AUD-19.2

Region-to-region kernel interpolation of acoustic transfer function with directional weighting

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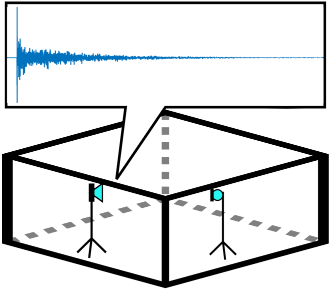
Background

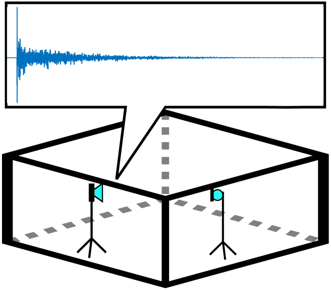
- Soundwave propagation inside an environment isn't predictable.
- The relation between source and receiver signal can be estimated with the **acoustic transfer function** (ATF) of the space.



Background

ATF interpolation with variable source and receiver

- Most ATF interpolation methods have a fixed source position.
- Our objective is to describe how the ATF changes for variable source and receiver within assigned regions. A **region-to-region** interpolation.



Background

⇒There are established region-to-region interpolation methods.

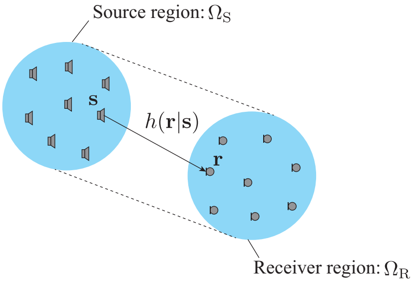
- In [Samarasinghe+, 2015], an ATF interpolation method using a spherical wavefunction expansion was proposed.
- This formulation was compared to a kernel ridge regression method with a specialized kernel in [Ribeiro+, 2020].
- The kernel method outperformed the wavefunction expansion for every frequency.

⇒The kernel method still has issues

- Neither method takes into consideration the distribution of the sources and receivers.
- The methods are rather susceptible to noise contamination, and as such are vulnerable to outliers in the data.

Problem statement

- Our objective is to accurately estimate the ATF between a variable receiver/source pair $\mathbf{r}|\mathbf{s}$ within regions in a space $\Omega \subset \mathbb{R}^3$.
- We distribute L loudspeakers in a source region $\Omega_\mathrm{S} \subset \Omega$ and M microphones in a receiver region $\Omega_\mathrm{R} \subset \Omega$.
- In order to interpolate the ATF between regions, we must derive an interpolation function from the $N = LM$ measurements.



Problem statement

Properties of the ATF

- The ATF is the superposition of two components:

$$h(\mathbf{r}|\mathbf{s},k) = h_{\mathrm{D}}(\mathbf{r}|\mathbf{s},k) + h_{\mathrm{R}}(\mathbf{r}|\mathbf{s},k).$$

- The direct component h_{D} is considered to be the equivalent of a recorded signal in the free-field, caused by a point source.

$$h_{\mathrm{D}}(\mathbf{r}|\mathbf{s},k) = G_0(\mathbf{r}|\mathbf{s},k) = \frac{e^{ik\|\mathbf{r}-\mathbf{s}\|}}{4\pi\|\mathbf{r}-\mathbf{s}\|}$$

- The reverberant component h_{R} satisfies the Helmholtz equation on both position variables:

$$(\nabla_{\mathbf{r}}^2 + k^2)h_{\mathrm{R}}(\mathbf{r}|\mathbf{s},k) = (\nabla_{\mathbf{s}}^2 + k^2)h_{\mathrm{R}}(\mathbf{r}|\mathbf{s},k) = 0$$

- Reciprocity for every pair $\mathbf{r}|\mathbf{s}$: $h(\mathbf{r}|\mathbf{s},k) = h(\mathbf{s}|\mathbf{r},k)$

Optimization objective

- Since h_{D} is considered known, we focus our efforts on h_{R}.
- The interpolation function $\hat{h}_{\mathrm{R}}$ obtained from our reverberant field measurements $\mathbf{y} = [y_1,\ y_2,\ \ldots,\ y_N]$.
- We create the cost to be minimized:

$$\mathcal{J}(f) := \sum_{n=1}^{N} |y_n - f(\mathbf{q}_n)|^2 + \lambda \|f\|_{\mathscr{H}}^2,\ f \in \mathscr{H}$$

- The vector $\mathbf{q}_n$ represents the n-th source/receiver position pair.
- The measurement vector $\mathbf{y}$ is obtained by removing the direct component from all impulse response recordings.

