Poster 9074

Unlimited Sampling with Local Averages

Recovery of High Dynamic Range Inputs from Average Samples

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Summary

- We address a fundamental bottleneck in physical sensors: **dynamic range limitation**.
- The limitation was mainly discussed in the traditional scenario of ideal point-wise sampling as part of the **Unlimited Sensing Framework**.
- In our hardware experiments we noticed a **deviation from the ideal sampling model**, particularly for signals with jump discontinuities.
- The observations are better described by a **sampling model with local averages**.
- We introduce a new average sampling architecture based on a nonlinear operator called ***modulo-hysteresis*** used recently for recovery from point-wise samples.
- We give **theoretical recovery guarantees** and show the performance of our method using hardware and synthetic data.

The Unlimited Sensing Framework (USF)

- A conventional ADC saturates for inputs outside its dynamic range $\Rightarrow$ permanent information loss; modulo encoding addresses this [2]: $z(t) = \mathscr{M}_\lambda(g(t))$, $k \in \mathbb{Z}$.

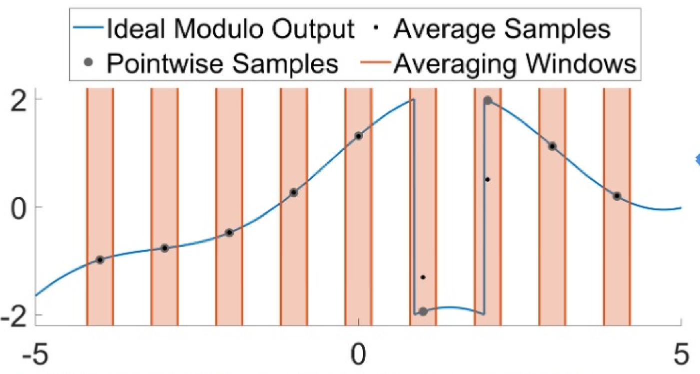
Theorem (**Unlimited Sampling Theorem** – [2]). *Let $g(t) \in \mathrm{PW}_\Omega$ and $y_k = \mathscr{M}_\lambda(g(t))|_{t=kT}$, $k \in \mathbb{Z}$ be the modulo samples of $g(t)$ with sampling period T. Then, a sufficient condition for recovery of $g(t)$ from the $\{y_k\}$ up to additive multiples of 2λ is*

$$T < \frac{1}{2\Omega e}.$$

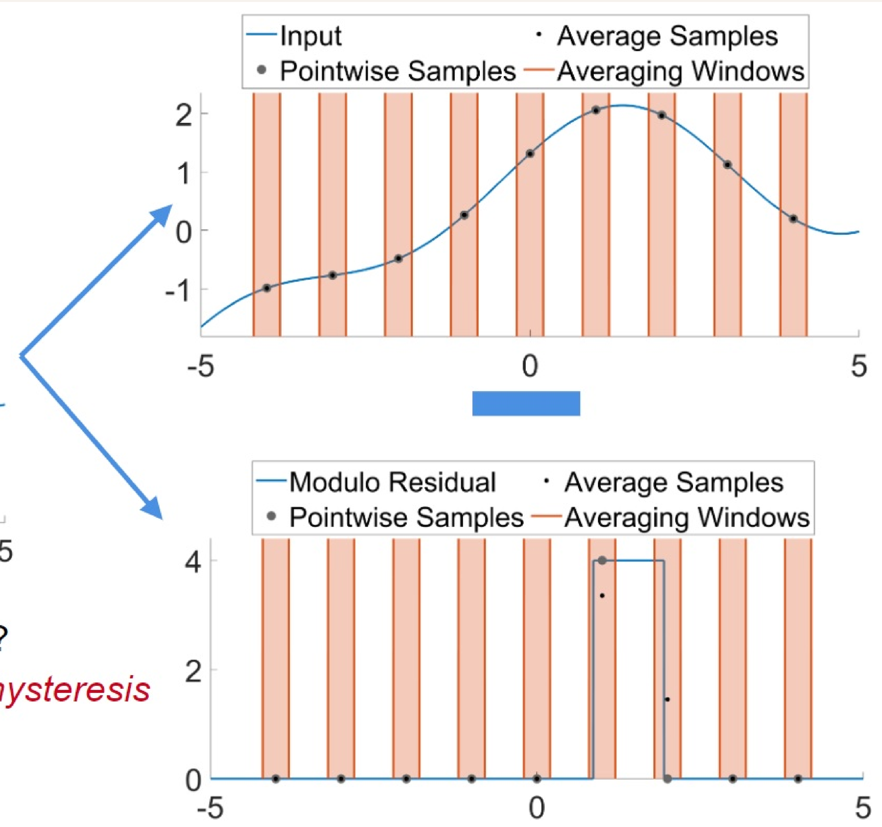
- If the sampling model is not ideal or point-wise (such as average sampling), then the USF approach is not applicable.

