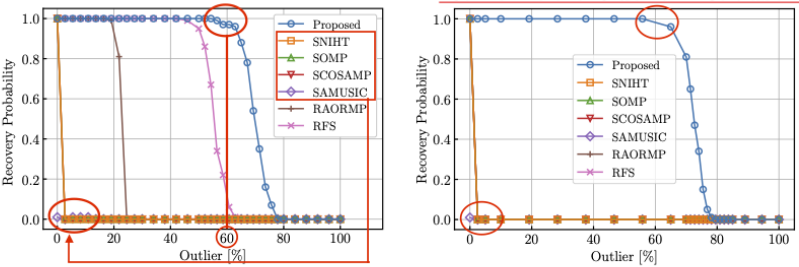

Remarkable Outlier–Robustness

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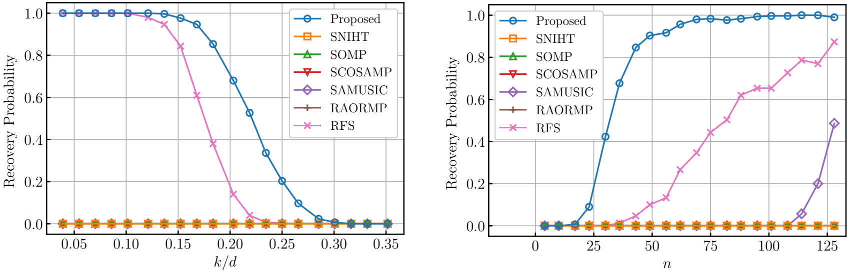
Experiment B: Results (2/2)



(a) $d = 256$, $m = 128$, $n = 32$.

(b) $d = 192$, $m = 128$, $k = 0.4d$.

Figure: Recovery probability as a function of k/d and n under outlier 30%, and $\mathrm{SNR_O}$ -30 dB.

Experiment B: Results (2/2)

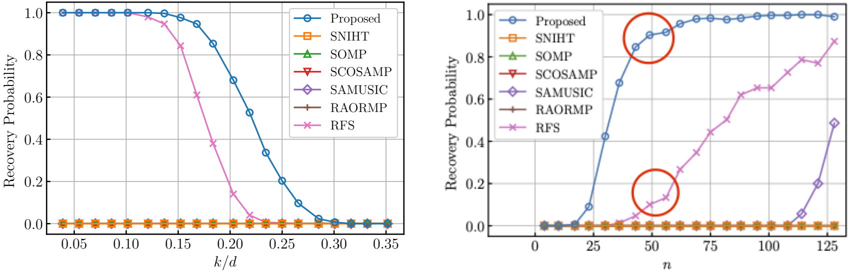


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Figure: Recovery probability as a function of k/d and n under outlier 30%, and $\mathrm{SNR_O}$ -30 dB.

Experiment C: MECG Signal Recovery

Signal model:

$$\begin{aligned} \boldsymbol{B} &= \boldsymbol{X}^{*}\boldsymbol{A} + \boldsymbol{E} + \boldsymbol{O} = \boldsymbol{W}\boldsymbol{\Psi}\boldsymbol{A} + \boldsymbol{E} + \boldsymbol{O} \\ &= \boldsymbol{W}\boldsymbol{\Theta} + \boldsymbol{E} + \boldsymbol{O} \end{aligned} \tag{9}$$

- $\boldsymbol{B} \in \mathbb{R}^{n \times m}$: publicly available database PTB [6]

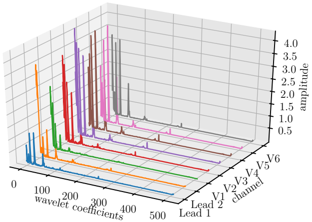
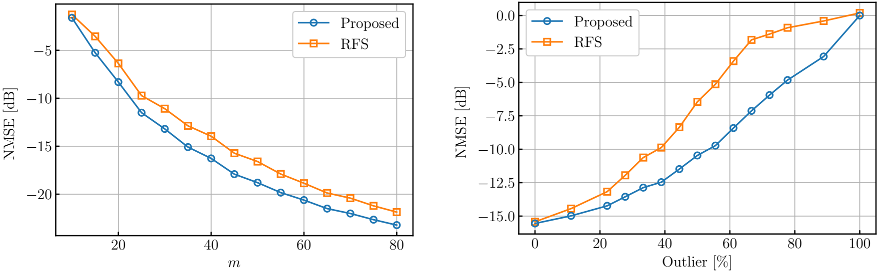


Figure: Amplitudes of ECG wavelet coefficients (**jointly sparse**)

- $\boldsymbol{A} \in \mathbb{R}^{d \times m}$: random sparse binary matrix
- $\boldsymbol{\Psi} \in \mathbb{R}^{d \times d}$: orthonormal wavelet basis
- $\boldsymbol{\Theta} := \boldsymbol{\Psi}\boldsymbol{A}$
- $\boldsymbol{W} \in \mathbb{R}^{n \times d}$: wavelet coefficient vectors
- SNR 30 dB
- $\boldsymbol{O}$: column sparse, $\mathrm{SNR_O} : -40$ dB

[6] A. L. Goldberger et al., "PhysioBank, PhysioToolkit, and PhysioNet: Components of a new research resource for complex physiologic signals," Circulation, vol. 101, no. 23, pp. e215—e220, 2000.

Experiment C: Results



(a) (b)

Figure: NMSE as a function of m under outlier rate 30%, NMSE as a function of outlier rate under $m = 30$.

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- We proposed a robust algorithm to recover jointly-sparse signals in the presence of outliers.
- We showed the convexity condition for the proposed formulation → this led to **the global optimality**.
    - The problem was reformulated via Moreau's decomposition for splitting the cost function into convex terms.
- The proposed approach is **scalable** to high dimensional settings by the use of the efficient primal-dual splitting method.
- Extensive simulation studies showed the **remarkable robustness** of the proposed method to outliers.

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  - The problem was reformulated via Moreau's decomposition for splitting the cost function into convex terms.
- The proposed approach is **scalable** to high dimensional settings by the use of the efficient primal-dual splitting method.
- Extensive simulation studies showed the **remarkable robustness** of the proposed method to outliers.

To the best of our knowledge, this is the first work leveraging the MC function in a robust framework while maintaining the overall convexity of the whole cost function.

Thank you very much for your kind listening