The functional space $\mathscr{H}$ must be defined to be representative of the data

Kernel ridge regression

- When $\mathscr{H}$ is a reproducing kernel Hilbert space (RKHS) of reproducing kernel κ, the minimizer of $\mathcal{J}$ has a closed form:

$$\hat{h}_{\mathrm{R}}(\mathbf{r}|\mathbf{s}) = \boldsymbol{\kappa}(\mathbf{r}|\mathbf{s})(\mathbf{K} + \lambda \mathbf{I})^{-1}\mathbf{y},$$

  where:

$$\boldsymbol{\kappa}(\mathbf{r}|\mathbf{s}) = [\kappa(\mathbf{r}|\mathbf{s}, \mathbf{q}_1), \ldots, \kappa(\mathbf{r}|\mathbf{s}, \mathbf{q}_N)],$$

$$\mathbf{K} = \begin{bmatrix} \kappa(\mathbf{q}_1, \mathbf{q}_1) & \kappa(\mathbf{q}_1, \mathbf{q}_2) & \ldots & \kappa(\mathbf{q}_1, \mathbf{q}_N) \\ \vdots & \vdots & \ddots & \vdots \\ \kappa(\mathbf{q}_N, \mathbf{q}_1) & \kappa(\mathbf{q}_N, \mathbf{q}_2) & \ldots & \kappa(\mathbf{q}_N, \mathbf{q}_N) \end{bmatrix},$$

  and $\lambda > 0$.

- In [Ribeiro+, 2020] we have shown that if the RKHS is defined with the properties of the ATF in mind, we can achieve accurate interpolations.

Definition of the generalized Hilbert space

- We express h_{R} using the Herglotz wavefunction $h_{\mathrm{R}}(\mathbf{r}|\mathbf{s}) = \mathcal{I}\left(\tilde{h}_{\mathrm{R}};\mathbf{r}|\mathbf{s}\right)$, where:

$$\mathcal{I}\left(f;\mathbf{r}|\mathbf{s}\right) := \int_{\mathbb{S}^2\times\mathbb{S}^2} e^{\mathrm{i}k(\hat{\mathbf{r}}\cdot\mathbf{r}+\hat{\mathbf{s}}\cdot\mathbf{s})} f(\hat{\mathbf{r}},\hat{\mathbf{s}})\mathrm{d}\hat{\mathbf{r}}\mathrm{d}\hat{\mathbf{s}}.$$

- We can thus define the Hilbert space $(\mathscr{H}, \langle\cdot,\cdot\rangle_{\mathscr{H}})$ as:

$$\mathscr{H} = \Big\{h_{\mathrm{R}} = \mathcal{I}\left(\tilde{h}_{\mathrm{R}};\mathbf{r}|\mathbf{s}\right) : \tilde{h}_{\mathrm{R}} \in L^2(W,\mathbb{S}^2\times\mathbb{S}^2), \quad \tilde{h}_{\mathrm{R}}(\hat{\mathbf{r}},\hat{\mathbf{s}}) = \tilde{h}_{\mathrm{R}}(\hat{\mathbf{s}},\hat{\mathbf{r}})\ \forall\hat{\mathbf{r}},\hat{\mathbf{s}}\in\mathbb{S}^2\Big\}$$

$$\langle f,g\rangle_{\mathscr{H}} = \int_{\mathbb{S}^2\times\mathbb{S}^2} \frac{\overline{\tilde{f}(\hat{\mathbf{r}},\hat{\mathbf{s}})}\tilde{g}(\hat{\mathbf{r}},\hat{\mathbf{s}})}{W(\hat{\mathbf{r}},\hat{\mathbf{s}})}\mathrm{d}\hat{\mathbf{r}}\mathrm{d}\hat{\mathbf{s}},\ \forall f,g\in\mathscr{H}$$

Directional kernel formulation

- The weight function W gives the reproducing kernel κ as

$$\kappa(\mathbf{r}|\mathbf{s},\mathbf{r}'|\mathbf{s}') = \mathcal{I}\left(W(\hat{\mathbf{r}},\hat{\mathbf{s}})\left(\frac{e^{-\mathrm{i}k(\hat{\mathbf{r}}\cdot\mathbf{r}'+\hat{\mathbf{s}}\cdot\mathbf{s}')}+e^{-\mathrm{i}k(\hat{\mathbf{r}}\cdot\mathbf{s}'+\hat{\mathbf{s}}\cdot\mathbf{r}')}}{2}\right);\mathbf{r}|\mathbf{s}\right)$$

- In [Ribeiro+, 2020], the relative position of the regions within Ω was not taken into account.
- The introduction of a weight function enables the choice of a kernel function better adapted for the region configurations.
- The weight will be separable, $W(\hat{\mathbf{r}},\hat{\mathbf{s}}) = w(\hat{\mathbf{r}})w(\hat{\mathbf{s}})$ in order to simplify calculations and guarantee reciprocity.
- As the direct component is removed, plane wave components in the direct path are expected to be less significant.