Ideal vs Average Sampling

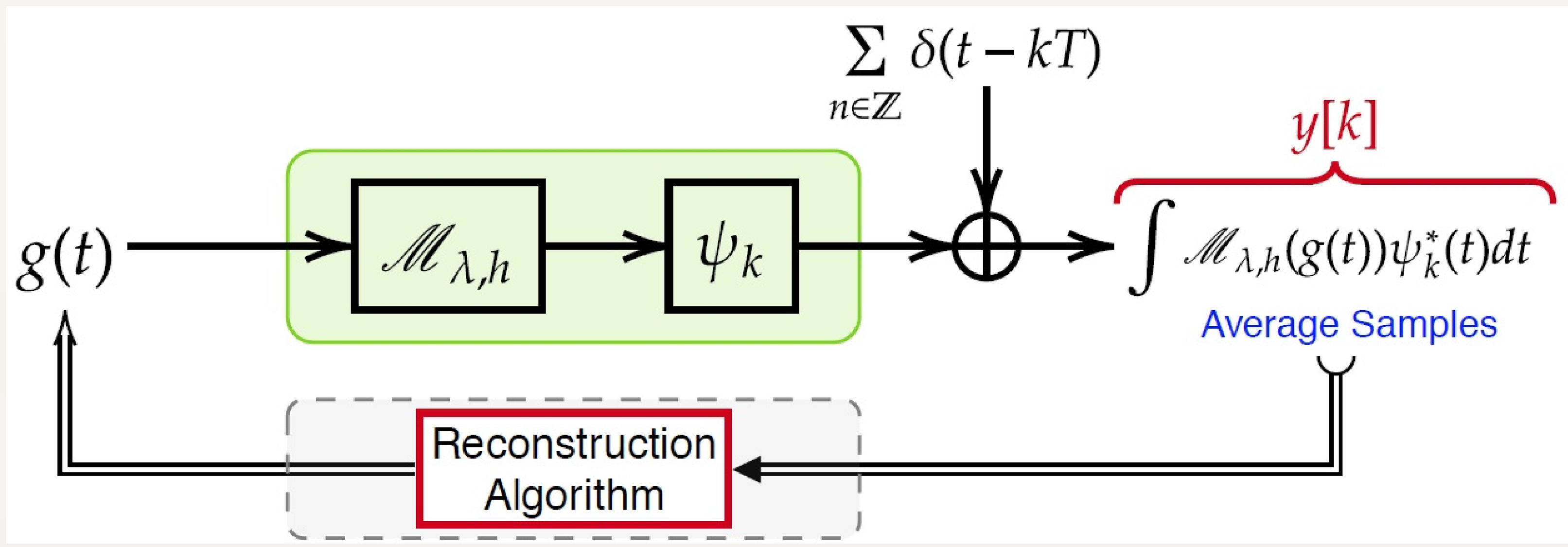
- Why not pointwise sampling?
- For high frequency inputs the assumption does not hold