Proposed directional kernel weight function

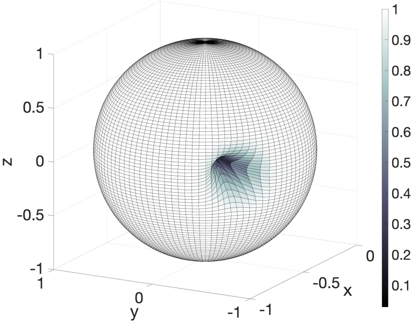
- The weight we propose for the interpolation method is:

$$w(\hat{\mathbf{v}}) = \frac{1}{4\pi}\left(1 + \gamma^2 - \frac{\cosh(\beta \hat{\mathbf{v}} \cdot \hat{\mathbf{v}}_0)}{\cosh(\beta)}\right), \ \hat{\mathbf{v}} \in \mathbb{S}^2$$

- The direction $\hat{\mathbf{v}}_0$ is the direction connecting the centers of Ω_{S} and Ω_{R}.
- The hyperparameter β adjusts the selectivity around the direct component.
- The hyperparameter γ adjusts the minimum gain baseline of the weight.

Proposed directional kernel weight function

- Below we have an example of the weight function, represented in a gain plot.
- For a direction $\hat{\mathbf{v}}$, the distance from the center to the surface is $w(\hat{\mathbf{v}})$.



$\Rightarrow$ But how do we choose β and γ?

Leave-one-out cross-validation

- Optimizing the loss function to our hyperparameters might over-condition the method to the measured data, which has noise.
- We opted instead to minimize the leave-one-out cross-validation error (LOO).

$$\mathrm{LOO}(\mathbf{y}, \ell) = \frac{1}{N} \sum_{n=1}^{N} \ell \left(\hat{f}_n(\mathbf{q}_n) - y_n \right)$$

- The loss ℓ was either square error (SQE) or Tukey loss.

$$\mathrm{SQE}(z) = |z|^2$$

$$\mathrm{Tukey}(z) = \begin{cases} \frac{\sigma^2}{6} \left(1 - \left(1 - \frac{|z|^2}{\sigma^2} \right)^3 \right), & |z| \leqslant \sigma \\ \frac{\sigma^2}{6}, & |z| > \sigma \end{cases}$$

The uniform weight kernel

- For $\gamma = 1$, $\beta = 0$, we have a uniform weight $w = 1/4\pi$, κ is known to be:

$$\kappa(\mathbf{r}|\mathbf{s}, \mathbf{r}'|\mathbf{s}') =$$
$$\frac{1}{2}\left(j_0(k\|\mathbf{r} - \mathbf{r}'\|)j_0(k\|\mathbf{s} - \mathbf{s}'\|) + j_0(k\|\mathbf{s} - \mathbf{r}'\|)j_0(k\|\mathbf{r} - \mathbf{s}'\|)\right)$$

- This kernel function coincides with the one used in [Ribeiro+, 2020], making this estimation identical.
- The weighted kernel is an extension of this kernel function.
- This method will be the standard of comparison.

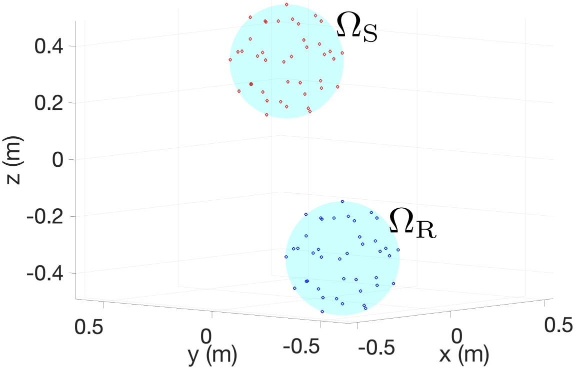
Experimental simulations

- We conducted numerical simulations with the image source method to compare both interpolation functions
- The arrays in both source and receiver regions had $L = M = 41$ points.
- Simulator conditions:

Room dimensions	$[3.2, 4.0, 2.7]$ m
Reverberation time T_{60}	0.45 s
Reflection coefficients	$[0.802, 0.866, 0.945]$
Inner radius of the array	0.19m
Outer radius of the array	0.20m
Signal-to-noise ratio	20 dB

Experimental simulations

- The center of the cartesian system is at the geometric center of the room.
- The centers of the source and receiver region were $\mathbf{s}_0 = [0.35, 0.43, 0.29]^\mathsf{T}$ m and $\mathbf{r}_0 = [-0.35, -0.43, -0.29]^\mathsf{T}$ m.



Error criteria

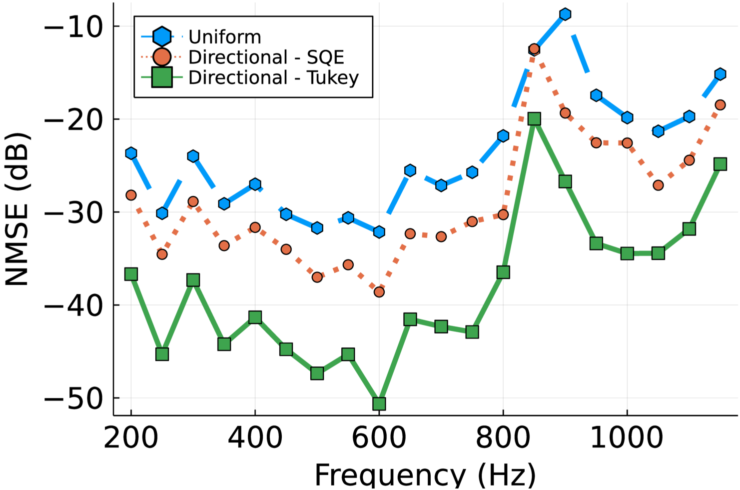
- The first criterion was the normalized mean square error (NMSE)

$$\mathrm{NMSE} = 10\log_{10}\left(\frac{\sum_n \left|\hat{h}(\mathbf{q}'_n) - h(\mathbf{q}'_n)\right|^2}{\sum_n |h(\mathbf{q}'_n)|^2}\right)$$

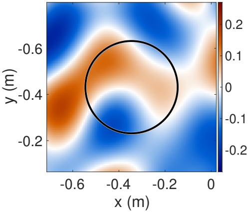
- We analyzed the NMSE for 9025 possible densely-distributed source-receiver evaluation pairs given as $\{\mathbf{q}'_n\}_{n=1}^{9025}$.
- The second criterion was the normalized square error (NSE) distribution in the regions.
- The frequency of analysis for the NSE was $950\ \mathrm{Hz}$

Normalized mean square error

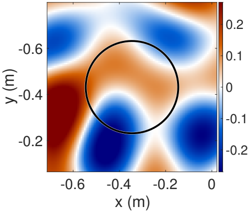
Normalized mean square error comparison:



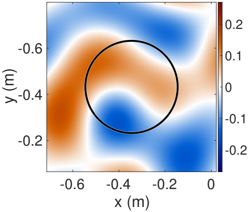
Colormaps of the real part, comparing the reconstruction of the signal generated by a single source in the center of Ω_{S}:



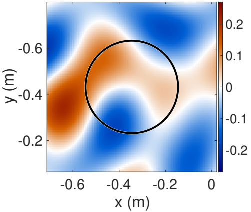
(a) Original



(b) Uniform

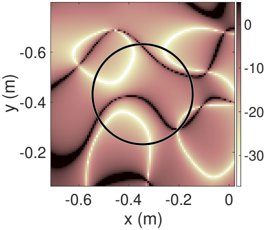


(c) SQE

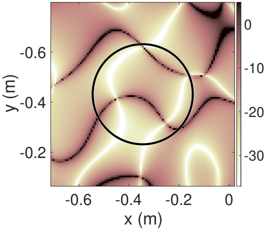


(d) Tukey

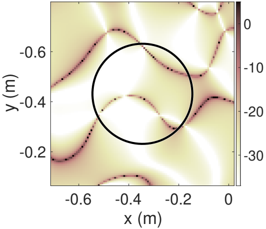
Normalized square error distributions:



(a) Uniform



(b) SQE



(c) Tukey

Conclusion

In summary:

- We defined a function space $\mathscr{H}$ using the physical properties of the ATF, which allowed us to interpolate its value for variable source and receiver positions.
- We generalized a previously established kernel formulation by adding directionality based on the expected profile of the ATF.
- This proposed formulation can be optimized using the same data points used to derive the model.
- The directional kernel estimations outperformed the uniform kernel in both mean error by frequency and in reconstructing the ATF spatially.
- Additionally, the use of a robust loss criterion also gave us better results than the standard square error.