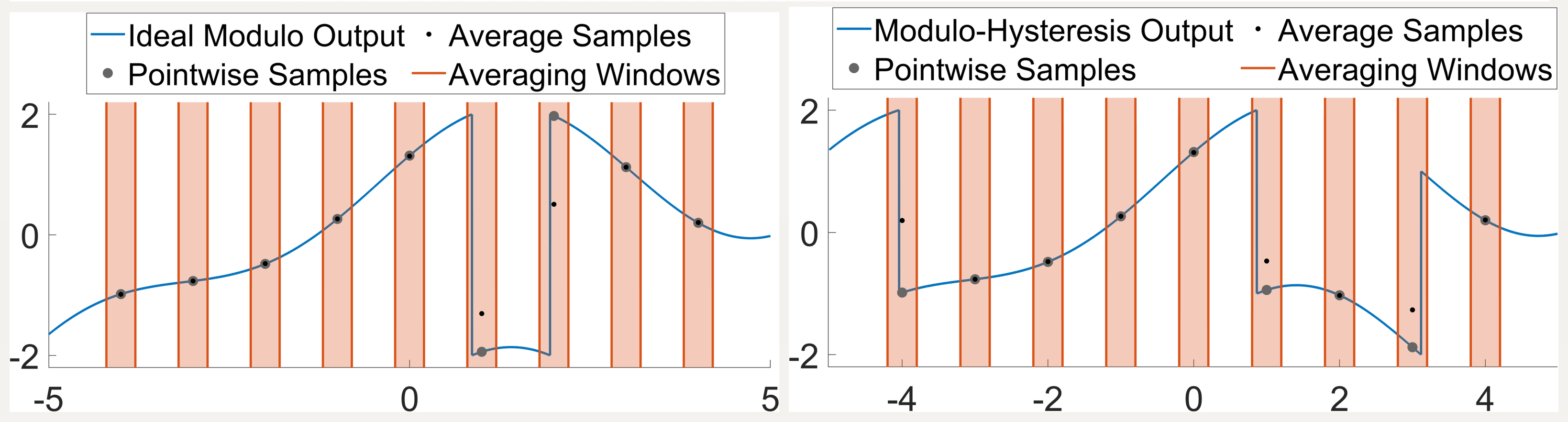


- Why are Ideal Modulo and USAlg not compatible with average sampling?
- Solution: a new model called *modulo-hysteresis* and *thresholding-based recovery [3]*



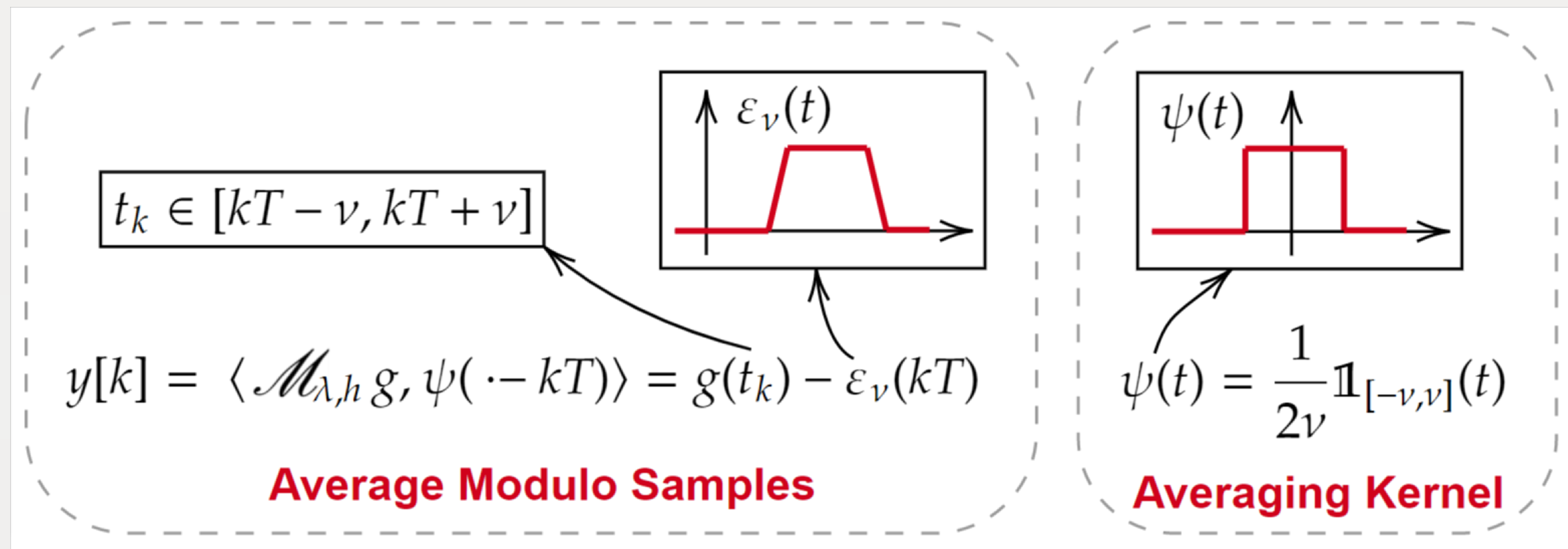
Modulo-Hysteresis Average Sampling





- Unlike ideal modulo where folds may be very close, **modulo-hysteresis** satisfies the following separation property, which enables recovery via thresholding [3]

$$\tau_{r+1} - \tau_r \geq \frac{\min\{h, 2\lambda_h\}}{\Omega g_\infty}$$



- $\mathbb{M}_N \triangleq \left\{k \in \mathbb{Z} \mid \left|\Delta^N y[k]\right| \geq \frac{\lambda_h}{2N}\right\}$ is the set of samples displaced by modulo

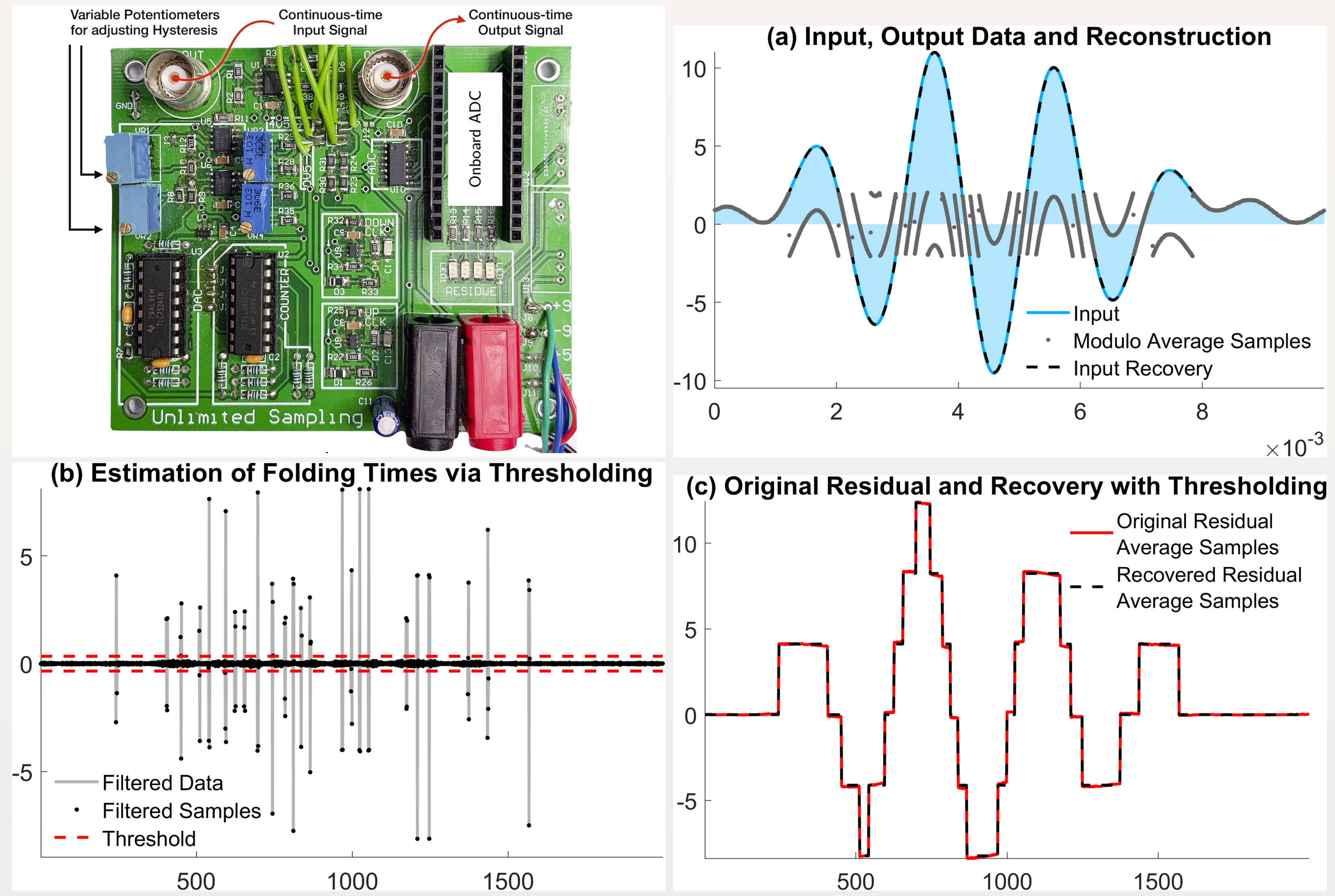
Theorem 1 (**Estimation of Folding Times**). *Let $k_m = \min \mathbb{M}_N$. If $\|\Delta^N g(t_k)\|_\infty < \frac{\lambda_h}{2N}$ then*

$$\widetilde{\tau}_1 = \widetilde{n}_1 T - 2\nu\widetilde{\beta}_1 + \nu \text{ and } \widetilde{s}_1 = -\mathrm{sign}\left(\Delta^N y[k_m]\right) \quad (1)$$

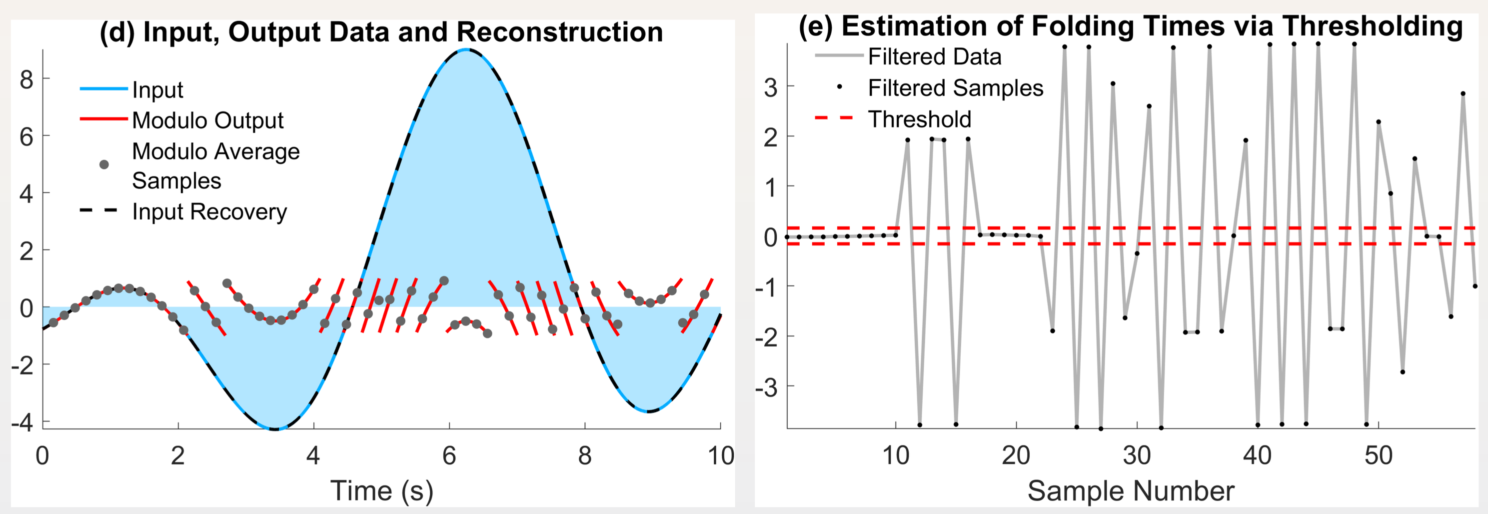
be the estimates of the folding time with $\widetilde{n}_1 = k_m + N$ and $\widetilde{\beta}_1$ where

$$\begin{cases} \widetilde{\beta}_1 = \frac{\Delta^N y[k_m]}{2\lambda_h \widetilde{s}_1} & \text{if } \left|\Delta^N y[k_m]\right| \leq 2\lambda_h - \frac{\lambda_h}{2N} \\ \widetilde{\beta}_1 = 1 & \text{if } \left|\Delta^N y[k_m]\right| > 2\lambda_h - \frac{\lambda_h}{2N}. \end{cases}$$

Hardware Experiment



Sampling at Low Rates



References

[1] A. Bhandari, F. Krahmer, and R. Raskar, " On unlimited sampling," in *Proc. Int. Conf. Sampling Theory Appl.*, IEEE, 2017.

[2] A. Bhandari, F. Krahmer and R. Raskar, "On Unlimited Sampling and Reconstruction," in *IEEE Transactions on Signal Processing*, doi: 10.1109/TSP.2020.3041955.

[3] D. Florescu, F. Krahmer and A. Bhandari, "The Surprising Benefits of Hysteresis in Unlimited Sampling: Theory, Algorithms and Experiments," in *IEEE Transactions on Signal Processing*.

[4] W. Sun and X. Zhou, "Reconstruction of band-limited signals from local averages," in *IEEE Transactions on Information Theory*.